**Exercise - Discriminating Monopoly**

Suppose that a monopoly can price discriminate between two markets: *market 1*, where the demand curve is given by \( q_1 = 2 - p_1 \), and *market 2* where the demand curve is given by \( q_2 = 4 - p_2 \). Suppose that once the product is sold, it cannot be resold in the other market. That is, assume that arbitrage is impossible, say, due to strict custom inspections on the border between the two markets. Assume that the monopoly produces each unit at a cost of \( c=1 \).

a) Calculate the profit-maximizing output level that the monopoly sells in each market. Calculate the price charged in each market.

b) Calculate the monopoly’s profit level.

c) Suppose that market 1 and 2 are now open, and all consumers are free to trade and to transfer the good costlessly between the markets. Thus, the monopoly can no longer price discriminate and has to charge a uniform price denoted by \( p \), \( p=p_1=p_2 \). Find the profit-maximizing value \( p \).

**Solution**

**Part (a)**

If the monopoly sells in each market:

\[
\text{Max } \pi_1(q_1) = q_1 \times p_1 - 1 \times q_1
\]

Hence, given that \( q_1 = 2 - p_1 \) or equivalently \( p_1 = 2 - q_1 \), hence:

\[
\text{Max } \pi_1(q_1) = q_1 \times (2 - q_1) - 1 \times q_1
\]

The first order condition with respect to \( q_1 \) is,

\[
\frac{\partial \pi_1(q_1)}{\partial q_1} = 2 - 2q_1 - 1 = 0
\]

Therefore, \( q_1 = 1/2 = 0.5 \). If the seller sells in market 2:

\[
\text{Max } \pi_1(q_2) = q_2 \times p_2 - 1 \times q_2
\]

Hence, given that \( q_2 = 4 - p_2 \) or equivalently \( p_2 = 4 - q_2 \), hence:

\[
\text{Max } \pi_1(q_2) = q_2 \times (4 - q_2) - 1 \times q_2
\]

The first order condition with respect to \( q_2 \) is,

\[
\frac{\partial \pi_1(q_2)}{\partial q_2} = 4 - 2q_2 - 1 = 0
\]

Therefore, \( q_2 = 3/2 = 1.5 \). Finally the equilibrium prices are:

\( p_1 = 2 - 1/2 = 3/2 = 1.5 \) and \( p_2 = 4 - 3/2 = 5/2 = 2.5 \).
Part (b)

\[ \pi_1(q_1) = \frac{1}{2} \times \left( 2 - \frac{1}{2} \right) - 1 \times \frac{1}{2} = \frac{1}{2} \times \frac{3}{2} - \frac{1}{2} = \frac{1}{4} = 0.25 \]

\[ \pi_1(q_2) = \frac{3}{2} \times \left( 4 - \frac{3}{2} \right) - 1 \times \frac{3}{2} = \frac{3}{2} \times \frac{5}{2} - \frac{3}{2} = \frac{9}{4} = 2.25 \]

Summing up, the monopoly’s profit under price discrimination is \( \pi = 2.5 \).

Part (c)

If the monopoly cannot discriminate:

\[ \text{Max } \pi_1(Q) = Q \times p - 1 \times Q \]

Where \( Q = q_1 + q_2 \) and and \( Q = 2 \times p_1 + 4 \times p_2 \). Note that \( p_1 = p_2 \), hence, \( Q = 6 - 2p \).

\[ \text{Max } \pi_1(Q) = (6 - 2p) \times p - 1 \times (6 - 2p) \]

The first order condition with respect to \( p \) is,

\[ \frac{\partial \pi_1(Q)}{\partial p} = 6 - 4p + 2 = 0 \]

Hence, \( p = 2 \) and \( Q = 2 \). If the price is equal to 2, the firm will have incentives to only produce in market 2 (Why?)

Finally, firm’s profits when it cannot discriminate are: \( \pi_1(Q) = (6 - 2 \times 2) \times 2 - 1 \times (6 - 2 \times 2) = 4 - 2 = 2 \)

Which is lower than part (b), that is, 2 < 2.5.