Hyperbolic Discounting

Tendency for people to choose a smaller-sooner reward over a larger-later reward, especially as the delay occurs sooner rather than later in time. When offered a larger reward in exchange for waiting a set amount of time, people act less impulsively if the reward is further in the future.

People avoid waiting more as the wait nears the present time. Explains a large range of phenomena, including lack of willpower, health outcomes, consumption and financial decisions.

At 10% interest, people should like \$100 now or \$110 in a year. Under exponential discounting, the discount depends only on the length of the wait and rate of discount is constant across different wait times. But there is much evidence that people don’t behave this way.

> eg. Many people prefer \$100 today to \$110 tomorrow, but few prefer \$100 in 30 days to \$110 in 31 days

Time inconsistency in choice

Example

Food experiment (Read and van Leeuwen, 1998)

Choice next week: Fruit (74%), Chocolate (26%)

Choice today: Fruit (30%), Chocolate (70%)

Example: People pledge to exercise, then don't follow through Solution: Precommitment at Gyms (Della, Vigna and Malmendier, 2004)

AC of membership: $75 month

Average number vists: 4 month

Average cost per visit: $19

Cost of "pay per visit": $10

Even though "pay per visit" is cheaper, people buy monthly memberships to "precommit" and make it more likely they will exercise

Hyperbolic Discounting vs Exponential Discounting

Exponential discounting: $\beta = \frac{1}{1+k}$ where k is the constant interest rate. A return or cost t years in the future is discounted by β^t. So at any time in the future, t*, a return r years ahead would be discounted β^r. It doesn't depend on how far into the future t* occurs.

Hyperbolic discounting reduces future values by $h^t = \frac{1}{(1+kt)^{\delta/\alpha}}$ where $\alpha, \delta > 0$ and usually assumed equal. k is again the constant interest rate.

This is the general form for a hyperbola. With a discount of this form, the rate of discounting decreases as the delay occurs further in the future so the discount depends on the length of the delay and when the delay occurs.

So, $h^{t1+s} \neq h^{t2+s}$ when $t1 \neq t2$. Hyperbolic discounting generally discounts future values more than exponential discounting for near values, but less for distant values.

Quasi-Hyperbolic Discounting is more tractable and nests exponential discounting

- Quasi-hyperbolic discounting (Phelps and Pollak 1968, Laibson 1997):
  $1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots$

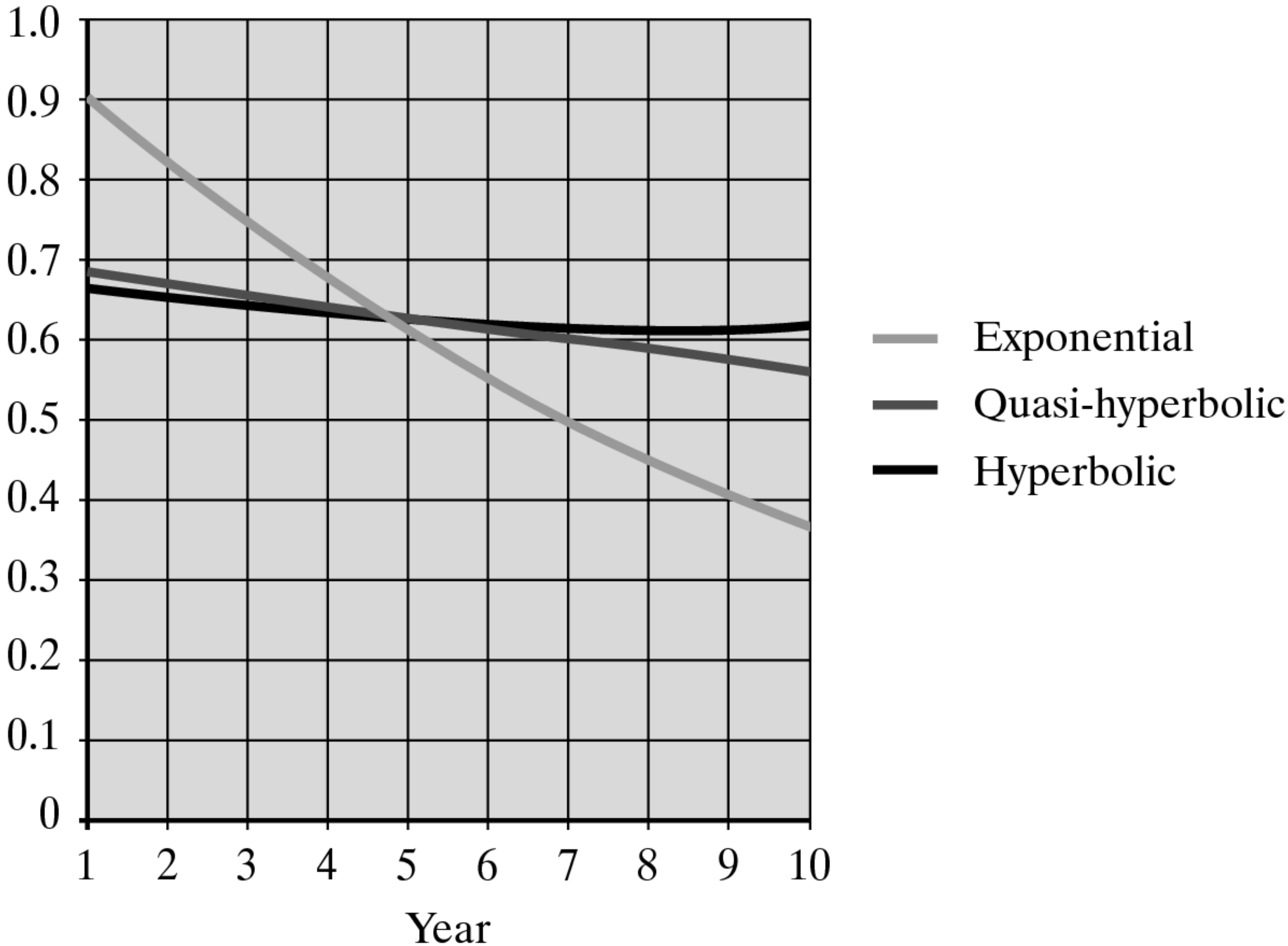
$$U_t = u(c_t) + \beta\delta u(c_{t+1}) + \beta\delta^2 u(c_{t+2}) + \beta\delta^3 u(c_{t+3}) + \ldots$$

- For exponentials: $\beta = 1$

$$U_t = u(c_t) + \delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \delta^3 u(c_{t+3}) + \ldots$$

- For "quasi-hyperbolics": $\beta < 1$

$$U_t = u(c_t) + \beta \left[\delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \delta^3 u(c_{t+3}) + \ldots\right]$$


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10
Year
Exponential
Quasi-hyperbolic
Hyperbolic

Example

- To build intution, assume that $\beta \simeq \frac{1}{2}$ and $\delta \simeq 1$

$$\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ...\} = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, ...\}$$

$$U_t = u(c_t) + \frac{1}{2}\left[u(c_{t+1}) + u(c_{t+2}) + u(c_{t+3}) + ...\right]$$

- Relative to the current period, all future periods are worth less (weight $\frac{1}{2}$).

- Most (for this example, *all*) of the discounting takes place between the current period and the immediate future.

- There is little (for this example, *no*) additional discounting between future periods.

What it means is this

$$U_t = u(c_t) + \frac{1}{2}[u(c_{t+1}) + u(c_{t+2}) + u(c_{t+3}) + ...]$$

- Preferences are dynamically inconsistent.
- At date t we prefer to be patient between $t+1$ and $t+2$.
- At date $t+1$ we want immediate gratification at $t+1$.

$$U_{t+1} = u(c_{t+1}) + \frac{1}{2}[u(c_{t+2}) + u(c_{t+3}) + u(c_{t+4}) + ...]$$

Laibson, "Golden Eggs and Hyperbolic Discounting"

Laibson cites Strotz (1956) who formalized a theory of commitment

- Strotz showed that when discount factors are nonexponential they will constrain their own future choices
- Laibson notes that all illiquid assets are a form of commitment
  - Generate a substantial benefit in the long run
  - Cannot be tapped immediately
  - Examples are housing, consumer durables, CDs, savings bonds
  - Most assets in US are illiquid
  - Equates it to the Goose who laid the golden eggs (1 every period)
    - Very valuable in the future
    - Hard to realize value now
  - Shows a commitment of investment
    - Shows actions to overcome dynamic inconsistency
    - Conflict between preferences now and in the future

- Time inconsistent discounting
  - A high discount between near periods now
  - A low discount between near periods far in the future
  - Think about exponential discounting
    - The discount between any adjacent period is β no matter how far in the future those adjacent periods might be (today and tomorrow or 1000 and 1001 days ahead)
  - Hyperbolic discounting is different
    - The discount between adjacent periods near term is much higher than the discount between adjacent periods far in the future
    - Quasi hyperbolic with $\beta=1/2$ and $\delta=1$, the discount between now and tomorrow is ½.
    - The discount between days 1000 and 1001 is 0.5

How does this apply to investing?

- This year I plan aggressive investing next year, but when next year comes, I defer for the future.
- Overcome it by committing to illiquid assets

The model predicts

1. Consumption tracks income
2. $MPC_{income} \neq MPC_{wealth}$
3. Ricardian equivalence does not hold
4. Financial innovations caused lower savings rates because the innovations increased liquidity
5. Financial market innovations lower overall welfare by inducing too much liquidity

The Model:

Two assets:

1) The liquid asset, x, can be disposed of and the assets spent immediately
2) The illiquid asset, z, must be sold one period before the assets are received. Can also borrow against z, but it also takes 1 period to receive money

Consumer problem

1. Receives a deterministic sequence of labor income, y_t
2. Liquid and illiquid assets, both chosen at t-1, return $R_t=1+r_t$
3. In time t the agent gets $y_t+R_t x_{t-1}$ and spends it on consumption so $c_t < y_t+R_t x_{t-1}$
4. The agent starts with x_0, z_0, and at each t chooses x_t and z_t so that

$$y_t+R_t(z_{t-1}+x_{t-1})-c_t=z_t+x_t \quad \text{with } z_t,x_t \geq 0$$

The choice of x_t and z_t determines asset liquidity next period. In addition, the constraint that assets be nonnegative removes future forced savings, and borrowing. Utility is given by

$$U_t = E_t\left[u\left(c_t\right) + \beta \sum_{\tau=1}^{T-t} \delta^{\tau} u(c_{t+\tau})\right]$$

so we are using a quasi hyperbolic discounting of the future. *<u>Preferences are dynamically inconsistent</u>*. Choices at *t* do not match preferences at *t+1.* The key is this

$$MRS_{t+1\,for t+2|t} = \frac{u'(c_{t+1})}{\delta u'(c_{t+2})} < \frac{u'(c_{t+1})}{\beta\delta u'(c_{t+2})} = MRS_{t+1\,for t+2|t+1}$$

since $0<\beta<1$. So self at t+1 is unlikely to follow plan set at t. The individual plays a dynamic game against future self.

Equilibrium strategies

The deterministic but time varying interest rate and labor income means we can't use the marginal condition to characterize the equilibrium. Existence also requires that the sequence of labor income be constrained inversely to β. This isn't important for the point of the paper. But properties P1-P4 of the equilibrium require

$$\text{P1} \quad u'(c_t) \geq \max_{\tau \in \{1,\ldots,T-t\}} \beta\delta^{\tau}\left(\prod_{i=1}^{\tau} R_{t+i}\right) u'(c_{t+\tau})$$

$$\text{P2} \quad u'(c_t) > \max_{\tau \in \{1,\ldots,T-t\}} \beta\delta^{\tau}\left(\prod_{i=1}^{\tau} R_{t+i}\right) u'(c_{t+\tau}) \Rightarrow c_t = y_t + R_t x_{t-1}$$

$$\text{P3} \quad u'(c_{t+1}) < \max_{\tau \in \{1,\ldots,T-t-1\}} \delta^{\tau}\left(\prod_{i=1}^{\tau} R_{t+i}\right) u'(c_{t+1+\tau}) \Rightarrow x_t = 0$$

$$\text{P4} \quad u'(c_{t+1}) > \max_{\tau \in \{1,\ldots,T-t-1\}} \delta^{\tau}\left(\prod_{i=1}^{\tau} R_{t+i}\right) u'(c_{t+1+\tau}) \Rightarrow z_t = 0.$$

P1 and P2 are restrictions on what is feasible.

P1 says the MU of consumption at time t can be too high, but can't be too low because the agent can always same. It is a standard Euler equation restriction when there are liquidity constraints

P2 says that when MU is too high it must be that liquidity is binding

P3 and P4 reflect strategic decisions to constrain future self

P3 says that self t will restrict self t+1 by limiting liquidity as much as possible (that is makes $x_t=0$) if expected consumption at t+1 is *higher* than self t would choose

P4 implies the opposite of P3. Self t will not limit self t+1 (so sets $z_t=0$) when expected consumption at t+1 is *below* what self t would choose

<u>Notice, in P3 and P4, self t *ignores* the β in the MRS relationship between period t+1 and any period after t+1</u>

Is the model realistic?
Empirical evidence that household consumption tracks income too much to be consistent with the lifecycle/Permanent Income Hypothesis model of consumption

Most importantly, consumption tracks expected income even when consumers have large stocks of wealth. Possible explanations include

- impatient consumers with a precautionary savings motive hold little wealth, and choose optimally to match consumption and income over the lifecycle
- Alternatively, it is a aggregation of demographic dynamics that explain much of the consumption-income comovement

Golden eggs offer an alternative explanation. Self at t-1 chooses x_{t-1} to constrain self's consumption at t by keeping most assets in illiquid form. Most importantly $c_t = y_r + R_t x_{t-1}$ does not imply that consumption tracks wage income because liquid savings, x_{t-1} is endogenous. In equilibrium liquid savings varies *inversely* with expected labor income to offset known variations in labor income.

High labor income, low liquid savings. But because self can only constrain the past (ie, can only deny access to assets accumulated from the past) self t cannot constrain how current income is spent. Hence y_t and c_t comove.

Implications for the economy

1. Golden egg generates wealth accumulation because self t cannot consume the illiquid asset z_t.
2. When planning for the future, includes use of z_{t+1} for c_{t+1+i} for $i>0$ when deciding the tradeoff between consumption at t+1 and beyond.
3. The value of β is superfluous since it equally affects all future periods, and so the steady state capital stock is independent of β

For example, with Cobb-Douglas production you get the following comovement

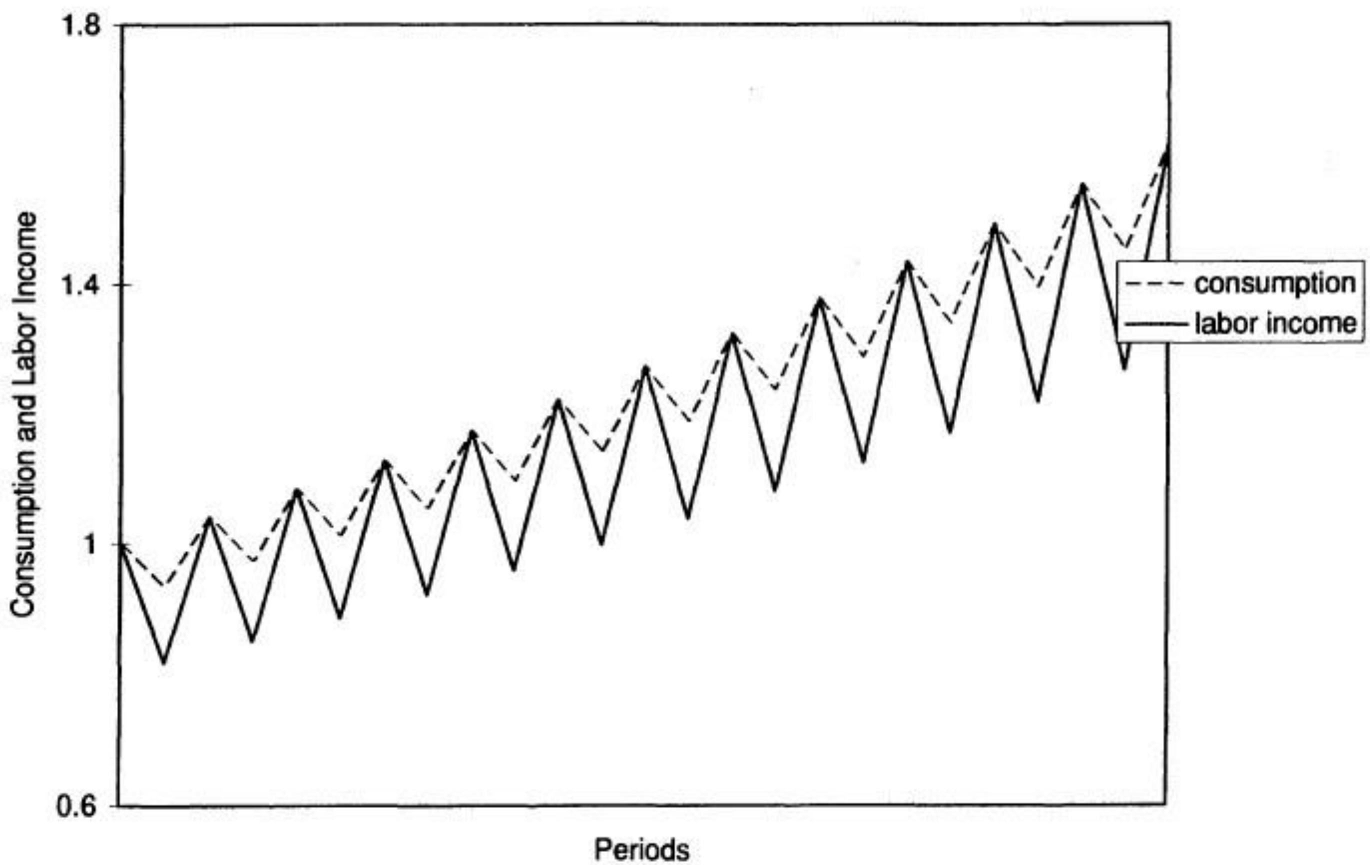


FIGURE II
Consumption and Labor Income

Golden Eggs gets the same result as a Thaler mental accounting argument. Thaler argued that agents had three mental accounts: current income, net assets, and future income, and treated them differently; MPC of current income≈1, MPC from future income≈0, and the MPC of assets is in-between

But Mental Accounting implies agents are less than fully rational because they ignore fungibility.

Golden eggs get the same result with a fully rational agent. MPC from current income≈1 because of the *endogenous* liquidity constraint. Moreover, if the growth of labor income is related to the return on capital, propositions 3 and 4 imply that the MPC out of illiquid savings≈0.

Ricardian equivalence: forward looking consumers internalize government budget constraints when making decisions, so the method of government financing its spending does not affect consumer spending. Golden Eggs violate Ricardian equivalence because the exogenous (labor and interest rate) cash flows drive behavior. If government tax policy disrupts these cash flows, behavior will change.

Evidence:

1. Decline in US savings rate in the 1980s, and the high MPC out of current income. Consumers cash flow increased, high MPC because of Golden Egg
2. New credit rules increased instantaneous credit, reducing the effectiveness of illiquid investing. Lowering commitment devices will lower capital accumulation

Instantaneous credit changes the consumption constraint from

$C_t \leq y_t + R_t x_{t-1}$ to $C_t \leq y_t + R_t x_{t-1} + R_t z_{t-1}$

Without commitment, the capital/output ratio is lower than with commitment – essentially there is no binding of future self's behavior. Current self becomes better off because of higher consumption. Future selves are worse off – less consumption and also have more freedom, so overall current welfare falls.

Drawbacks

1. No explanation about how initial assets are accumulated. Since consumption always exceeds labor income on the equilibrium path, that requires some assets to start. There are fixes if you allow non-discretionary borrowing like life insurance and mortgage payments.
2. Predicts consumers always self-impose a binding liquidity constraint, ie, $x_t \equiv 0$ for all t. Can address this with precautionary savings
3. Ignores that some consumers don't need external controls like illiquid savings; they have self-control. Adding these people breaks down the equilibrium
4. Ignores social commitment devices stronger than illiquid assets.