A reader's guide to Lucas' paper

Two period OGE

Population is constant, each period N agents are born, and people live two periods, young and old. Young agents are endowed with labor, n, and use their labor to produce the consumption good, while old agents are not endowed at all. The consumption good is not storable. This sets up an incentive to design an institution to allow for trade in some manner.

One institution is where young agents exchange the value of their labor pn for fiat money, m, so pn = m, where p is the price level. Old agents use their cash to buy consumption, m = p'c, where c is second period consumption, and p' is the price level in the future.

Production: y = n
Utility: U = u(c) - n
(In our notation, $c_1 = 0$, $c_2 = c$.)

The socially optimal allocation, or social planner's allocation, satisfies, u'(c) = 1, where c = y = n. Why? The resource constraint is y = c and the technology is y = n so c = n. Solve the problem: Max {u(n) - n}. The necessary condition is u'(n) - 1 = 0.

Suppose there is a **fixed** stock of money and markets are competitive. A young agent exchanges the product of her labor for money and then when old uses the money to buy the consumption good. She solves Max{u(pn/p') - n}, which requires (p/p')u'(pn/p') - 1 = 0. A reasonable expectation for the price ratio is that p' = p. Therefore, u'(n) = 1, which is socially optimal. But we're really interested in what happens when the government starts injecting money into the economy.

Now suppose the money supply is growing at rate x > 1, following Lucas, M' = xM. This is a **money supply rule** and it can be written as M' – M = xM – M or

$$\Delta M = (x - 1)M. \qquad \text{(MSR)}$$

Given this rule, what should agents expect about the price level over time? It seems reasonable that agents in this economy should expect p' = xp. Suppose the new money is transferred to the young,

$$\Delta M = pTN, \qquad \text{(GBC)}$$

where the N young agents each get an **anonymous** transfer T that is worth pT. This is the government's budget constraint (GBC) in this simple economy. If we combine these two equations governing the government's policy,

$$pTN = (x - 1)M,$$

or,

$$T = (x - 1)(M/pN). \qquad (*)$$

This is the anonymous transfer.

Alternatively, the transfer can be tailored to the individual. If the individual acquires m on his own, then the transfer can be made proportional to m according to,

$$T = (x - 1)(m/p), \qquad (**)$$

Where m/p is the real balances the individual chooses to hold, which equals n, n = m/p.

First, consider injecting money into the economy via an anonymous transfer where everyone gets the same transfer. For an individual, the budget constraints are

$$m = pn, \quad (1)$$

and

$$p'c = m + pT, \quad (2)$$

where T is the transfer, which is also $p'c = pn + pT$ by (1). Solve to get

$$c = pn/p' + pT/p'. \quad (3)$$

Use this in utility to get the decision problem when the transfer is anonymous:

$$\text{Max}\{u(pn/p' + pT/p') - n\}$$

taking T as given. The max is given by

$$(p/p')u'(c) = 1, \quad (4)$$

where c is given by (3). If $p'/p = x$, (4) is $u'(c) = x$. But what is c? Consumption is equal to one's own cash plus the anonymous per capita transfer. We need to simplify the transfer.

Let's develop equation (2). Suppose we substitute (*) into (2),

$$p'c = m + (x-1)(M/N).$$

But in equilibrium supply equals demand: $M = Nm$, or $m = M/N$ so

$$p'c = m + (x-1)m = m + xm - m = xm.$$

Since $m = pn$, this becomes

$$p'c = xpn.$$

If expectations are correct, then $p' = xp$, and we have

$$xpc = xpn,$$

or, $c = n$. Substitute this and $p/p' = 1/x$ into equation (4) to get,

$$u'(n) = x. \quad (5)$$

Inflation acts as a tax on future consumption that causes people to consume more now and less in the future.

(As an aside, differentiate (5) and solve to get $dn/dx = 1/u'' < 0$; an increase in the growth of the money stock leads to inflation which reduces labor productivity and reduces production as well since $y = n$.)

Next, suppose the transfer is person specific and proportional to the individual's own holdings of cash, $pT = (x - 1)m$. From (2)

$$p'c = m + pT = xm = xpn.$$

So $c' = pnx/p' = n$ if $p' = xp$. The decision problem becomes $\text{Max}\{u(n) - n\}$ and the Max is the same as the socially optimal allocation! This follows because the agent no longer takes T as given but recognizes it is equal to $(x-1)m/p$, i.e., their own cash balances. One interpretation is that the Central Bank is paying interest on the individual's own cash balances. In fact, the Fed has been paying interest on the cash reserves of member banks for several years now.

Summary

Seek to build a model that connects money supply to productivity. Exchange is forced through the assumption of who is endowed with production and who is not. Money is transferred into the economy in different ways. If it is anonymous, then the agent, i.e., bank, will take the transfer as given when optimizing and inflation will affect productivity. If it is individual specific so the transfer is tailored to an individual account for each bank, for example, then it is socially efficient. This can be interpreted as paying competitive interest on reserves. We can interpret the “agent” in this framework as a bank.

In the last part of the paper Lucas modifies the model so inflation stimulates production by introducing a trading friction with uncertainty. Imagine a set of islands. Production takes place on each island, which isolates trading. Sellers observe the local price and changes in the local price, but do not observe prices elsewhere in the economy as a whole. Suppose $\Delta p > 0$ for the local price and suppose it is large. The question is why? There are two possibilities. It could be that local sellers are producing less at that location and there is a short supply. Or, it could be because of the government inflating the currency causing the monetary transfers into the banking system at all locations to be high and hence inflation might be high at all locations. Sellers selling a lot gain if market supply is low, but not if there is inflation. If sellers mistake inflation across all locations for a price increase at their specific location, then they will be tempted to produce more. This leads to a positive correlation between inflation and production, i.e., Phillips curve. However, the monetary authority cannot exploit this tradeoff. If it does, this will be anticipated and the relationship will change.