Game Theory and the Environment

Henry

Not Guilty Guilty

Dave

Not Guilty

2 Years 5 Years 1 Yr.

Guilty

5 Years 1 Yr. 3 Years
Game theory attempts to mathematically capture behavior in strategic situations.

- Normal Form Game: Each Player simultaneously choose a strategy, for instance,
  - Eliminate externalities: Noise, Garbage, emissions, etc.
Classical Example: *Prisoners’ Dilemma* "Two suspects are arrested and charged with a crime"

<table>
<thead>
<tr>
<th></th>
<th>Not Confess</th>
<th>Confess</th>
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<tbody>
<tr>
<td><strong>Prisoner 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not Confess</td>
<td>$-1, -1$</td>
<td>$-6, 0$</td>
</tr>
<tr>
<td>Confess</td>
<td>$0, -6$</td>
<td>$-3, -3$</td>
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Normal Form Game

The Normal Form representation of a game specifies:

1. The players in the game
2. The strategies available to each player
3. The payoff received by each player for each combination of strategies

“Chicken Game”: Assumptions:

1. Two Countries: Country X and Country Y
2. Countries must choose whether or not to abate pollution.
3. Abatement cost $7
4. Benefits $5 (To both countries)
5. Doing nothing exposes both countries to serious pollution damage. Cost $4
### Chicken Game

<table>
<thead>
<tr>
<th></th>
<th>Pollute</th>
<th>Abate</th>
</tr>
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<tbody>
<tr>
<td><strong>X</strong></td>
<td>(-4, -4)</td>
<td>(5, -2)</td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td>(-2, 5)</td>
<td>(3, 3)</td>
</tr>
</tbody>
</table>

**Country X’s Strategies**

- Pollute
- Abate

**Country Y’s Strategies**

- Pollute
- Abate

**Country X’s Pay-off**

- (-4, -4)
- (5, -2)

**Country Y’s Pay-off**

- (-2, 5)
- (3, 3)
How do we obtain the **Nash equilibrium solution**?

We have to keep fix the strategy of the other player (or country) and to analyze what is the best response for the player who does not have its strategy fixed. For instance:

Let us assume that country $Y$ always **pollutes**, so what is country $X$’s best response in this specific case?

$$U_X(\text{Abate} \backslash \text{Pollute}) > U_X(\text{Pollute} \backslash \text{Pollute})$$

since $-2 > -4$

<table>
<thead>
<tr>
<th></th>
<th>$Y$</th>
<th>Pollute</th>
<th>Abate</th>
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</thead>
<tbody>
<tr>
<td>Pollute</td>
<td>(-4, 4)</td>
<td>(5, -2)</td>
<td></td>
</tr>
<tr>
<td>Abate</td>
<td>(-2, 5)</td>
<td>(3, 3)</td>
<td></td>
</tr>
</tbody>
</table>
Now, let us assume that country $Y$ always abates, so what is country $X$’s best response in this specific case?

$$U_X(\text{Abate}\backslash\text{Abate}) < U_X(\text{Pollute}\backslash\text{Abate}),$$

since $3 < 5$
The second step: Analyze what happens when country X always abates.

So the question will be: what is country Y’s best response in this specific?

\[ U_Y(\text{Abate} \backslash \text{Abate}) < U_Y(\text{Pollute} \backslash \text{Abate}), \]

since \( 3 < 5 \)
Finally, let us assume that country $X$ always pollutes, so what is country $Y$’s best response in this specific?

$$U_Y(\text{Abate} | \text{Pollute}) < U_Y(\text{Pollute} | \text{Pollute}),$$

since $-2 > -4$

<table>
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<td>Pollute</td>
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</tbody>
</table>

Therefore, we have can say that the Nash equilibrium of the Chicken game is $\{(5, -2) \text{ and } (-2, 5)\}$ or $\{(\text{abate, pollute)} \text{ and } (\text{pollute, abate})\}$. 
Sequential game:

- Country X chooses first (First Mover of the game)
- Country Y then observes country X’s choice and decides on its own action.

The **extensive form** of the game is:

```
Country X
  /  
 Abate Pollute
    /       
Country Y Pollute Abate
  /   
(5, -2) (3, 3)
```

```
Country Y
  /   
Pollute Abate
    /   
(5, -2) (3, 3)
```

```
Country X
  /   
Abate Pollute
    /   
(1, -4) (0, 0)
```

```
Country Y
  /   
Pollute Abate
    /   
(1, -4) (0, 0)
```

```
Country X
  /   
Abate Pollute
    /   
(5, -2) (3, 3)
```

```
Country Y
  /   
Pollute Abate
    /   
(5, -2) (3, 3)
```

```
Country X
  /   
Abate Pollute
    /   
(1, -4) (0, 0)
```

```
Country Y
  /   
Pollute Abate
    /   
(1, -4) (0, 0)
```
The solution of this game can be found by the method of backward induction.

We will start analyzing the bottom of the tree, which means, country Y’s best response.

If country Y observes that country X has chosen to pollute, then country Y’s best response is to abate (since $-4 < -2$).

If country Y observes that country X has chosen to abate, then country Y’s best response is to pollute (since $3 < 5$).
Now, let us analyze country $X$’s best response (assuming that country $X$ knows country $Y$’s best response).

- Country $X$ knows that when it chooses to pollute country $Y$ will abate.
- and when it chooses to abate country $Y$ will decide to pollute.
- then country $X$ best response is to abate (since $5 > -2$).

Therefore the Nash equilibrium of this game is $(5, -2)$ where country $X$ pollutes and country $Y$ abates.

First Mover Advantage