Commercial plantation forestry

(Chapter 18) Perman et al. "Natural Resource and Environmental Economics"
Introduction

- What harvest programme is required in order that the present value of the profits from the stand of timber is maximized?
- The optimal rotation time depends on what model is being used.
- Let us analyze more formally the single-rotation commercial forest model
Suppose there is a stand of timber of uniform type and age. Planted at the same time. The forest will not be replanted (one cycle of rotation). The land has no alternative uses (Opportunity cost?).

\( k \): planting cost
\( c \): marginal harvesting cost
\( P \): price of the timber

no external effects and value only through the timber
What is the optimal time at which to cut the trees? Choosing the age at which the PV of profits is maximized

\[(P - c) S_T e^{-iT} - k = pS_T e^{-iT} - k\]

- $S_T$ is the volume of timber available for harvest at time $T$
- $p$ (in lower case) is the net price of the harvested timber
- $i$ is the private consumption discount rate (the opportunity cost of capital to the forestry firm)
The PV of profits is max. when *the rate of growth of the NV of the resource stock* is equal to the *private discount rate*. 

\[
\begin{align*}
\frac{d}{dT} \left(pS_T e^{-iT} - k\right) &= p e^{-iT} \frac{dS_T}{dT} + pS_T \frac{de^{-iT}}{dT} = 0 \\
p e^{-iT} \frac{dS_T}{dT} - ipS_T e^{-iT} &= 0 \\
\text{or } p \frac{dS_T}{dT} - ipS_T &= 0 \\
p \frac{dS_T}{dT} &= ipS_T \\
i &= \frac{p \frac{dS_T}{dT}}{pS_T}
\end{align*}
\]
Assume: \( P = 10, c = 2, p = 8, k = 5,000 \) and \( i = 0 \) or \( i = 3\% \).

Growth function \( S_T = at + bt^2 + ct^3 \) where \( a = 40, b = 3.1 \) and \( c = -0.016 \).
The optimal time for cutting will depend upon the discount rate used!

A rise in the discount rate from zero to 3% not only dramatically lowers the profitability of the forest but also significantly changes the shape of the present-value profile, reducing the age at which the forest should be harvested (135-50 years).
Infinite- Rotation Forestry Models

- A rational owner would consider further planting cycles if the land had no other uses.
- Indefinite quantity of rotations.
- What is the optimal length of each rotation?
- First: A delay in harvesting has an opportunity cost in the form of interest forgone on the (delayed) revenues from harvesting.
- Second: Delay in establishing the next and all subsequent planting cycles.
- Optimal harvesting and replanting program:

  Benef. of deferring H = the cost of deferring P
Infinite- Rotation Forestry Models

Assumptions:

- $k$: planting cost
- $c$: marginal harvesting cost
- $P$: price of the timber (net price: $p = P - c$)
- the first rotation begins with the planting at $t_0$
- $t_1, t_2, t_3, \ldots$ is the infinite sequence of points in time that are ends of the rotations
Infinite- Rotation Forestry Models

**Net present value of profits**

\[ NPV = pS(t_1-t_0)e^{-i(t_1-t_0)} - k \]

- The volume of the timber growth between the start and the end of the cycle \( S(t_1-t_0) \) \( \times \) by the discounted net price of a unit of timber \( pe^{-i(t_1-t_0)} \) less the forest planting cost \( k \)
- The net present value of profits over this infinite sequence is

\[
\Pi = \left[ pS(t_1-t_0)e^{-i(t_1-t_0)} - k \right] + \\
\quad e^{-i(t_1-t_0)} \left[ pS(t_2-t_1)e^{-i(t_2-t_1)} - k \right] + \\
\quad e^{-i(t_2-t_0)} \left[ pS(t_3-t_2)e^{-i(t_3-t_2)} - k \right] + \\
\quad e^{-i(t_3-t_0)} \left[ pS(t_4-t_3)e^{-i(t_4-t_3)} - k \right] + \ldots
\]

(Right hand side) Sum of the present values of the profits from each of the individual rotations
Infinite- Rotation Forestry Models

- Optimal rotation $T$

\[
\Pi = \left[ pS_T e^{-iT} - k \right] + e^{-iT} \left[ pS_T e^{-iT} - k \right] + e^{-2iT} \left[ pS_T e^{-iT} - k \right] + e^{-3iT} \left[ pS_T e^{-iT} - k \right] + \ldots \]

- Factorizing (use $e^{-iT}$)

\[
\Pi = \left[ pS_T e^{-iT} - k \right] + e^{-iT} \left\{ \left[ pS_T e^{-iT} - k \right] + e^{-iT} \left[ pS_T e^{-iT} - k \right] + e^{-2iT} \left[ pS_T e^{-iT} - k \right] + \ldots \right\} \]
Note that this identical to $\Pi$ in equation (1). Therefore we can rewrite it as

$$\Pi = \left[ pS_T e^{-iT} - k \right] + e^{-iT} \Pi$$

Solving for $\Pi$

$$\Pi = \frac{\left[ pS_T e^{-iT} - k \right]}{\left[ 1 - e^{-iT} \right]}$$

It gives the present value of profits for any rotation length, $T$, given $p$, $k$, $i$ and $S$
### Infinite- Rotation Forestry Models

<table>
<thead>
<tr>
<th>i</th>
<th>Optimal T (infinite)</th>
<th>Optimal T (single)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>99</td>
<td>135</td>
</tr>
<tr>
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<td>71</td>
<td>98</td>
</tr>
<tr>
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<td>38</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>26*</td>
<td>26*</td>
</tr>
<tr>
<td>7</td>
<td>24*</td>
<td>22*</td>
</tr>
<tr>
<td>8</td>
<td>22*</td>
<td>19*</td>
</tr>
</tbody>
</table>

6% or higher result in negative present values at any rotation, and the * are those which minimize present value losses.
Comparisons

- Single rotation
  \[ i = \frac{p \frac{dS_T}{dT}}{pS_T} \]

- Infinite rotations
  \[ i + \frac{i \Pi}{pS_T} = \frac{p \frac{dS_T}{dT}}{pS_T} \]

- \( i + \frac{i \Pi}{pS_T} \): rate of interest that could be earned on the capital tied up in the growing timber plus the interest that could be earned on the capital tied up in the site value of the land.