EconS 450 – Advanced Farm management

Lecture 14 – Risk Part 2
Game Theory

Basic Valuation Model

Diminishing marginal utility leads directly to risk aversion.

Risk aversion is, therefore, reflected in the basic valuation model

$$V = \sum_{t=1}^{n} \frac{\pi_t}{(1+i)^t}$$

Basic Valuation Model

$$V = \sum_{t=1}^{n} \frac{\pi_t}{(1+i)^t}$$

This model basically states that the value of the firm is equal to a discounted stream of profits where *i* is the risk-free rate of return.

Basic Valuation Model

Under conditions of uncertainty, the π_t in the numerator is the expected value of profits during each future period.

Thus we need some technique to choose between alternative courses of action with different risk exposure.

Basic Valuation Model

There are two basic approaches to adjusting the valuation model for risk.

1. Expected profits are adjusted to account for risk
2. The interest rate is increased to reflect risk considerations

Certainty Equivalent Adjustments

The certainty equivalent method is an adjustment to the numerator of the valuation model to account for risk.

Here a decision maker must specify a sum that they regard as comparable to the expected value of a risky investment alternative.

Certainty Equivalent Adjustments

Suppose you face the following choices:

1. Invest 100,000 if you are successful you receive 1,000,000 if not, you receive 0, the probability of either outcome is 50%
2. Keep the 100,000

If you are indifferent, then your certainty equivalent is $100,000

Certainty Equivalent

In this example, any certainty equivalent less than $500,000 indicates risk aversion.

If the maximum you are willing to invest in the project is only $100,000 you are exhibiting a very risk averse behavior.

Certainty Equivalent

In this example each certain dollar is *worth* 5 times as much as each risky dollar.

Alternatively, each risky dollar of expected return is worth only 20 cents in terms of certain dollars.

Certainty Equivalent

Any expected risky amount can be converted to an equivalent certain sum using a certainty equivalent adjustment factor.

This factor, called α is calculated as the ratio of a certain sum divided by an expected risky amount where both dollar values provide the same level of utility.

Certainty Equivalent

$$\text{Adjustment Factor} = \alpha = \frac{\text{Equivalent Certain Sum}}{\text{Expected Risky Sum}}$$

The important point is that the numerator and denominator provide the same reward in terms of utility.

Certainty Equivalent

In general, we can classify risk attitudes using this factor as follows:

If	Then	Implies
Certain < risky	$\alpha < 1$	Risk averse
Certain = risky	$\alpha = 1$	Risk neutral
Certain > risky	$\alpha > 1$	Risk loving

The basic valuation model

The basic valuation model can be converted to a risk-adjusted valuation model by using alpha.

$$V = \sum_{t=1}^{n} \frac{\alpha E(\pi_t)}{(1+i)^t}$$

Risk Adjusted Discount Rate

The alternative to the certainty equivalent factor is to adjust the discount rate of the basic valuation model.

As risk increases, higher expected returns are required to compensate for additional risk.

Risk Adjusted Discount Rate

The basic valuation model is now written:

$$V = \sum_{t=1}^{n} \frac{\pi_t}{(1+k)^t}$$

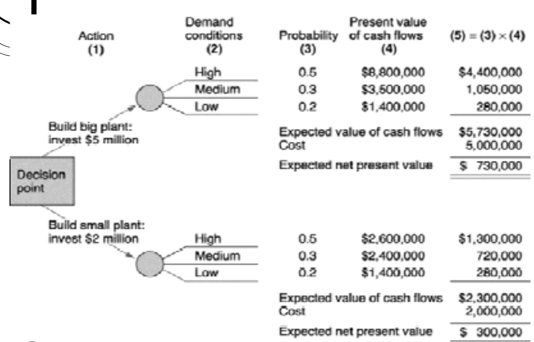
Where k = R_f + R_p is the sum of the risk free rate of return plus the required risk premium.

Sequential Decision Making

A Decision Tree is a technique for analyzing sequential decisions in a risky environment.

A decision tree gets its name from the characteristic shape, and traces outcomes from a decision point through each subsequent action.

Decision Tree Example



Game Theory

In an uncertain environment, value maximization using risk-adjusted valuation is appropriate.

Under certain circumstances when faced with a possibly hostile decision environment, game theory approaches may be more appropriate.

Game Theory

Game theory dates to the 1940's and originates from von Neuman and Morgenstern.

Like a lot of risk literature, this was developed due to a poker game and the decision of when to bluff, fold, stand pat or raise.

Game Theory

Game theory was initially applied to analyzing participant behavior in auctions.

English auctions – familiar to everyone, the auctioneer sells to the highest bidder.

Game Theory

In an English auction is widely regarded as a fair and open process, participants see and hear what the competition is doing.

The openness of this process can lead bidders to act in an overly aggressive manner.

Game Theory

The so-called "Winner's Curse" results when overly aggressive bidders pay more than the economic value of auctioned items.

Game Theory

Sealed-bid auctions are also common. Here all bids are secret and the highest bid wins.

Such an approach should be free from collusion, but it may result in less income as bidders may act cautiously.

Game Theory

Dutch Auctions are also used at times. This is essentially a reverse auction where the price is systematically lowered until a winning bidder emerges.

A disadvantage here is that bidders may act cautiously

Competitive Decisions

Game theory behavior involves strategic considerations. In some sequential conflict situations, systematic action becomes predictable and can be exploited by rivals. Take football for example.

Game Theory

To successfully implement game theory concepts, decision makers must understand the benefits of concealing or revealing useful information.

Prisoner's Dilemma

The most famous game is the prisoner's dilemma.

		Suspect #2	
		Not Confess	Confess
Suspect #1	Not Confess	Freedom Freedom	5-years Fine and probation
	Confess	Fine and probation 5-years	2-years 2-years

Prisoner's Dilemma

This is a one-shot game. The underlying interaction between competitors occurs only once.

While each suspect can control the range of sentencing outcomes, neither can control the ultimate outcome.

Prisoner's Dilemma

In this situation there is no dominant strategy that creates the best result for either player regardless of the action taken by the other.

A secure strategy, also called the *maximin* strategy guarantees the best possible outcome given the worst possible scenario.

Maximin Criterion

The *maximin* criterion states that the decision maker should select the alternative that provides the best of the worst possible outcomes.

Assume the worst will happen, and choose the alternative the offers the best outcome.

U-Pump Example

Decision Alternative	States of Nature: Competitor reduces prices	Competitor maintains price
Reduce Price	$2,500	$3,000
Maintain Price	$1,000	$5,000

Maximin criterion would require that U-Pump lower its price.

Maximin Criterion

The Maximin criterion obviously suffers from the shortcoming of focusing on the most pessimistic outcome.

Thus this criterion assumes a very strong aversion to risk.

Minimax Regret

A decision criterion that focuses on the opportunity loss associated with a decision is known as the *minimax regret* criterion.

Minimax Regret

The *minimax regret* criterion states that a decision maker should minimize the maximum possible regret (lost opportunity) associated with a wrong decision after the fact.

Minimax Regret

	States of Nature	
Decision Alternative	Competitor reduces prices	Competitor maintains price
Reduce Price	$2,500	$3,000
regret	0	$2,000
Maintain Price	$1,000	$5,000
regret	$1,500	0

To find regret find the max. outcome for a state of nature and subtract other outcomes from it.

Minimax Regret

The *minimax regret* criterion would cause U-Pump to maintain its current price.

Expected Opportunity Loss

Using the regret matrix and assuming that the probability of each state of nature is 50% we can calculate the expected opportunity loss.

Cost of Uncertainty

	States of Nature	
Decision Alternative	Competitor reduces prices	Competitor maintains price
Reduce Price	0 × 0.5 = 0	2000 × 0.5 = 1000
Maintain Price	1500 × 0.5 = 750	0 × 0.5 = 0

Minimum expected opportunity loss = $750

Cost of Uncertainty

Here the cost of uncertainty is measured by the minimum expected opportunity loss.

Firms often engage in activities to reduce uncertainty before making irrevocable decisions. These can alter the probabilities and therefore the cost of uncertainty.

Game Theory

We learned from the prisoners' dilemma that there may not be a dominant strategy solution to the game.

But the problem, a bargaining between to individuals, is one with broad application to the business world.

Game Theory

Competitors like Coca-Cola and Pepsi-Cola confront similar bargaining problems on a regular basis.

Suppose the two have to decide whether to offer a special discount to a large grocery retailer.

Game Theory

		Pepsi-Cola	
Coca-Cola		Discount Price	Regular Price
	Discount Price	4,000 2,000	10,000 1,000
	Regular Price	1,500 6,500	12,500 9,000

When faced with these potential payoffs, what would you do if you were Pepsi-Cola?

Nash Equilibrium

In the cola example, the secure strategy is to offer a discount, regardless of the competitor's actions.

The outcome is that both firms offer discounts and earn modest profits. This is also the Nash Equilibrium.

Nash Equilibrium

In a Nash Equilibrium, given the strategy of it's competitor, neither firm can improve its own payoff by unilaterally changing its strategy.

Clearly, from this example, profits are less than if the companies collude and charge regular prices.

Nash Equilibrium

From the individual firm's point of view the Nash equilibrium is inferior to the result of collusion.

This is the business manifestation of the prisoners' dilemma.

If firms collude, consumers would be made worse off. This is precisely why collusion is illegal in the United States.

Dominant Strategies

In the Pepsi/Coke example, the optimal strategy for each firm depended on what the competition decided.

In some situations, one firm's best strategy *may not* depend on the choice made by the other participants in the game.

Dominant Strategies

Consider the following payoff matrix for two firms facing a decision on prices:

		Firm 2	
		No price change	Price Increase
Firm 1	No price change	10,000 10,000	100,000 -30,000
	Price Increase	-20,000 30,000	140,000 25,000

Dominant Strategies

Based on this table, if firm 1 increases its price, firm 2 is still better off with no price change because profit will be $30,000 compared to $25,000 if it increases prices.

Hence, firm 2's dominant strategy is to hold prices at existing levels, regardless of what firm 1 does.

Dominant Strategies

A dominant strategy occurs when the best course of action for a firm is not affected by competitors choices.

When one player has a dominant strategy, the game will *always* have a Nash equilibrium.

Dominant Strategies

Nash equilibriums will occur in games where a player has a dominant strategy, because that player will always choose the dominant strategy and the other will respond with its best alternative.

Mixed Strategies

So far we have assumed that each participant in a game selects only one course of action. This approach is called a *pure strategy.*

In many games, however, a pure strategy may be a very poor choice.

Mixed Strategies

Consider the game of baseball, and the duel between a pitcher and a hitter.

If a pitcher throws all curves or all fastballs, the hitter would have a good chance of getting a hit – to be effective, the pitcher must keep the hitter off-balance.

Mixed Strategies

In this example, the pitcher must adopt a mixed strategy, and throw a mixture of curves and fastballs.

Consider this payoff matrix:

		Pitcher	
		Throws fastball	Throws curveball
Hitter	Anticipates fastball	40%	20%
	Anticipates curveball	20%	40%

Mixed Strategies

The table indicates the percent base hits, so that if a hitter anticipates a fastball, and the pitcher throws one, then the hitter will hit .400 (the same applies for the curve).

Thus if the pitcher only throws one pitch repeatedly, in this example he will always be facing a .400 hitter.

Mixed Strategies

Clearly, the best strategy for the pitcher in this case is to throw a mixture of curveballs and fastballs.

For the payoff table presented in this problem, there is no Nash equilibrium, and no dominant strategy.

There are, however, equilibrium mixed strategies.

Mixed Strategies

If the hitter randomly alternates between anticipating a fastball and a curveball on a 50-50 basis, and the pitcher likewise throws a 50-50 mixture of fastballs and curveballs, the hitter's batting average will be .300

Mixed Strategies

Action	Percent	Outcome
Expect fastball, fastball thrown	0.5×0.5=0.25	0.25×0.4 = 0.1
Expect fastball, curveball thrown	0.5×0.5=0.25	0.25×0.2 = 0.05
Expect curveball, fastball thrown	0.5×0.5=0.25	0.25×0.2 = 0.05
Expect curveball, curveball thrown	0.5×0.5=0.25	0.25×0.4 = 0.1
Sum		0.300

Mixed Strategies

If the pitcher throws a random 50-50 mix, and the hitter adopts a strategy other than anticipating that fastballs and curveballs are equally likely, then the hitter will guess wrong more often than right, and his average will drop below .300

Thus the best approach for the hitter give the pitchers strategy is to anticipate a random 50-50 mix

Mixed Strategies

In contrast, if the batter continues to assume a random 50-50 mix, but the pitcher throws a different mix, the hitters average will still be .300

Surprisingly, this result does not depend on the mixture of fastballs and curveballs thrown, as long as the hitter doesn't change.

Mixed Strategies

Assume the pitcher throws a 60-40 mix

Action	Percent	Outcome
Expect fastball, fastball thrown	0.5×0.6=0.30	0.30×0.4 = 0.12
Expect fastball, curveball thrown	0.5×0.4=0.20	0.20×0.2 = 0.04
Expect curveball, fastball thrown	0.5×0.6=0.30	0.30×0.2 = 0.06
Expect curveball, curveball thrown	0.5×0.4=0.20	0.20×0.4 = 0.08
Sum		0.300

Mixed Strategies

So, as long as the batter continues to anticipate that curves and fastballs are equally likely, the hitters average will be .300 regardless of the pitchers strategy.

Thus, this game has many Nash equilibriums.

Mixed Strategies

When mixed strategies are allowed, every game with a finite number of players and a finite number of strategies has *at least one Nash equilibrium*

Nash Bargaining

The prisoner's dilemma is an example of a non-cooperative game.

In that example, the players cannot bargain, negotiate or enforce agreements.

Nash Bargaining

A Nash bargaining game is another application of the simultaneous-move one-shot game.

In this case, the competitors "bargain" over some item of value having only one chance to reach an agreement.

Nash Bargaining

Example:

A company has $1M profit sharing to distribute. It can be done only in increments of $0, $500K, or $1M.

Management and labor have to agree on how to split the money, but if the sum of the amounts requested exceeds $1M, neither gets anything.

Nash Bargaining

		Management		
	Request Strategy	$0	$500,000	$1,000,000
Labor	$0	$0 $0	$0 $500,000	$0 $1,000,000
	$500,000	$500,000 $0	$500,000 $500,000	$0 $0
	$1,000,000	$1,000,000 $0	$0 $0	$0 $0

Where are the Nash Equilibriums in this game?

Nash Bargaining

In this game there are three Nash equilibriums. *Where are they?*

In the bargaining process, requesting $0 is dominated by requesting $500K or $1M. If you ask for nothing, that is the most you will get.

This is true for both management and labor.

Nash Bargaining

		Management		
	Request Strategy	$0	$500,000	$1,000,000
Labor	$0	$0 $0	$0 $500,000	$0 $1,000,000
	$500,000	$500,000 $0	$500,000 $500,000	$0 $0
	$1,000,000	$1,000,000 $0	$0 $0	$0 $0

In each Nash equilibrium, the entire-profit sharing pool is paid out

Nash Bargaining

It is clear that given this potential pay-off matrix, the only sensible request from each party is $500,000

- To request less guarantees $0
- To request $1M, risks getting $0

Repeat Games

The one-shot game leads to the conclusion that tacit collusion is possible.

However, *competitors often interact on a continuous basis*. That is, firms are involved in repeat games.

Repeat Games

The repeat nature of competitor interaction can sometimes harm consumers.

The repetitive interaction also provides incentives for firms to produce high-quality goods and services and to *maintain product consistency*.