Optimal Cooperative and Non-Cooperative equilibrium levels of Abatement
The world consists of two countries, X which is poor and Y which is rich. The total benefits (B) and total cost (C) of emissions abatement (A) are given by the functions:

\[
B_X = 8(A_X + A_Y) \quad B_Y = 7(A_X + A_Y)
\]

\[
C_X = 10 + 2A_X + 0.5A_X^2 \quad C_Y = 10 + 2A_Y + 0.5A_Y^2
\]

Obtain the cooperative equilibrium levels of abatement for X and Y
Both Countries cooperate

Let us denote Country $X$’s Net Benefit as $\Pi_X$ and Country $Y$’s Net Benefit as $\Pi_Y$. Therefore:

$\Pi_X = B_X - C_X = 8(A_X + A_Y) - (10 + 2A_X + 0.5A_X^2) = 8A_X + 8A_Y - 10 - 2A_X - 0.5A_X^2$

Rewriting equation (1) we have that Country $X$’s net benefits can be expressed as follows:

$\Pi_X = -0.5A_X^2 + 6A_X + 8A_Y - 10$
First Step

Both Countries cooperate

And Country Y’s Net benefit is:

\[(2) \quad \Pi_Y = B_Y - C_Y = 7(A_X + A_Y) - (10 + 2A_Y + 0.5A_Y^2) = 7A_X + 7A_Y - 10 - 2A_Y - 0.5A_Y^2 \]

Rewriting equation (2) we have that Country Y’s net benefits can be expressed as follows:

\[(2') \quad \Pi_Y = -0.5A_Y^2 + 5A_Y + 7A_X - 10 \]
## Second Step

<table>
<thead>
<tr>
<th></th>
<th>Pollute</th>
<th>Abate</th>
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<tbody>
<tr>
<td><strong>X</strong></td>
<td><strong>Y</strong></td>
<td></td>
</tr>
<tr>
<td>Pollute</td>
<td>(1) (0,0)</td>
<td>(2) ($8A_Y$; $7A_Y - 10 - 2A_Y - 0.5A_Y^2$)</td>
</tr>
<tr>
<td>Abate</td>
<td>($8A_X - 10 - 2A_X - 0.5A_X^2$; $7A_X$)</td>
<td>(3) ($-0.5A_X^2 + 6A_X + 8A_Y - 10$; (4) $-0.5A_Y^2 + 5A_Y + 7A_X - 10$)</td>
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Third Step

**Cooperative Case**

\[ U_X(ABATE \setminus ABATE) > U_X(POLLUTE \setminus ABATE) \]

\[-0.5A_X^2 + 6A_X + 8A_Y - 10 > 8A_Y \]

\[-0.5A_X^2 + 6A_X - 10 > 0 \]

Quadratic equation: \(-ax^2 + bx - c > 0\)

\[ U_Y(ABATE \setminus ABATE) > U_Y(POLLUTE \setminus ABATE) \]

\[-0.5A_Y^2 + 5A_Y + 7A_X - 10 > 7A_X \]

\[-0.5A_Y^2 + 7A_Y - 10 > 0 \]

\[ X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Equilibrium

Emission abatement levels that will make both countries to cooperate
Non-Cooperative

\[ U_x(\text{pollute} \mid \text{pollute}) > U_x(\text{abate} \mid \text{pollute}) \]

\[ 0 > -0.5A_x^2 + 6A_x - 10 \]

\[ U_y(\text{pollute} \mid \text{pollute}) > U_y(\text{abate} \mid \text{pollute}) \]

\[ 0 > -0.5A_y^2 + 5A_y - 10 \]
# Countries Pollution Costs

\[ C_x^p = -0.5A_x^2 - 8A_y + 6A_x \]
\[ C_y^p = 0.10A_y^2 + 5A_y - 10 \]

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
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<tr>
<td>X</td>
<td>Y</td>
<td>((-0.5A_x^2 - 8A_y + 6A_x; 0.10A_x^2 + 5A_y - 10))</td>
<td>((8A_y; -0.5A_y^2 + 5A_y - 10))</td>
</tr>
<tr>
<td>Y</td>
<td>X</td>
<td>((-0.5A_x^2 + 6A_x - 10; 7A_x))</td>
<td>((-0.5A_x^2 + 6A_x + 8A_y - 10; -0.5A_y^2 + 5A_y + 7A_x - 10))</td>
</tr>
</tbody>
</table>
Non-Cooperative

\[ U_x(\text{pollute}) > U_x(\text{abate}) \]

\[ -0.5A_x^2 - 8A + 6A_x > -0.5A_x^2 + 6A_x - 10 \]

\[ U_Y(\text{pollute}) > U_Y(\text{abate}) \]

\[ 0.10A_Y^2 + 5A_Y - 10 > -0.5A_Y^2 + 5A_Y - 10 \]