Ch 5 Monetary Equilibria

We will study the determinants of monetary equilibria in simple models where there is some trading friction. Use of money can overcome such frictions as we have seen. However, we will also present models where the competitive equilibrium cannot be improved upon. In that case, agents will not want to hold fiat money when there are private assets that clearly dominate it.

5.1 The classic OGE

We will begin with a version of Samuelson's classic model. The economy lasts forever and time is discrete t = 1, 2, ... Population is constant. At time t, N identical new agents are born and each lives two periods, young and old. At any given moment in time there are N young agents and N old agents coinciding. For example, in the first period there are old agents in the '0' generation and some young agents in the '1' generation. In the second period the old from the first period leave, the young from the first period become old, and there is a new generation of young born, generation '2.' The process continues as time progresses.

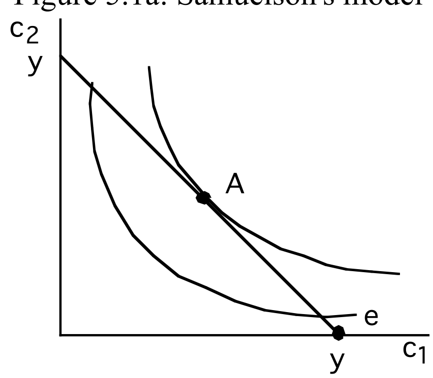
Table: Population structure

	t=1	t=2	t=3	t=4	....
generation 0	old				
generation 1	young	old			
generation 2		young	old		
generation 3			young	old	
generation 4				young	
.					.
.					.

Each young person is endowed with y units of the one good available. The old are not endowed with anything. Each person wants to consume in both periods of life and has a well behaved utility function, $u(c_1, c_2)$. We know from the last chapter that the resource constraint is $Ny = Nc_1 + Nc_2$, or $y = c_1 + c_2$. We also know that the social planner would choose an allocation such that the ratio of the marginal utilities is equal to one, $u_1/u_2 = 1$.

For example, suppose $u = Log(c_1) + \beta Log(c_2)$. Then $u_1 = 1/c_1$, $u_2 = 1/\beta c_2$, and we have two equations, $c_2 = \beta c_1$, and $y = c_1 + c_2$. Combining, $c_1 = y/(1+\beta)$ and $c_2 = \beta y/(1+\beta)$. This is the social optimum, point A in the figure.

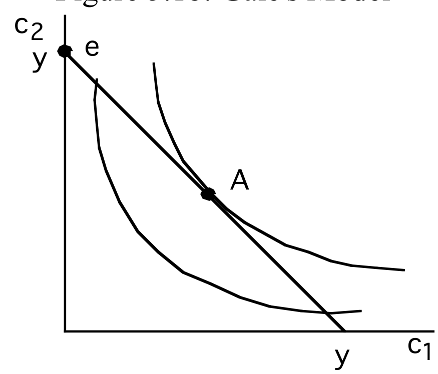
Figure 5.1a: Samuelson's model



What happens under competition? The young are endowed with some of the good but the old have nothing to trade that the young. The competitive equilibrium is autarky where no trading occurs, $c_1 = y$ and $c_2 = 0$, point e in Figure 5.1. This is a famous example where the competitive equilibrium is not Pareto Optimal. Everyone can be made better off by moving to another allocation. Suppose we take a unit of the good away from each young person and give it to an old person moving along the budget line in a northwest direction away from e. The initial old receive some of the good and are better off as a result. The initial young and all future generations move toward a higher indifference curve and are also better off.

Next suppose that each old person is endowed with y units of the good and the young are not endowed with anything. The endowment point is now at e. Agents would like to trade and move to a point like A in the diagram.

Figure 5.1b: Gale's Model



What happens under competition? Once again trades cannot take place and the competitive equilibrium is autarky. Is this equilibrium Pareto optimal? The answer is yes. Can we find an allocation where everyone is at least as well off as before and no one is worse off? Suppose we take some of the good from the initial old and give it to the young. The young are moving toward A and are better off. However, the initial old lose some of the good and are worse off.

5.2 Money

Suppose individuals are endowed with y units of the good when young and none when old. The government prints M pieces of fiat money that is not backed by anything, and gives them to the old so each old person receives M/N pieces of paper. The old can trade the paper to the young in exchange for the good. The young acquire the money and use it later to buy the good when old. The young agent's budget constraints are $y = c_1 + m/p$ and $m/p' = c_2$, where p is the price level today and p' is the price level in the future. Solve the second equation for m and substitute into the first, $y = c_1 + (p/p')c_2$. This equation describes a budget line with slope $- p/p'$.

The next step is to seek a stationary monetary equilibrium. In such an equilibrium real money balances are constant, $M/Np = M'/N'p'$, where primes denote next period's value. If the population is constant and the stock of money is constant, then this last equation implies $p/p' = 1$. The agent can move from the endowment point e to A in Figure 5.1a. Money allows trading that in turn supports the optimal allocation.

5.3 Inside money

Private agents can make contracts with one another and the resulting trades that are made make agents better off. In an environment where there is such contracting, can fiat money improve matters? This turns out to be a complicated question.

To get at this issue suppose there are lenders and borrowers. A lender is endowed with y units of the good in the first period and none in the second. A borrower is endowed with y of the good in the second period but none in the first. First, we'll look at private credit arrangements and then see what money adds. To that end, suppose everyone has the same preferences, $u = \log(c_1) + \beta\log(c_2)$. The lender chooses how much to save and allocates saving to loans. We know from our previous work that the lender's saving is $s_L = \beta y/(1+\beta)$. This is equal to loans when that is the only asset. The borrower's "saving" is $s_B = - yR/(1+\beta)$, where $R = 1/(1+r)$. Suppose there are ϕN lenders and $(1-\phi)N$ borrowers.

In a non-monetary equilibrium total savings sums to zero, i.e., loans equal borrowing, $\boldsymbol{\phi N s_L + (1-\phi)N s_B = 0}$. Substituting,

$$\phi N\beta y/(1+\beta) - (1-\phi)NyR^{nm}/(1+\beta) = 0. \quad (!)$$

Simplify and solve for 1+r,

$$1+r^{nm} = (1-\phi)/\phi\beta,$$

where the 'nm' stands for non-monetary. More generally, suppose the lender's endowment is y_L and the borrower's endowment is y_B. We have instead

$$1+r^{nm} = (1-\phi)y_B/\phi\beta y_L. \quad (*)$$

This interest rate clears the credit market. To see this note that the supply of credit is $\phi\beta N y_L/(1+\beta)$ and the demand for credit is $(1-\phi)Ny_B R^{nm}/(1+\beta)$. Substitute equation (*) into the demand function for 1+r to get

$$(1-\phi)Ny_B\phi\beta y_L/(1+\beta)(1-\phi)y_B = N\phi\beta y_L/(1+\beta),$$

which is equal to supply. This proves that the $1+r^{nm}$ in (*) clears the credit market.

The private credit in this economy is backed by the endowment of the borrowers. The credit acts like money in allowing trades to take place. This sort of "money" is sometimes referred to as ***"inside" money***. In contrast, fiat money, which is not backed by anything, is referred to sometimes as ***"outside" money***.[1]

There are two important cases to consider. Case 1: $1+r^{nm} > 1$; Case 2: $1+r^{nm} < 1$. As we saw in the last section, the return to holding money is $p/p' = 1$. Let r^m be the return to holding money in a monetary equilibrium, We can restate the cases as Case 1: $1+r^{nm} > 1+r^m$; Case 2: $1+r^{nm} < 1+r^m$. In the first case lenders will not hold fiat money. Only in case 2 will lenders hold fiat money. If the value of inside money or private credit is too low, fiat money can improve on the equilibrium for lenders by offering a higher rate of return.

Example 5.3a: Suppose $\phi = 1/3$, $y_B = (2/5)y_L$, and $\beta = 1/2$. Then according to equation (*), $1+r^{nm} = 8/5 > 1$.

Example 5.3b: Suppose $\phi = 1/2$, $y_B = (2/5)y_L$, and $\beta = 1/2$. Then $1+r^{nm} = 4/5 < 1$.

Example 5.3c: Let $\phi = 1/2$, $y_B = (3/2)y_L$, and $\beta = 1/2$. Then $1+r^{nm} = 6/2 > 1$.

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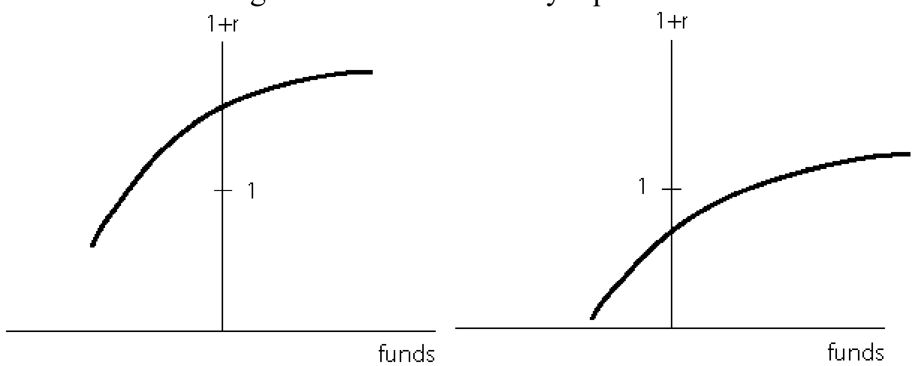
[1] In a sense the phrase "non-monetary" may be a mild misnomer. Inside money, or private credit, allows agents to trade just like outside money. For example, checking accounts at banks act just like currency in facilitating transactions and are included in some definitions of "money" like M2. We'll use the phrase "non-monetary" to refer to such equilibria where fiat money is not used and "monetary" to refer to equilibria where government issued currency has value and is used.

Example 5.3d: Let $\phi = 1/2$, $y_B = (2/5)y_L$, and $\beta = 1/3$. Then $1+r^{nm} = 6/5 > 1$.

In the first example, the ratio of borrowers to lenders is higher than in the second example. In 5.3a, $(1-\phi)/\phi = 2$, while in the second example, $(1-\phi)/\phi = 1$. More borrowers relative to lenders leads to greater demand for credit, and a higher interest rate, i.e., 8/5 > 4/5. In the third example, borrowers receive a larger endowment. This also increases their demand for credit raising the interest rate. Finally, in the last example, the discount factor β of the saver is lower. This means the saver cares less about the future and saves less as a result. A lower supply of credit translates into a higher interest rate in the non-monetary equilibrium.

To summarize, anything that raises the demand or lowers the supply of private credit raises the interest rate in the non-monetary equilibrium. A higher the interest rate on private credit, the less likely it is that lenders will want to use fiat money.

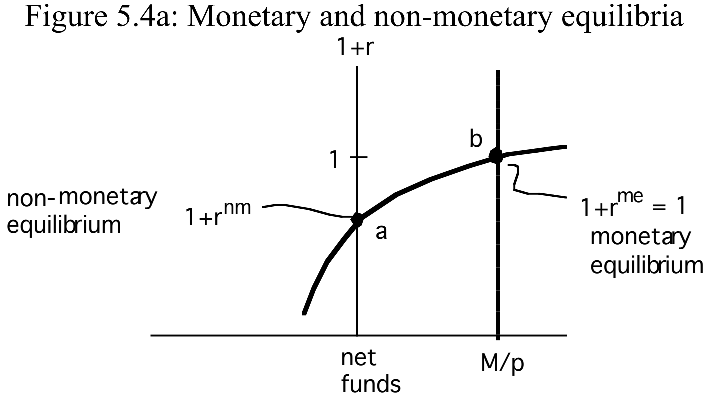
Figure 5.3: Non-Monetary equilibria



5.4 Inside money and fiat money

First, we consider a geometric analysis and then use some equations to make the same argument. First, consider the case where the non-monetary equilibrium involves an interest rate below one, $1+r^{nm} < 1$. The non-monetary equilibrium involving inside money or private credit occurs at point a in Figure 5.4a, where net funds are zero, i.e., total savings is zero, equation (!) holds.

Figure 5.4a: Monetary and non-monetary equilibria

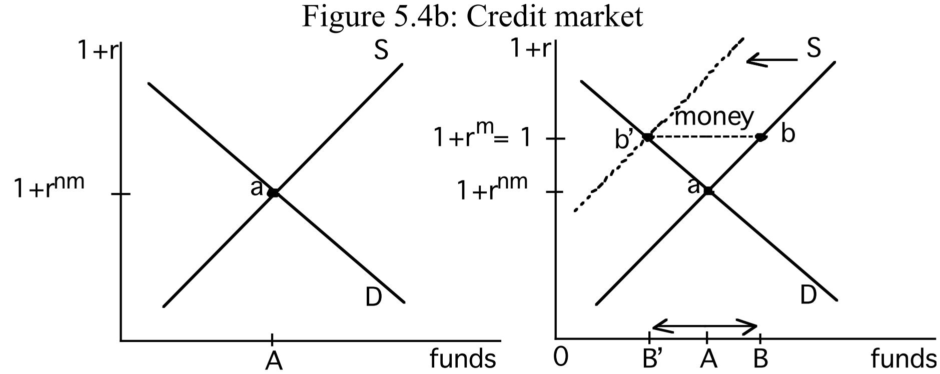


In a monetary equilibrium real per lender money balances must be constant, $M/\phi Np = M'/\phi N'p'$. If the stock of lenders and money supply are both fixed, then rearranging, p/p' = 1. This is the rate of return to holding fiat money. If lenders are holding both assets, the returns must be equal. In that case, $1+r^{me}$ = p/p' = 1. This is point b in the figure.

Intuitively, before the government issues fiat money, the interest rate on loans $1+r^{nm}$ is low, below one, at point a. The government issues M pieces of paper to the initial old generation and they use it to buy some of the good from young lenders. When a stationary monetary equilibrium is established, agents expect the return to fiat money to be equal to one. Young lenders then have a choice. They can make loans at a rate below one, or acquire money that they believe will have a return equal to one. Young lenders will put some of their savings into the asset with the higher return, fiat money. As savings shifts from loans to fiat money, fewer loans are made to borrowers and the return on loans to borrowers will increase until it is equal to one. When the return on loans is equal to one, lenders are indifferent between making private loans and thus holding inside money, or holding fiat money.

This is illustrated in the following figure depicting the credit market. Initially, the private credit market is in equilibrium at point a. When the government issues currency with a higher rate of return, some of the supply of credit shifts to currency and the return on loans increases. The market shifts from point a to points bb'. Distance 0-B' is private loans and distance B-B' is fiat currency.

Figure 5.4b: Credit market



How do you think the lenders and borrowers will be affected by this? Initially, the outside money has a greater return than loans. As lenders start holding the outside money, the interest rate on loans will rise. Lenders will be better off since they are getting a higher return in the monetary equilibrium. On the other hand, borrowers are worse off. So there is a conflict between the two types of agent as the interest rate on loans rise. In moving from a non-monetary equilibrium to a monetary equilibrium in this framework, borrowers are made worse off so this shift is not Pareto optimal!

Next consider the case where the non-monetary interest rate is above one, as in the left panel in Figure 5.3. Since $1+r^{nm} > 1$ and the fiat currency has a return equal to one, $1+r^{me} = 1$, lenders will choose not to hold the fiat money and money will not have value. What would cause the non-monetary interest rate to be greater than one? Large ratio of borrowers to lenders, $(1-\phi)/\phi$, large ratio of borrower income to lender income, y_B/y_L, or a small discount factor, β. In a phrase, anything that raises the demand for credit relative to the supply will cause the non-monetary interest rate to be high and make it less likely that fiat money will have value.

5.5 Mathematical demonstration

Our strategy is the following. We know that private individuals can always make private contracts with one another to lend and repay loans. If the government tries to introduce a new asset, bonds or money, into the financial markets, private agents can refuse to use the new asset and continue issuing private credit instead. Therefore, the private credit equilibrium, or non-monetary equilibrium, is always available, barring totalitarian government control over financial markets.[2] First, we calculate the private credit, or non-monetary equilibrium. Then, we calculate the monetary equilibrium. Finally, we compare the returns in the two equilibria to see if private agents will hold the government's new asset.

Suppose there are two assets for lenders to acquire, inside money or private loans (L) and outside money or fiat money (m/p). In that case, in a monetary equilibrium where both assets are held, they must have the same rate of return. In that case, the individual lender's saving is allocated between the two assets. The lender's budget constraints are

$y - c_1 = L + m/p$ and $(1+r)L + m/p' = c_2$,

where 1+r is the return on loans, $m/p' = (p/p')(m/p)$, (p/p') is the return to holding money, and m/p is real money balances. By definition income minus consumption is saving, i.e., $y - c_1 = s_L$, and saving is allocated between the two assets so it follows that $\mathbf{s_L = L + m/p}$.

Suppose we add up across all savers using this last equation,

$\phi N s_L = \phi NL + \phi Nm/p$.

This says that the total supply of savings in the economy is allocated to making loans in the private credit market, ϕNL, and fiat money, $\phi Nm/p$. What must be true in the credit market? Supply equals demand in equilibrium, $\phi NL = (1-\phi)NB$. Substitute this into the previous equation,

$\phi N s_L = (1-\phi)NB + \phi Nm/p$.

Next, note that M is the supply of money and ϕNm is the demand for money by lenders. Supply of money must equal demand in equilibrium. Use this in the last equation to get

$\phi N s_L = (1-\phi)NB + M/p$,

and solve for M,

$M = p[\phi N s_L - (1-\phi)NB]$.

Borrowing is negative savings, $B = - s_B$,

$M = p[\phi N s_L + (1-\phi)N s_B]$.

Next, substitute for the savings functions and divide by N,

$$M/N = p[\phi s_L + (1-\phi)s_B] = p[\phi\beta - (1-\phi)/(1+r^{me})]y/(1+\beta), \qquad (**)$$

where r^{me} is the interest rate in the monetary equilibrium.

How do we determine the interest rate? We look of a stationary monetary equilibrium where $M/Np = M'/N'p'$. Rearrange, $p/p' = (M/M')(N'/N)$ is the rate of return to holding money. The no-arbitrage condition has to hold, $1+r^{me} = p/p'$, or

$1+r^{me} = (M/M')(N'/N)$.

If the stock of money is constant and the population is constant, $1+r^{me} = 1$.

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[2] The government could force agents to use its currency in a number of ways. For example, it could demand that taxpayers make tax payments in units of its currency. It could ban private credit altogether. Of course, enforcement would be an issue. More subtle control occurs when the government imposes minimum denomination requirements, reserve requirements, and credit controls on foreign currencies.

What is going on intuitively? The government announces it is going to supply a constant amount of money to the initial old, who can decide whether to use it or not. The initial young lenders can decide whether they will accept it or not. If they believe that the stock of money will be fixed, they can calculate the rate of return to money using (M/M')(N'/N). If this return is greater than the non-monetary return, they will hold the money as long as they believe that future agents will want to acquire the money.

Using $1+r^{me} = 1$ in our condition we obtain

$$M/N = p[\phi\beta - (1-\phi)]y/(1+\beta).$$

Several comments are in order. This is a version of the quantity theory equation of exchange. It follows that once the equilibrium is established a change in the stock of money will feed through directly to prices. To see this note,

$$\Delta M/N = \{[\phi\beta - (1-\phi)]y/(1+\beta)\}\Delta p.$$

Divide by the equation of exchange, $\Delta M/M = \Delta p/p$.

Second, from the equation of exchange,

$$M/Np = [\phi\beta - (1-\phi)]y/(1+\beta).$$

This confirms our assertion about the value of real cash balances in equilibrium,

$$M/Np = \Phi \text{ and } M'/N'p' = \Phi \text{ implies } M/Np = M'/N'p',$$

where Φ is a positive constant. The constant is given by the equation of exchange,

$$\Phi = [\phi\beta - (1-\phi)]y/(1+\beta).$$

Third, in order for money to have value the supply of credit has to be greater than the demand at the monetary equilibrium interest rate,

$$N\phi\beta y/(1+\beta) > N(1-\phi)y/(1+r^{me})(1+\beta).$$

The excess of the supply of credit over the demand for credit at r^{me} (distance b-b' in Figure 5.4b) is held in fiat money. If this is not true, so the excess of supply over demand at r^{me} is negative, then money won't have value. Let's see what happens in that case. Suppose demand is greater than supply at r^{me},

$$N\phi\beta y/(1+\beta) < N(1-\phi)y/(1+r^{me})(1+\beta).$$

Simplify and rearrange,

$$(1+r^{me}) < (1 - \phi)/\phi\beta.$$

From equation (!), $(1 - \phi)/\phi\beta = 1+r^{nm}$. We have the implication that money won't have value in this environment when

$$\mathbf{1+r^{me} < 1+r^{nm}}.$$

This should make intuitive sense.

5.6 Population growth

How do our results change if the population is growing? Suppose the stock of lenders and the stock of borrowers both grow at the same rate, 1+n. The ratio of borrowers to lenders remains fixed in that case. So N'/N = 1+n. The non-monetary equilibrium in the credit market at time t is $N_t\phi s_L + N_t(1-\phi)s_B = 0$, and the term N_t drops out so the non-monetary equilibrium is not affected by this extension.

How about the monetary equilibrium? Our stationarity condition becomes p/p' = (M/M')(N'/N) = 1+n when population grows but the stock of money is fixed. By arbitrage, $1+r^{me} = p/p' = 1+n$, hence $r^{me} = n$. If population is growing, $1+r^{me} = 1+n > 1$. This makes it more likely that money will have value in this economy.

Why? Because it is now possible for the non-monetary interest rate to be greater than one in magnitude but still less than 1+n.

Substitute $1+r^{me} = 1+n$ into equation (**),

$M/Np = [\phi\beta - (1-\phi)/(1+n)]y/(1+\beta)$.

By taking the derivative of this last equation, we can show that real money balances are increasing in the population growth rate,

$d(M/Np)/dn = (1-\phi)/(1+n)^2 y/(1+\beta) > 0$.

Intuitively, when there are more young people, there is greater demand for money. If there is an increase in population growth, each new subsequent generation is larger, and the demand for real money balances increases thus increasing the value of the money.

Exercise: $\phi = 2/3$, $\beta = 1/2$, $y = 1$. Show that real money balances are zero when $n = 0$. Suppose $n = 1/2$. What happens to real money balances? Will people hold money? (Hint: calculate the non-monetary equilibrium interest rate and the interest rate in a stationary monetary equilibrium.)

5.7 Monetary expansion

Suppose the money stock is growing at rate $1+z$, $M_{t+1} = (1+z)M_t$, i.e., $M' = (1+z)M$. From before,

$p/p' = (M/M')(N'/N) = (1+n)/(1+z)$.

Population growth increases the return to money while an increase in the rate of growth of supply lowers the return to money. If $z = n$, $p/p' = 1$. In this case, the stock of money matches the increase in population.

As can be seen in the chart below, M2, a broad measure of the money stock, has been increasing steadily since the mid 1960s. The same is true of narrower measures of money like M1, which is made up of currency and demand deposits.