Mixed Strategies

November 18, 2013

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heads</td>
<td></td>
</tr>
<tr>
<td>1, -1</td>
<td>-1, 1</td>
</tr>
<tr>
<td>Tails</td>
<td></td>
</tr>
<tr>
<td>-1, 1</td>
<td>1, -1</td>
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</tbody>
</table>
Let us assume that $S_i$ is the set of strategies available to player $i$, and the combination of strategies $(s_1^*, ..., s_n^*)$ is the Nash equilibrium if, for each player $i$, $s_i^*$ is player $i$’s best response to the strategies of $n-1$ other players:

$$u_i(s_1^*, .., s_i^*, .., s_n^*) \geq u_i(s_1^*, .., s_i, .., s_n^*)$$ (1)
Using the above definition (1), there is no NE in the following game (Known as *Matching Pennies*):

- Assume that Canada generates a negative externality on U.S. when it pollutes.
- US emissions do not affect Canada.
- If US decides to abate and Canada does not, Canada international image is damaged.
- If Canada abates and US does not, Canada has to incur in an abatement cost and US gets the benefits of the environmental quality.
- When both countries abate Canada is favored by the international opinion and US has to incur in the abatement cost.
## Matching Pennies

<table>
<thead>
<tr>
<th>Canada</th>
<th>Pollute</th>
<th>Abate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollute</td>
<td>1, -1</td>
<td>-1, 1</td>
</tr>
<tr>
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In any game in which each player would like to outguess the other, there is no NE, because the solution to such game involves uncertainty.

Mixed Strategies will be interpreted in terms of one player’s uncertainty about what another player will do. (It is a probability distribution over the strategies in $S_i$).

Let us analyze our previous game.

<table>
<thead>
<tr>
<th></th>
<th>Pollute (q)</th>
<th>Abate (1-q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollute (p)</td>
<td>1, 1</td>
<td>-1, 1</td>
</tr>
<tr>
<td>Abate (1-p)</td>
<td>-1, 1</td>
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Suppose that Canada believes that US will chose to pollute with probability $q$ and abate with probability $1 - q$. ($0 \leq q \leq 1$)

Solution........
Canada’s (player 1) best response is to pollute if $q > \frac{1}{2}$ and abate if $q < \frac{1}{2}$.

Canada is indifferent between polluting and abating if $q = \frac{1}{2}$.

Now let us analyze the possible mixed-strategy response by Canada.

Let $(p, 1-p)$ denote the mixed strategy in which Canada plays pollute with probability $p$.

And U.S.?
Solution

- Canada’s BRS is pollute with probability 1 if $q > \frac{1}{2}$
- Canada’s BRS is abate with probability 1 if $q < \frac{1}{2}$
- If $q = \frac{1}{2}$ Canada randomizes between pollute and abate with $p = \frac{1}{2}$

Graphically ($p^*(q)$ is the best response correspondence):

```
\begin{tikzpicture}

% Axes
\draw[->] (-0.5,0) -- (1.5,0) node[anchor=north west] {q};
\draw[->] (0,-0.5) -- (0,1.5) node[anchor=south east] {p};

% Grid
\draw[thin, dashed] (0,0) grid (1,1);

% Points
\node[below left] at (0,0) {Abate};
\node[above right] at (1,0) {Pollute};

% Best Response Correspondence
\draw[dashed] (0,0) -- (0,1) node[midway, below] {$p^*(q)$};
\draw[dashed] (0,1) -- (1,1);
\draw[dashed] (1,1) -- (1,0);
\draw[dashed] (1,0) -- (0,0);

\end{tikzpicture}
```