Chapter 6.

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Cournot Market Structure

- Oligopolistic Market structures
- Consumers cannot differentiate among brands
- Firms are not price-takers
- Two-seller game
- \( TC_i(q_i) = c_i q_i \), where \( i = 1, 2 \) and \( c_2, c_1 \geq 0 \)
- Demand: \( P(Q) = a - bQ \), \( a, b > c_i \) where \( Q = q1 + q2 \)
Let each firm’s action be defined as choosing its production level.

Simultaneous game ($q_i \in A_i \equiv [0, \infty)$ where $i = 1, 2$)

\[ \pi_i(q_1, q_2) = p(q_1, q_2)q_i - TC_i(q_i) \]

Definition: The triplet \(\{p^C, q_1^C, q_2^C\}\) is a Cournot-Nash equilibrium if

- given $q_2 = q_2^C$; $q_1^C$ solves $\max_{q_1} \pi_1(q_1, q_2^C)$
- given $q_1 = q_1^C$; $q_2^C$ solves $\max_{q_2} \pi_2(q_1^C, q_2)$
- $p^C = a - b(q_1^C + q_2^C)$, $p^C, q_1^C, q_2^C \geq 0$
Cournot Market Structure

- Best response functions:
  - \( q_1 = R_1(q_2) = \frac{a-c_1}{2b} - \frac{1}{2} q_2 \)
  - \( q_2 = R_2(q_1) = \frac{a-c_2}{2b} - \frac{1}{2} q_1 \)
Cournot Market Structure

- \( q_1^C = \frac{a-2c_1+c_2}{3b} \), \( q_2^C = \frac{a-2c_2+c_1}{3b} \)
- \( Q^C = \frac{2a-c_1-c_2}{3b} \) and \( p^C = a - bQ^C = \frac{a+c_1+c_2}{3} \)
- Note that if \( c_2 \geq c_1 \) then \( q_1 \geq q_2 \)
- \( \pi_i^C = b(q_i^C)^2 \)
- *Process innovation* firm 1. \( \bar{c}_1 < c_1 \), effects on \( q_1^C \)?
N-Seller game

- **N** firms
  - **N** identical firms
  - Heterogeneous firms
- Assume \( c_i = c \) for every \( i = 1, 2, \ldots, N \)
- Calculate the BRF of a representative firm

\[
\max_{q_1} \pi_1 = p(Q) - cq_1
\]

\[
= \left[ a - b \sum_{i=1}^{N} q_i \right] q_1 - cq_1
\]

- \( R_1(q_2, q_3, \ldots, q_N) = \frac{a-c}{2b} - \frac{1}{2} \sum_{i=2}^{N} q_i \)
- \( q^c = \frac{a-c}{(N+1)b}, Q^c = \frac{a-c}{b} \frac{N}{(N+1)}, p^c = \frac{a+Nc}{N+1} \) and \( \pi_i^c = b(q^C)^2 \)
- What will happen if \( N \to \infty \)?
Sequential Moves

- Firms move in sequence (Leader-Follower, Stackelberg)
- Assume the order of moves is given
- Two-stage game:
  - First stage: firm 1 chooses quantity (irreversible)
  - Second stage: after observing $q_1$, firm 2 chooses $q_2$
- Example: Toyota was the first automaker to offer hybrid cars in 1997, then Honda introduced the Insight in 1999.
- First Mover Advantage?
- Comparison between Cournot and Stackelberg
- Assume firms have identical cost, $c_1 = c_2 = c$
The second and first-period games

- Only firm 2 moves: \( q_2 \) \( (BRF_2 : R_2(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1) \)
- Firm 1 chooses \( q_1^s: \max_{q_1} \pi_1^s = p(q_1 + R_2(q_1))q_1 - cq_1 = \left[ a - b(q_1 + \frac{a-c}{2b} - \frac{1}{2}q_1) \right] q_1 - cq_1 \)
  \[ q_1^s = \frac{a-c}{2b} \]
  \[ q_2^s = \frac{a-c}{4b} \]

- Hence the leader produces a higher output level than the Cournot market structure
- the follower’s output level falls compared with the Cournot output level.

\[ Q^s = \frac{3(a-c)}{4b} > Q^C \text{ and } p^s = \frac{a+3c}{4} < p^C \]

\[ \pi_1^s = \frac{(a-c)^2}{8b} > \pi_1^C \text{ and } \pi_2^s = \frac{(a-c)^2}{16b} < \pi_2^C \]
Bertrand Market Structure

- Firms set prices rather than quantities
- Why Bertrand is more attractive than Cournot market structure?
- Two main assumptions:
  - Consumers always purchase from the cheapest seller
  - If two seller charge the same price, half of the consumers purchase from firm 1 and the other half from firm 2.
- Assume

\[
q_i = \begin{cases} 
0 & \text{if } p_i > a \\
0 & \text{if } p_i > p_j \\
\frac{(a-p)}{2b} & \text{if } p_i = p_j \equiv p < a \\
\frac{(a-p)}{b} & \text{if } p_i < \min\{a, p_j\}
\end{cases}
\]
Proposition: If the medium of exchange is continuous and if the firms have the same cost structure \((c_2 = c_1 = c)\), then a Bertrand equilibrium is \(p^b_1 = p^b_2 = c\), and \(q^b_1 = q^b_2 = \frac{(a-c)}{2b}\).

Proof:

In equilibrium \(\pi_i \geq 0\). Hence \(p^b_i \geq c\), \(i = 1, 2\).

By contradiction assume \(p^b_1 > p^b_2 > c\), then \(\pi_1 = 0\). Hence \(p^b_2 > \tilde{p}_1 > c\) and \(q_i = \frac{(a-\tilde{p}_1)}{b}\).

By contradiction assume \(p^b_1 > p^b_2 = c\), then \(\pi_1 = 0\), but firm 2 has incentives to deviate!

Assume \(p^b_1 = p^b_2 > c\). This cannot constitute a Nash equilibrium. Why?
Bertrand under capacity constraints

- In the short run firms are constrained by capacity.
- "Reservation Prices"
- assume \( c_1 = c_2 = 0 \)

Bertrand equilibrium prices under no capacity constraints need not be NE prices under capacity constrained