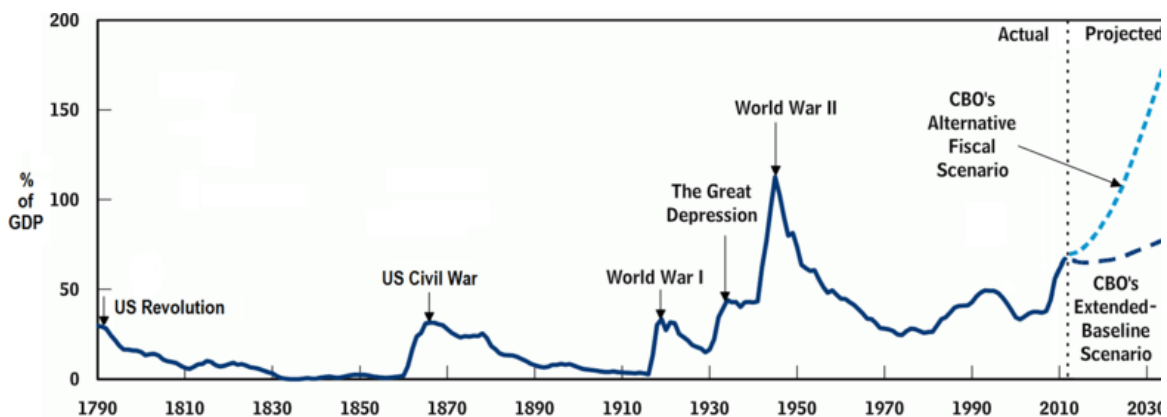


## Government Debt

There are typically three ways of financing government spending, taxes, debt, and money. Some governments also charge for the use of certain services. User fees for national parks, toll roads, and charging for public transportation are also used. However, they tend to be vastly dominated by taxes, bonds, and money, which is what we will focus on. And governments can sell off assets, e.g., buildings. But this is very limited.

From the chart one can easily see the pattern of US Federal debt since 1790 as a percent of GDP. The government dramatically increased the stock of debt to finance its wars. This was usually followed by a long period of declining debt as the US paid off the debt incurred during the war. However, a significant change occurred in the early 1980s. The Reagan tax cuts, e.g., ERTA, led to large deficits that were mainly financed by bonds at a time of relative peace. Tax increases and strong economic growth in the 1990s led to budgetary surpluses by 1999. However, the Bush tax cuts of the early 2000s followed by increased spending on the wars in Afghanistan and Iraq, prescription drug coverage for seniors in Medicare, and a farm bill that was very generous in subsidizing farming, led to large deficits once again, which added to the stock of debt. It should be noted that the chart does not include intergovernmental bonds, i.e., bonds transacted between agencies in the government. This would add about \$2.3 trillion to the current debt. One of the main sources of these bonds is the social security trust fund, which began accumulating in the early 1980s and will continue to grow until the so-called baby boomer generation retires.



Government debt is not necessarily a bad thing. Typically, a family that wants to buy a house cannot simply pay for it all at once, but must borrow much of the purchase price and then gradually pay it back over time. The same is true for governments. For example, a local government may issue bonds to pay for a new school, library, or computer equipment and then gradually pay off the bonds. This is a socially useful purpose for government debt. As a second example, an unusual event may occur that must be financed. It would be inefficient to raise taxes by a large amount to finance the event. For example, wars are an unusual event and the US essentially financed its war efforts by issuing bonds. A third use for government bonds is to smooth the government's cash flow over the business cycle. Typically, revenue falls in a recession as people lose their jobs and stop paying taxes, and expenditures rise as people apply for unemployment benefits. A deficit occurs and the government can sell bonds to borrow the money it

needs to meet its obligations. The opposite tends to happen as the economy recovers from a recession.

A final reason for government debt is a bit more controversial. It involves Keynesian "leaning against the wind" of the business cycle, i.e., countercyclical use of debt. This policy requires the government to cut taxes and increase spending during recessions thus running deficits, and reverse course when inflation heats up running surpluses instead. Presumably, the deficits are balanced by the surpluses. Unfortunately, there appears to be a bias toward deficit spending since it is hard to raise taxes when inflation becomes a problem. Second, it may take too much time to get a tax cut or tax increase passed. In addition, many economists now believe that such policies do not have their intended effect especially if private agents come to expect such a policy to be pursued. Robert Lucas pointed out that agents can anticipate such policies and adjust their behavior so as to increase the variability of output rather than reduce it as intended.<sup>1</sup>

Recently, however, many governments in advanced countries have begun to rely increasingly on debt financing and rolling over the debt when it comes due thus increasing their outstanding stock of debt rather than paying it off. Data from the OECD indicates that many advanced countries have increased their reliance on debt in recent years. For example, from 1980 to 2006, Germany increased its stock of outstanding debt by 10.3%, Italy by about 7.5%, Japan by 14.7%, the US by about 5.8%, and the OECD as a whole by about 7%.<sup>2</sup> This widespread use of debt for financing government spending is a relatively new phenomenon. Most of these governments were running balanced budgets until the late 1970s and debt was used mainly to smooth the flow of receipts and government expenditures over the business cycle, or for financing infrastructure projects. This new reliance on debt may have implications for world credit markets. For example, it may lead to higher world interest rates that may squeeze less developed countries out of the credit market.

## **2. Government debt: the crowding out effect**

We will begin by illustrating the impact of debt on the economy geometrically. We imagine there is a market for credit where the supply of credit and the demand for it determine the going interest rate or the cost of credit. Suppliers are made up of lenders making loans. The demand comes from borrowers and the government. Alternatively, one can think of the borrowers as issuing bonds and the lenders buying the bonds. However, we will stick to the so-called loanable funds interpretation of analyzing the credit market.

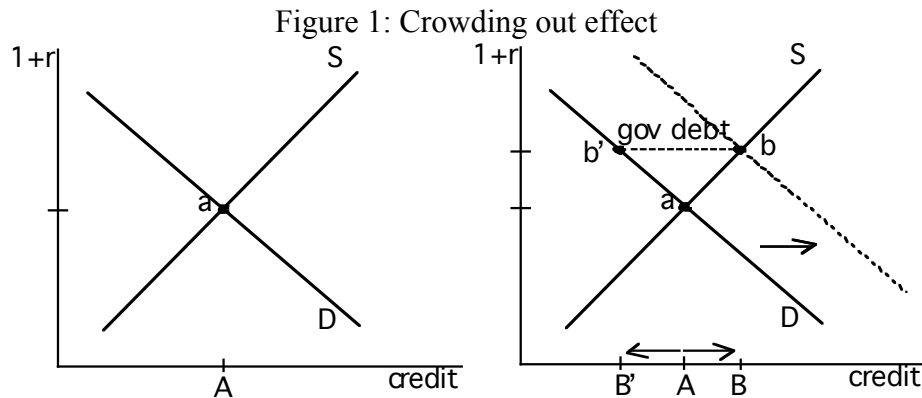
The economy is initially in equilibrium at point a in Figure 1 and the interest rate has been established as an equilibrium between the supply of credit and the demand for it.

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<sup>1</sup> Consider a company that is going to invest in new plant and equipment. They read in the newspaper that the economy is sinking into a recession and the company holds back investment waiting for a tax cut with an investment credit to be passed. Investment drops before the credit is passed and increases after thus possibly increasing the variability of investment and possibly GDP relative to what would have happened had the government not been expected to pursue a countercyclical policy.

<sup>2</sup> Source OECD data: [http://stats.oecd.org/wbos/default.aspx?datasetcode=GOV\\_DEBT](http://stats.oecd.org/wbos/default.aspx?datasetcode=GOV_DEBT). This does not include intergovernmental bonds in the US not held by the public, which pushes the total stock of debt to about \$9 trillion by 2007. If this is included, then the stock of debt grew by 8.8% in the US during this period. Unfortunately, the Great Recession, which began in 2008 added over \$5 trillion in federal debt so the total stock had reached about \$16 trillion by 2012.

Then the government comes along and issues new debt hence increasing its borrowing. The demand for credit shifts out raising the interest rate.



The total volume of loans increases from A to B in the right hand diagram. However, the total amount of credit available to the private sector diminishes from A to B'. This is the so-called crowding out effect of government debt. Some borrowers may be unable to get loans and some may be unwilling to take out loans at the higher interest rate. Imagine that you are running a small business and would like to borrow to expand your operation. This might make sense at the original interest rate. However, as the new government debt crowds out private borrowing and raises the interest rate, the cost of borrowing increases. If the cost increases enough, it may mean that the expansion of your business doesn't make sense from a profitability standpoint. In that case, the company decides not to invest in new plant and equipment and business investment falls as a result.

Exercise: Suppose the supply of loans to the credit market is perfectly inelastic. Is there any crowding out when the government introduces debt? Can you demonstrate this in a diagram like that of the last figure? Suppose supply slopes upward but demand for loans is perfectly inelastic. Now is there any crowding out? Can you draw the appropriate graph and demonstrate this in the graph?

### 3 A permanent increase in debt financed by a tax on young savers

Many governments appear to have permanently increased their outstanding stock of debt. We can ask what the impact is of this. Consider the government's budget constraint in period 1,

$$G_1 - T_1 = B^g_1.$$

and for period 2,

$$G_2 - T_2 = B^g_2 - (1+r) B^g_1,$$

and so on. Government spending minus taxes, e.g.,  $G_1 - T_1$ , is one definition of the deficit. If the deficit is fixed in period 2 at the same level as period 1, there is an immediate problem, how do we pay interest on the debt? Since  $G_2 - T_2$  is positive,

$$B^g_2 - (1+r) B^g_1 > 0.$$

So

$$B^g_2 - B^g_1 > r B^g_1 > 0.$$

But if  $B^g_2 > B^g_1$ , then the stock of debt is growing. In fact if the deficit is constant, the stock of debt will grow at the rate of  $1+r$ . If deficits continue forever, then the stock of debt grows to infinity! This would literally bankrupt the economy since lenders would not believe the government could repay such a large stock of debt. The government might have to default on its debt.

There have been some famous cases of default like Russia in the late 1990s. And this is precisely what happened in a number of countries after 2010. Greece, Cyprus, Spain, Portugal, and Italy, among others, had to implement severe austerity programs to pay down their debt.

Suppose we consider a permanent increase in debt which is sustainable, i.e., can be paid back. How would this work? Suppose the government spending, taxes and debt are constant. Then the government's budget constraint becomes

$$G - T = B^g - (1+r) B^g,$$

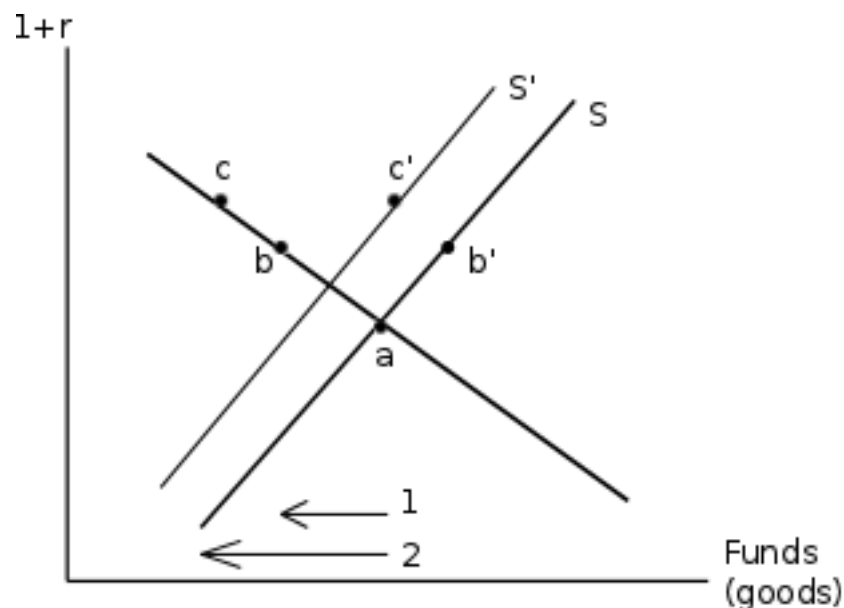
or, in the long run,

$$T - G = rB^g.$$

The government must eventually run a surplus ( $T > G$ ) to pay interest on its bonds.

How does this affect the credit market? The first part of the figure below replicates the earlier diagram. Because  $L = B + B^g$  the new government debt increases demand for loanable funds raising the interest rate from  $a$  to  $b - b'$ . The distance between  $b$  and  $b'$  is exactly  $B^g$ . Private borrowing is crowded out by the distance 1 in the diagram.

Figure 2: Permanent increase in debt



However, taxes must rise to finance the interest rate on the debt. If these taxes reduce savings, the loan function or supply of funds shifts back and the interest rate rises again to  $c - c'$ , where the gap  $c - c'$  is  $B^g$ . The total amount of crowding out is distance 2 in the long run. This is the classic analysis due to Diamond (1965) in a landmark paper that helped win him a Nobel prize.

We can specialize this. There are different kinds of borrowing. People have life cycle reasons for borrowing and others borrow to invest. So, suppose we have some life cycle borrowers endowed  $(0, w)$ , i.e., have income only in the second period, who borrow  $B$  to consume in the first period. We know  $B(w, 1+r)$ , which is increasing in future income  $y$  and decreasing in  $1+r$ . In addition, there are investors who wish to borrow to invest,  $b = k$  where  $k$  produces  $f(k)$ . Under profit maximization,  $f'(k) = 1+r$ , where a bank charges  $1+g = 1+r$  for the loan. Differentiate the profit maximization condition, to get  $dk/dr = 1/f'' < 0$  so the demand for investment loans is decreasing in the interest rate, and hence this demand is “downward sloping.” Since  $b = k$ ,  $b(1+r)$  is the demand for borrowing as a function of the cost of borrowing and  $db/dr < 0$ .

Equilibrium is given by

$$S(w-T, 1+r) = B(w, 1+r) + b(1+r) + B^g.$$

An increase in  $1+r$  reduces borrowing for life cycle reasons,  $B$ , and business borrowing,  $b$ . This second effect is the **crowding out** effect. The tax effect enters through savings. A permanent increase in taxation to finance interest on the bonds reduces savings and also increases the interest rate as in the last Figure, the move from  $b$  to  $c$ . However, when people reduce life cycle borrowing and savings falls thus raising the real interest rate, this also has implications for the economy. For example, it makes it harder to buy a house and this can have adverse consequences for the housing industry and for the real estate industry as well. It also becomes harder to borrow to go to college, which affects human capital accumulation.

#### 4. Temporary debt

In some cases the government tries to use a temporary change in debt to either stimulate the economy or to cool it down. Temporary changes in debt can be matched to temporary changes in taxes, e.g., tax cut coupled with an increase in government bonds, or temporary changes in money, e.g., open market operation. We'll look at the taxes first and then open market operations later.

Consider a simple two period framework. The government's budget is

$$G_1 = T_1$$

$$G_2 = T_2$$

when its budget balances. We will fix government spending in what follows and examine different fiscal policies.

Suppose the government wants to cut taxes in the first period and float some bonds  $B^g$ . The budgets become

$$G_1 = T^*_1 + B^g$$

$$G_2 = T^*_2 - (1+r)B^g,$$

where  $T^*_1 < T_1$  by the tax cut. In period 2 the government can either raise taxes or cut spending, but we have assumed spending is fixed. So there is a tax increase so the future tax is higher,  $T^*_2 > T_2$ .

Suppose the government cuts taxes today and raises them in period 2. How does this affect a saver? The saver's constraints are

$$c_1 = w - T_1 - s,$$

$$c_2 = (1+r)s - T_2.$$

Then the government comes along and lowers  $T_1$  to  $T_1^*$  and raises  $T_2$  to  $T_2^*$ . What should the consumer do? Suppose the saver increases her saving to  $s^*$ , but doesn't change her consumption,

$$\begin{aligned}c_1 &= w - T_1^* - s^*, \\c_2 &= (1+r)s^* - T_2^*.\end{aligned}$$

Will she alter her consumption pattern? Maybe not. Notice that  $T_1$  falls by  $T_1 - T_1^*$ . But this is the extra debt issued by the government,  $B^g = T_1 - T_1^*$ . Suppose the saver saves this so the increase in savings is  $s^* - s = T_1 - T_1^* = B^g$ . The extra savings earn her  $(1+r)B^g$ . Then next period, taxes go up to pay off the debt plus interest by  $T_2^* - T_2 = (1+r)B^g$ . So the increase in savings pays the higher taxes and consumption is not affected at all!

Another way to see this is to look at the present value constraints for the government,

$$G_1 + G_2/(1+r) = T_1 + T_2/(1+r).$$

This says the present value of government spending is equal to the present value of taxes, and the government uses credit markets to achieve this. The saver confronts her lifetime budget constraint,

$$W - T_1 - T_2/(1+r) = c_1 + c_2/(1+r).$$

The left hand side is the present value of her disposable income, the right side is the present value of her consumption. If there is no change in her disposable income, then there won't be any change in her consumption pattern.

Now suppose there is a change in tax policy with government spending fixed, then it follows that  $dT_1 + dT_2/(1+r) = 0$ . But this implies that if there is a tax cut today ( $dT_1 < 0$ ) there must be a tax increase in the future ( $dT_2 > 0$ ), or if there is a tax increase now ( $dT_1 > 0$ ), there must be a tax cut in the future ( $dT_2 < 0$ ) as long as government spending is fixed. Thus, a tax cut today,  $dT_1 < 0$ , financed by selling bonds, must be followed by a tax increase tomorrow,  $dT_2 > 0$ . However, if it is true that  $dT_1 + dT_2/(1+r) = 0$ , then the consumer's disposable income doesn't change and she has no reason to change her consumption pattern. The tax cut has failed to stimulate consumption!

This is known as *The Ricardian Neutrality Theorem* after David Ricardo.

Ricardo and Malthus carried on dialogs on a number of issues. One such issue was how to finance the Napoleonic Wars; Britain and France were locked in a death struggle. How should Britain finance its war effort, bonds or taxes? Ricardo argued that bonds issued today would come due in the future. This meant higher taxes in the future to pay off the bonds plus interest. Forward looking consumers could save the bonds and use the interest to pay the future taxes. If so, it didn't matter how the government financed its war spending. The equilibrium consumption is unaffected by the method of financing the war.<sup>3</sup>

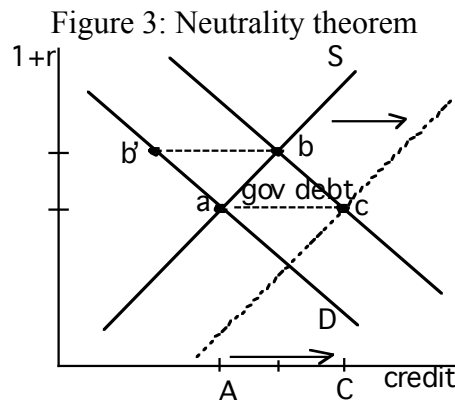
The evidence suggests that perfect neutrality probably doesn't hold one for one. In taxpayer surveys done by Gallup in 2001 after a massive tax cut under President Bush about half the respondents in the surveys said they would use the cut to pay down debt,

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<sup>3</sup> As an aside, it should be noted that Ricardo didn't believe that consumers would act that way; they weren't forward looking enough for complete neutrality to work. So Ricardo was not a Ricardian. This idea was independently discovered by Martin Bailey in the early 1960s and published in his graduate macro text, rediscovered by Earl Thompson in an article in the late 1960s, and re-discovered by Robert Barro in his famous 1974 article, "Are Government Bonds Net Wealth?"

about 30% said they would save it, and only about 20% said they would spend it. Remarkably, the breakdown in surveys in 2008 was the same. So a tax cut will generate about \$0.20 in spending.<sup>4</sup> Neutrality may fail for various reasons. Some may not be forward looking. Some may feel a tax increase is too far in the future to care about. And some may pass away or leave the country and not bear the burden of a future tax increase. Such individuals may very well spend their tax cut.

To demonstrate neutrality geometrically consider the following diagram. Recall that the government cuts taxes and has issued debt to finance its spending, which is unchanged. Initially, our analysis suggested that the demand for credit would shift out and lead to a rise in the interest rate as the credit market shifted from point a to points b and b'. Private credit of volume A-B' is crowded out. However, suppose individuals believe that the tax cut today will be followed by a tax increase in the near future. How should they respond? It might be optimal for them to save the tax cut today in order to pay the future tax increase, say, next year. In that case, the supply of savings or credit shifts out because of the additional saving. If the government pays the same interest rates on its debt as private agents can get, then the supply curve can shift out so as to completely offset the initial impact of the debt. In that case, the interest rate might not rise at all. Also notice that the supply and demand for credit have both increased by the same amount so private credit is now at point A and the gap A-C is the amount of government debt. No crowding out has occurred!



So instead of ending up at points b - b', the economy ends up moving from a to c instead. The interest rate is unaffected by the increase in demand for credit by the government due to the tax cut and there is no private crowding out since the interest rate has not increased.<sup>5</sup>

There are a number of examples of "temporary" tax cuts or tax increases that may not have had the intended effect. Inflation was starting to increase in 1967 due to spending on the war in Vietnam, the Apollo space program, and the war on poverty under the Johnson Administration. Johnson's advisors warned him that a tax increase would be

<sup>4</sup> Shapiro and Slemrod (2008) found that only about 20% of survey respondents stated they would spend their tax cut in 2008.

<sup>5</sup> Under the life cycle permanent income hypothesis some of you covered in earlier classes, a temporary increase in income should be spent quickly. The fact that temporary increases in income are not spent quickly implies there is something else going on for many consumers. One possibility is the expectation of a future tax increase.

necessary and he demurred since he wanted to run for re-election. After a poor showing in the New Hampshire primary, he dropped out of the presidential race in 1968 opening the way to the temporary tax surcharge of 10% in 1968. Taxpayer's were told the surcharge would be temporary. This tax increase did not stop the inflation since people generally didn't alter their consumption.

A second example occurred when the economy slowed down due to military base closings in the early 1990s. When the economy went into the so-called "double dip" recession, President G. H. W. Bush signed an executive order that reduced withholding so taxpayers observed a slight increase in their take home pay. This, however, was not a change in their actual tax liability. Many taxpayers simply used the money to pay off credit cards or saved it to pay the tax later the following April 15. It did not have the intended consequences for the economy; consumption hardly changed at all.

Basically, the response of the credit market depends critically on expectations about taxes and when they are paid in the life cycle. If people pay taxes when young and the changes in fiscal policy are permanent, then our earlier analysis in figures 1 and 2 is appropriate. If the change in policy is temporary, then figure 3 is more accurate, although the outward shift in savings may not be complete;  $1+r$  may rise slightly if only 80% of those receiving a tax cut save it rather than 100%.

## 5. Open market operations

### 5.1 Perfect neutrality of government asset swaps

Consider the government's deficit defined as  $G - T$ , spending minus taxes. It can finance it by bonds or money. Consider two different economies. Suppose there are savers in each and they want to acquire the government's assets. In economy #1, they acquire bonds  $B^g$  that pay  $1+r$ . The government constraint is

$$G - T = B_t^g - (1+r)B_{t-1}^g.$$

The equilibrium in the asset market is

$$S(w - T, 1+r) = B_t^g,$$

where  $S(w - T, 1+r)$  is savings. These two equations tell us everything we need to know about this economy.

Now consider the second economy. In economy #2 the government only uses money. So its constraint is

$$G - T = M/p - (1+r_m)M_{t-1}/p_{t-1}$$

And the equilibrium in the asset market is

$$S(w - T, 1+r_m) = M/p.$$

We have the following result: If  $B^g = M/p$  and  $1+r = 1+r_m$ , then the two economies yield the same consumption allocation,  $c_1$  and  $c_2$ . In economy #1 the budget constraint has slope  $-(1+r)$  and this is the same as the slope in economy #2,  $-(1+r_m) = -(1+r)$ , and the intercepts of the budget line are the same. So the budget line for the consumer is the same in the two economies. It follows that  $(c_1, c_2)$  is the same.

What does this mean? Imagine we are in economy #1 and the Fed engineers a massive open market operation replacing government debt with money and convinces markets that it will have the same rate of return as the bonds. If  $1+r_m = 1+r$  and consumers were willing to acquire the bonds, then they must be willing to acquire the money. The swap of money for bonds will have no impact on consumption  $(c_1, c_2)$ .

The government's complete budget constraint is



$$G - T = M_t/p_t - (1+r_m)M_{t-1}/p_{t-1} + B_t^g - (1+r)B_{t-1}^g.$$

Notice that the terms  $(1+r_m)M_{t-1}/p_{t-1}$  and  $(1+r)B_{t-1}^g$  are given at time  $t$ . The terms  $M_t/p_t$  and  $B_t^g$  are not given but can respond. An open market operation satisfies

$$0 = dM_t/p_t + dB_t^g.$$

One term must be positive and the other term negative. This is a swap of one asset for another. The following thinking governs these operations:

$$\text{Contractionary policy: } dM_t/p_t < 0 \text{ and } dB_t^g > 0.$$

$$\text{Expansionary policy: } dM_t/p_t > 0 \text{ and } dB_t^g < 0.$$

However, if the two assets are perfect substitutes these policies can't affect the interest rate and therefore can't affect savings and consumption. To see this, note that the equilibrium with just savers acquiring assets of government bonds and money is

$$S(w - T, 1+r) = M/p + B^g,$$

where  $1+r = 1+r_m$ . The right hand side doesn't change under an open market operation. So the left hand side can't change. It follows that  $1+r$  is unaffected.

## 5.2 Non-neutrality of government asset swaps

To begin, we'll assume there are two assets, government bonds and money and two types of saver, rich and poor, with endowments  $(w^r, 0)$  and  $(w^p, 0)$ , respectively. They are only different in their endowment, and  $w^r > w^p$  by definition. The government issues the two assets. Debt pays  $1+r$  while money pays  $1+r_m$  and  $1+r > 1+r_m$ .

Both agents would like to acquire bonds. However, the government issues the bonds in a large denomination such that  $B^g > w^p$  so the poor cannot afford them. The government also imposes a law or restriction that the poor agents cannot get together, form an investment company, and intermediate a bond thus earning the high rate of return.<sup>6</sup> Rich agents can acquire government bonds paying  $1+r$ , while poor agents can only acquire money. We know from our earlier work that the savings of the rich will depend on income and the return,  $S^r(w^r, 1+r)$ , and similarly for the poor,  $S^p(w^p, 1+r_m)$ . The equilibrium in the asset markets is given by

$$S^r(w^r, 1+r) = B^g$$

$$S^p(w^p, 1+r_m) = M/p$$

if there are no other assets, for simplicity. The government's budget constraint is

$$G - T = M/p + B^g - (1+r_m)M_{t-1}/p_{t-1} - (1+r)B_{t-1}^g.$$

$G - T$  is fixed,  $(1+r_m)M_{t-1}/p_{t-1}$  is fixed, and  $(1+r)B_{t-1}^g$  is also fixed. The open market operation satisfies  $dM/p + dB^g = 0$ .

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<sup>6</sup> How might the poor intermediate a government bond? Suppose there are ten poor agents and  $10w^p > B^g$ . Then the poor agents could get together and use their pooled income to buy a bond and each poor saver would receive a share of the return on the bonds in the future earning  $1+r$  rather than holding money and only earning  $1+r_m$ . The legal restriction prohibits this.

How does this operation affect the bond market? An increase in  $B^g$  increases the demand for loanable funds and this raises the interest rate. Alternatively, a decrease in government bonds reduces the demand for funds and lowers the interest rate.<sup>7</sup>

Also notice that if money grows at rate  $1+z$ , then  $1+r_m = 1/(1+z)$ . Thus,

$$S^p(w^p, 1/(1+z)) = M/p.$$

But this is the equation of exchange in the Quantity Theory. Income is fixed,  $z$  is fixed, therefore, the left hand side is fixed. So an open market operation implies  $dM/p - Mdp/p^2 = 0$  so prices change according to  $dp/p = dM/M$ . An increase in money raises prices and a decrease in money lowers prices. We have the following result:

Contractionary policy:  $dM < 0$ ,  $dB^g > 0$  implies  $dr > 0$  and  $dp < 0$ .

Expansionary policy:  $dM > 0$ ,  $dB^g < 0$  implies  $dr < 0$  and  $dp > 0$ .

We can add the other elements of the credit model. There may be life cycle borrowers who are endowed  $(0, w)$  so their borrowing is  $B(w, 1+r)$ , with  $B_r < 0$ , i.e., borrowing demand is “downward sloping.” And there may be business investors borrowing to invest so  $b = k$ . Profit is  $f(k) - (1+r)k$  so under profit maximization,  $df/dk = 1+r$ . So borrowing to invest is a function of the cost of borrowing  $1+r$ ,  $b(1+r)$ , and  $b_r = db/dr < 0$ .

Equilibrium in the credit market is

$$S^r(w^r, 1+r) = B(w, 1+r) + b(1+r) + B^g.$$

And equilibrium in the money market is the same as before,

$$S^p(w^p, 1/(1+z)) = M/p.$$

Once again the interest rate and government debt will be positively correlated; an increase in  $B^g$  will cause the interest rate to rise and vice versa for a decrease in debt.<sup>8</sup>

So if the government’s assets are not perfect substitutes, then an open market operation can have real effects on the economy. This raises an interesting conundrum. If the government deregulates the financial markets, money and government bonds may become closer substitutes. In that case, it may become more difficult for the government to utilize open market operations to affect the economy.

### 5.3 Another alternative

Suppose there are banks and both rich and poor savers can put deposits in banks. The banks hold the government’s bonds and make loans to various borrowers. There are two types of deposit, high yield deposits and low yield deposits, which earn less. There is a minimum denomination required by the bank for the high yield asset; the deposit has to be larger than the minimum for the high yield asset. Assume the minimum is such that

<sup>7</sup> Differentiate the equilibrium in the bond market:  $S_r dr = dB^g$ , or  $dr/dB^g = 1/S_r > 0$  so  $\text{corr}(r, B^g) > 0$ , i.e., the correlation of the interest rate and government debt is positive.

<sup>8</sup> Differentiate the credit market condition to get  $(S_r - B_r - b_r)dr = dB^g$  so  $dr/dB^g = 1/[S_r - B_r - b_r] > 0$  since  $S_r > 0$  and  $B_r, b_r < 0$ .

rich deposit into the high yield account and the poor only deposit into the low yield account. High yield accounts pay  $1+r$ , low yield accounts pay  $1+q$ , with  $r > q$ .

Suppose the bank must hold money  $M/p$  as reserves by law. No one else holds money. The bank's balance sheet is

Bank	
Assets	Liabilities
Reserves = $M/p$	$d^r$
Loans = $b + B$	
$B^g$	$d^p$
$M/p + b + B + B^g = d^r + d^p$	

Suppose the bank imposes the further restriction:  $b + B + B^g$  are loans that provide income for a high yield account paying  $1+r$  and  $M/p$  is a low yield account paying  $1+q = 1+r_m$ , with  $1+r > 1+r_m$ .

For the rich agent, the constraints are

$$c_1 = w^r - d^r$$

$$c_2 = (1+r)d^r$$

and for the poor agent,

$$c_1 = w^p - d^p$$

$$c_2 = (1+r_m)d^p$$

Effectively, the poor agent is holding money at the bank, while the rich agent is earning higher returns on a variety of assets held at the bank. So  $S^r(w^r, 1+r) = d^r$  and  $S^p(w^p, 1+r_m) = d^p$ .

The bank's restriction gives us the following breakdown for the equilibrium.

$$d^r = S^r(w^r, 1+r) = b + B + B^g$$

$$d^p = S^p(w^p, 1+r_m) = M/p$$

and we get the same result as in the last section. The assets are not perfect substitutes and open market operations will have their intended effects.

Another possibility is for the bank to charge a fixed fee to "join" the high yield asset. Then for the rich,

$$c_1 = w^r - d^r - F,$$

$$c_2 = (1+r)d^r,$$

where  $F$  is the fee. If  $F > w^p$ , the poor cannot join the high yield asset.