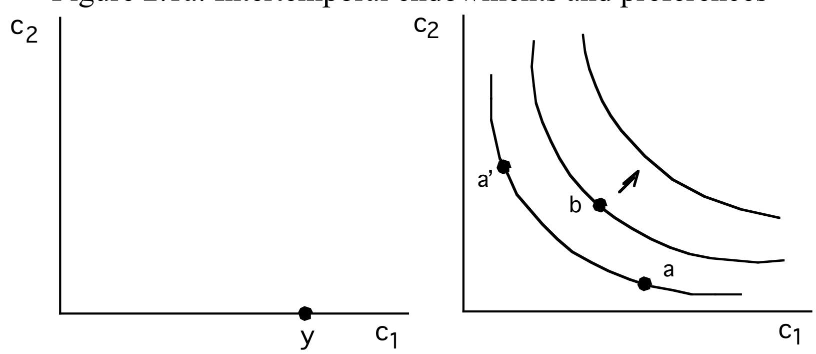
Ch 2. A Two Period Economy

We'll start by considering a simple two period economy. A lot can be learned from such a simple model. We can set up a decision problem for the agent and learn some simple tools that will help us solve the problem. We can introduce an equilibrium condition and determine how prices respond to various parameters. We can add a menu of assets and see how that alters the model. In addition, since there are two periods, something can happen in one period can affect what happens in the other period. Expectations about what will happen in the second period can be critically important. If people expect one thing to happen next period based on the fundamentals and the government is promising something else, the government's plan may be thwarted as a result. This may make it difficult, if not impossible, to impose a particular policy.

2.1 A simple model of savings

The economy lasts two periods. Suppose there is one good available and there are N identical individuals. Each individual lives for two periods and is endowed with y units of the good in the first period and none in the second period. Think of y as income. Let c_1 be the individual's consumption in the first period and c_2 be the individuals consumption in the second period. We have graphed the endowment in the left hand figure, where c_1 is on the horizontal axis and c_2 is on the vertical axis.

Figure 2.1a: Intertemporal endowments and preferences



Let (c_1, c_2) be a consumption bundle, a point in the right hand figure. Each individual also has tastes or preferences over bundles of consumption over time that are represented by a utility function, $u(c_1, c_2)$. We have depicted this in the right hand diagram. The indifference curves have two important properties, more is preferred to less so utility is increasing in the direction of the arrow where the individual receives more of both goods, and tradeoffs exist so the consumption bundles a and a' are equivalent in the sense that they yield the same satisfaction. If the agent is at point a and some c_2 is taken away, we can give him a free gift of c_1 that moves him to point a and he is as well off as before. Notice that a move from a bundle like a or a' to the middle of the figure increases utility. If the consumer is at a point like y, moving to a point to the northwest makes them better off since they are moving to a higher indifference curve.

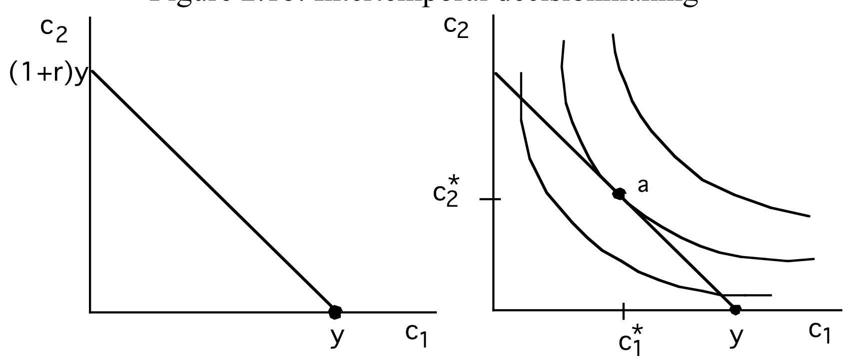
Next, suppose there is a storage technology such that if k units of the good are stored in the first period, $(1+r)k$ units become available in the second period where $r > -1$ and is fixed. Think of storage as ***saving*** for future consumption. We can write the consumer's budget constraints as $y - c_1 - k = 0$ and $(1+r)k - c_2 - 0$. Solve the first equation for k, $k = y - c_1$, substitute into the second equation, $(1+r)(y - c_1) - c_2 = 0$, divide by 1+r, $y - c_1 - c_2/(1+r) = 0$, and take the consumption terms to the right hand side,

$y = c_1 + c_2/(1+r)$.

This is the wealth constraint of the consumer. It equates the value of the endowment on the left to the present value of consumption on the right. We have graphed the budget line in the figure on the left. The two intercepts are (y, 0) and (0, (1+r)y). It has slope - (1+r). The higher r is, the steeper the slope, the lower r is, the shallower the slope. Obviously, the consumer wants r to be higher rather than lower. We'll look at a simple model in a moment where this may not be true.

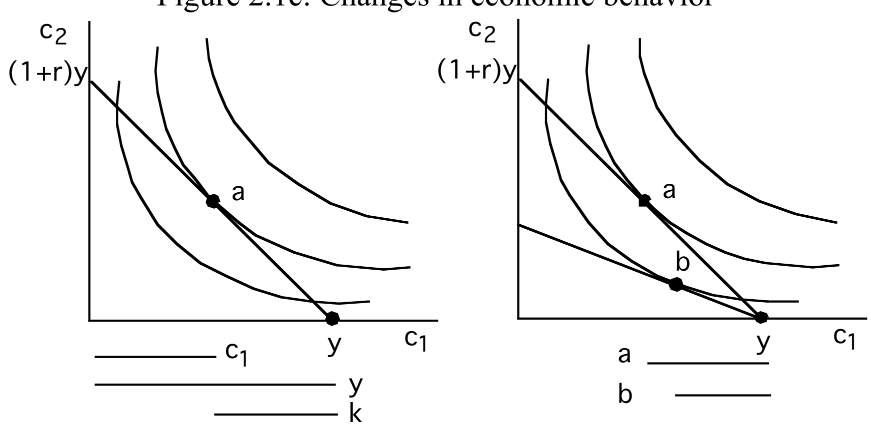
The consumer will maximize utility subject to the wealth constraint. Notice that one of the variables, r, occurs in the future, i.e., second period, when the individual is solving their decision problem in the first period. Since r was assumed fixed, it is easy to calculate what r will be next period. The result is a point of tangency like a in the right hand diagram. The solution to the consumer's decision problem is (c^*_1, c^*_2), as depicted in Figure 2.1b. Bundle a maximizes utility subject to the consumer's budget. No other bundle that is affordable will provide greater utility.

Figure 2.1b: Intertemporal decisionmaking



From the diagram below we can see that storage, or savings, geometrically is the gap between length y and length c_1. This follows since $k = y - c_1$. We can also see what happens when r changes. Suppose r decreases, as in the diagram on the right. The budget line swivels down and the consumer will move from point a to point b. Notice that savings or capital accumulation falls as we go from a to b.

Figure 2.1c: Changes in economic behavior



At the solution of the decision problem savings is a function of the interest rate, k(r). Changes in the interest rate will cause the consumer to alter her savings. Under certain circumstances, an increase in the interest rate increases saving and a decrease in the interest rate lowers saving.

2.2 Examples

Example 2.2a: $u = Log(c_1) + c_2$. Substitute the budget constraints for c_1 and c_2 to get $u = Log(y - k) + (1+r)k$. To maximize utility, differentiate and set the result to zero,

$$- 1/(y - k) + (1+r) = 0.$$

Solve this equation to get $k = y - (1/(1+r))$. This is the saving function. It tells us how much the consumer will store in the first period in order to consume in the second period. Notice that as r increases, $1/(1+r)$ decreases and $- 1/(1+r)$ increases so k increases as r increases, i.e., the supply of savings is "upward sloping." We can also obtain this result by differentiating the savings function,

$$\partial k / \partial r = (1/(1+r))^2 > 0.$$

Also, as y increases, savings increases. We can make this more precise by taking the derivative of the saving function with respect to y, $\partial k / \partial y = 1$.

Since $k = y - (1/(1+r))$ and $c_1 = y - k$, we can combine to get $c_1 = y - [y - (1/(1+r))] = 1/(1+r)$. This is a special version of the consumption function, $c_1 = 1/(1+r) > 0$. It follows that current consumption c_1 is decreasing in the interest rate.

Example 2.2b: $u = Log(c_1) + \beta Log(c_2)$. This is the case of the ***log utility function,*** which is quite popular in applied research. Show that $k = \beta y/(1+\beta)$. Can you also show that $c_1 = y/(1+\beta)$? (Hint: maximize $\{Log(y - k) + \beta Log((1+r)k)\}$ and solve the first order condition to get the savings function for k. Then substitute that into the first period constraint to get the consumption function, $c_1 = y - k$.)

As an aside, what would happen if the government taxed interest income? A tax on interest income is "like" a drop in r. How do you think the consumer will respond to a drop in r, hence to a tax on r? We'll discuss taxation of capital gains and interest income later in these notes. This is sometimes a major political issue, at least in most presidential elections in the US. When we lower such taxes the government must find other ways to finance spending. Creating money is one such method.

Example 2.2c: $u = (c_1^{1-\sigma} + \beta c_2^{1-\sigma})/(1-\sigma)$. This is the case of ***isoelastic utility function***, which is also widely used in applied work. Another way of setting u the mathematical optimization problem is to use the Lagrangean approach. The Lagrangean for this decision problem is

$$L = (c_1^{1-\sigma} + \beta c_2^{1-\sigma})/(1-\sigma) + \lambda[y - c_1 - Rc_2]$$

where $R = 1/(1+r)$ and λ is the Lagrange multiplier for the constraint. We choose c_1 and c_2 to maximize the objective function and we choose the multiplier λ to minimize the impact of the constraint on the maximization problem. The first order conditions are

$$c_1^{-\sigma} = \lambda,\ c_2^{-\sigma} = \lambda\beta R, \text{ and } y - c_1 - Rc_2 = 0.$$

These can be combined and solved to obtain the consumption function and the storage or saving function, $c_1 = [1+R^{1-1/\sigma}\beta^{1/\sigma}]^{-1}y$ and $k = \{1 - [1+R^{1-1/\sigma}\beta^{1/\sigma}]^{-1}\}y$. The parameter $1/\sigma$ is the so-called elasticity of substitution of intertemporal consumption for this utility function. Formally, this elasticity is defined in the following way. Suppose utility for c_1 and c_2 is $u(c_1, c_2)$ and the marginal utilities are defined as u_1 and u_2, respectively, i.e., $u_1 = \partial u / \partial c_1$. The elasticity is defined as

$$-[\frac{c_1/c_2}{u_1/u_2}\frac{d(u_1/u_2)}{d(c_1/c_2)}]^{-1}.$$

For the isoelastic utility function, $u_1 = c_1^{-\sigma}$, $u_2 = c_2^{-\sigma}$, $u_1/u_2 = (c_2/c_1)^{\sigma}$. Plugging into the formula and differentiating, the elasticity is $1/\sigma$.

The larger this elasticity is, the greater the substitutability of consumption over time. For a two period model, where a period is an economic span of thirty years, we would expect this elasticity to be low. For an individual with a T period planning horizon where T is large, e.g., T = 40 years, the elasticity might be much higher.

2.3 An extension

Suppose the storage technology is a bit more complicated. Assume that the return to storage depends on how much is being stored and diminishing returns exists. Suppose that the technology is such that $r = \eta/k$. Now the return falls with the amount of storage. With the utility function of example 2.2b we know that storage is $k = \beta y/(1+\beta)$. Combine this with the equation for r, $r = \eta(1+\beta)/\beta y$. The return to storage is increasing in the parameter η and decreasing in y and β. An increase in η improves the productivity of a unit of storage and raises the return. The parameter β for the log utility function is the discount factor. Higher β means the future matters more, savings and hence storage are higher, and the return is lower. Lower β means the future means less, savings and storage are lower and the return is higher as a result.

Later we will consider cases where the consumption good is produced via a neoclassical technology where capital is used to produce output. Profit maximizing firms will choose capital to equate the marginal product of capital to the cost of capital, r. Diamond (1965) considered such a model.

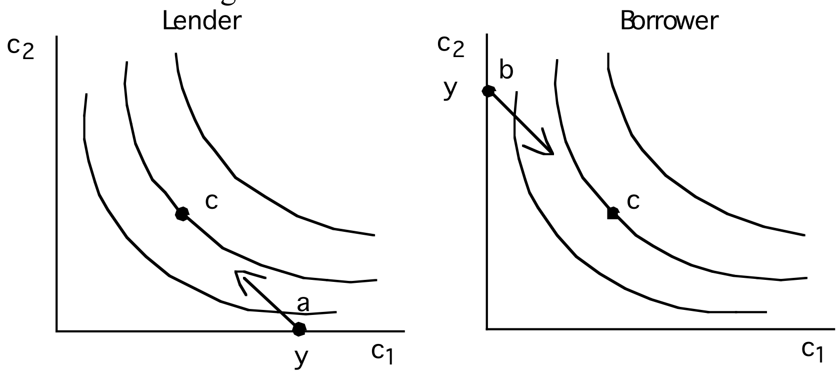
2.4 Borrowers and lenders[1]

Now assume there is no storage available. There are two kinds of people: lenders who are endowed with y units of the good in the first period and borrowers who are endowed with y in the second period. Suppose population is constant; there are N people, ϕN are lenders and $(1 - \phi)N$ are borrowers. A lender is endowed at point a in Figure 2.2a while a borrower is endowed at point b. Notice that a lender is better off moving to bundle c. A borrower is also better off moving to bundle c as well.

Both individuals have an incentive to trade with one another. Here's how it works. The lender has some of the good in the first period and the borrower doesn't but wants to get some of it so he can consume some of the good in the first period. The borrower has some of the good in the second period and the lender doesn't but wants to get some of it so he can consume in the second period. The lender gives some of his first period endowment to the borrower in exchange for receiving some of the good from the borrower in the second period, i.e., the lender makes a loan to the borrower and receives repayment next period plus interest. The loan contract moves both agents toward bundle c. Both are better off through trading.

[1] Thomas Sargent (1983) studied a more complex model of borrowing and lending than the one we will describe below in his graduate macro lecture notes.

Figure 2.4a: Borrowers and lenders



A lender is like the consumer of the last section. In fact, Figure 2.2b describes her decisions. Her budget constraints are $y - c_{1L} - L = 0$ and $(1+r)L - c_{2L} = 0$, where L is the amount of loans made and loans pay interest of r. Notice that she cannot consume in the second period unless she makes a loan. Loans are similar to the storage of the last section. However, there is a difference. The interest rate is no longer fixed but will be determined by the supply and demand for credit in the credit market. For the moment, assume that everyone knows what the interest rate will be in the second period when they are making their decisions in the first period.

The representative lender chooses consumption and loans to maximize utility subject to their budget constraint. Solve the first budget equation of the lender for L, substitute into the second equation, and rearrange to get the lender's wealth constraint just as we did in the last section, $y = c_{1L} + c_{2L}/(1+r)$. This is the same wealth constraint as in the last section. If we use the utility function in example 2.2b, her loan function, which we can also think of as her savings function, is $L = \beta y/(1+\beta) = S^L$, where 'S' stands for saving.

A borrower has almost the same decision problem. His constraints are $c_{1B} = B$ and $y - (1+r)B = c_{2B}$, where B is the amount borrowed. Notice, that he cannot consume in the first period unless he borrows. This is important. We can collapse his budget constraints to get a wealth constraint, $y/(1+r) = c_{1B} + c_{2B}/(1+r)$. Notice the difference between the borrower and lender; y enters the lender's constraint because she receives her endowment, or income, when young while $y/(1+r)$ enters the borrower's constraint since he receives his endowment when old so we discount his endowment by 1+r to write his constraint in present value terms. For the utility function in Example 2.2b, his decision problem can be written as $\max\{\text{Log}(-B) + \beta\text{Log}(y - (1+r)B)\}$. The first order condition is $-1/B + \beta(1+r)/(y - (1+r)B) = 0$. Rearrange to get, $y - (1+r)B = \beta(1+r)B$, and solve to get the borrowing function, $B = y/(1+r)(1+\beta)$. Notice that if r is higher, $1/(1+r)$ is lower so B is lower, i.e., the demand for credit is decreasing in its price r so demand slopes downward.

We assumed that everyone knew what the interest rate would be in the second period when they solved their decision problems in the first period. This is an example of ***perfect foresight.*** More generally, an agent has an expectation about a future variable that depends on the information they have available now. For example, you know what your salary is right now. What is your expectation about your future salary? Individuals can take a pretty good guess once they enter the permanent job market after school. They can see what salaries are earned by coworkers and friends who are further into their careeers. There is plenty of information available that allows them to make such a calculation.

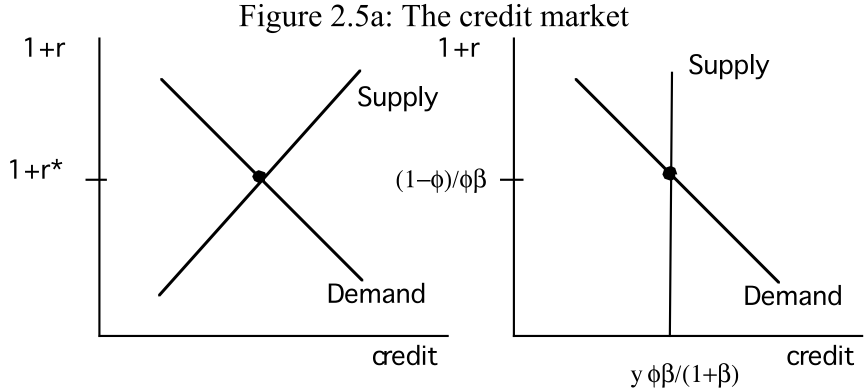
What is a bit harder is forecasting aggregate variables like interest rates, future inflation, and future tax policy. Typically, researchers employ the ***rational expectations*** hypothesis. This hypothesis states that rational agents collect and use information efficiently to make their forecasts using economic theory and statistics, and then choose their behavior accordingly. For example, if a person believes that a tax cut today is only temporary and will most likely be followed by a tax increase next year, then it may be optimal for them to save most of the tax cut today to pay the future tax increase. On the other hand, if they do not expect a tax increase next year, then they may consume most of the tax cut today.

As another example, if an individual expects demographic pressures on social security to force benefit cuts in the future, they may save more in private accounts like IRAs now for their own retirement. We will assume individuals have perfect foresight for much of what we do in the rest of these notes. They "expect" a certain value for the interest rate, for example, and then choose behavior that brings that expectation about in equilibrium. So we can calculate an equilibrium, assume the agents in the model understand what the equilibrium is, and that they will choose actions to bring it about. This is sort of a "self fulfilling" prophecy for the economy.

2.5 Equilibrium in the borrowing lending economy

An equilibrium is a situation where market prices are such that the supply of credit provided by the lenders equals the demand for it on the part of the borrowers. Suppose we normalize on the population by assuming N = 1. Then the total amount of supply is equal to ϕL and the total amount of demand for credit is $(1 - \phi)B$. If there is an equilibrium interest rate it will occur where $\phi L = (1 - \phi)B$ as depicted in Figure 2.3a on the left. This is the ***equilibrium condition***. If we didn't normalize on population we would have $\phi LN = (1 - \phi)BN$ instead and the N's cancel anyway. In general, supply slopes upward and demand slopes downward. In the special case considered in the right hand diagram, supply is fixed.

Figure 2.5a: The credit market



With the log utility function we have $L = \beta y/(1+\beta)$, which is a constant relative to the interest rate and $B = y/(1+r)(1+\beta)$, which slopes downward. These are depicted on the right in Figure 2.3a. It follows that $\phi L = \phi\beta y/(1+\beta)$ and $(1 - \phi)B = (1 - \phi)y/(1+r)(1+\beta)$. Substituting into the equilibrium condition we have

$$\phi\beta y/(1+\beta) = (1 - \phi)y/(1+r)(1+\beta).$$

Solve for 1+r to get

$1+r = (1 - \phi)/\phi\beta$.
So the equilibrium interest rate that clears the market depends on the ratio of borrowers to lenders, $(1 - \phi)/\phi$, and also on β. If the ratio of borrowers to lenders goes up, i.e., if there is an increase in demand for credit relative to supply, the interest rate goes up. To see this, shift the demand curve up.

The solution also depends on β. What is β? The utility function is $u = Log(c_1) + \beta Log(c_2)$. The parameter β informs us of the psychological tradeoff between present and future consumption. If $\beta = 1$, then c units of consumption today delivers utility of Log(c) today and c units consumed in the future delivers Log(c) in the future. If $\beta = 1$, consuming c today or c in the future yield the same utility from today's perspective. However, if $\beta < 1$, c units of consumption today still delivers Log(c) utility today but c units consumed in the future only delivers $\beta Log(c) < Log(c)$ in the future. When β is less than one, future utility means less today than utility today. The parameter β is referred to as the ***discount factor***. It tells us how important the future is to the individual. The larger β is the more important the future is to the person.

If β is large, then $1/\beta$ is small so the interest rate is small when β is large. What is the intuition? When β is large, the future matters more to both the borrower and lender so the borrower borrows less and the lender lends more. Demand shits down and supply shifts out as a result. Both reactions lead to a decrease in the interest rate. To see this, shift the appropriate curves.

We assume that each agent understands the workings of the credit market and has enough information to allow them to calculate the interest rate and then act accordingly. The formula for doing so is the equilibrium condition, $1+r = (1 - \phi)/\phi\beta$. So an agent needs to know ϕ and β and can then use the formula for 1+r to calculate the equilibrium interest rate. Since everyone is identical in the model this is easy to do in this particular case. More generally, we assume people understand economic theory and statistics and use them to calculate their expectations of important variables like interest rates and such. These assumptions constitute our model of their behavior.[2]

2.6 Two assets

An issue arises as to what happens if there is more than one asset. A theme that we will see time and again is that the rates of return across assets must be equal in some sense if consumers are going to hold the various assets. If the rate of return for one asset is higher than another asset, no one will hold the asset with the lower return. There may be exceptions to this but it is typically true.

To see this suppose consumers have access to a storage technology and can also make loans in the credit market. Once again, we will assume there are two types of agent, lenders and borrowers. For a lender there are two vehicles for saving, storage and making loans. Let storage pay a return of $1 + \rho$ and assume a loan pays 1+r. The budget

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[2] We model their behavior in this manner to develop our theories and focus on the predictions. This is very similar to Milton Friedman's famous example of the pool player. A pool player doesn't know anything about physics or Newtonian mechanics and yet behaves as if he does when playing the game. We could write down a mathematical model involving physics and use the model to explain how the pool game is played even though the actual players playing the game do not know this math. It is the predictions of the model that matter, not the assumptions.

constraints for a lender are $y - L - k - c_1 = 0$ and $(1+r)L + (1+\rho)k - c_2$. If $r > \rho$, no one will store the good and we will only observe loans. If $r < \rho$, no one will make loans and we will only observe storage. Suppose $r = \rho$. In this case, the two assets are ***perfect substitutes*** and the lender is perfectly indifferent between the two. The two budget constraints become $y - c_1 = k + L$ and $(1+r)(k + L) = c_2$. Since the two assets are perfect substitutes, we cannot determine separate demands for each of them. We can only determine the total, $k + L$. We will see this problem arise again when we consider a group of assets including money.

2.7 A credit crunch

What happens when credit dries up all of a sudden? How can we model this? What does the model predict? One simple way of modeling this is to assume a dramatic decline in the resources of the lender, which is similar to a bank.

For this purpose, it is useful to distinguish the variables between borrowers and lenders to look at their influence individually. Let each lender be endowed with y_L units of the good in the first period and none in the second, and let each borrower be endowed with y_B units of the good in the second period and none in the first. In general, we'll assume that $y_L \neq y_B$. We can go through the same exercise as in section 2.3 and find that $L = \beta y_L/(1+\beta)$ and $B = y_B/(1+r)(1+\beta)$ when borrowers and lenders choose optimally. The equilibrium condition is

$$\phi\beta y_L/(1+\beta) = (1 - \phi)y_B/(1+r)(1+\beta).$$

Simplify and solve for 1+r,

$$\phi\beta y_L = (1 - \phi)y_B/(1+r),$$

$$1+r= [(1 - \phi)/\phi\beta][y_B/y_L].$$

This last equation tells us everything we need to know about how the equilibrium will adjust to a change in circumstances. Once again, the ratio of borrowers to lenders is given by the term $(1 - \phi)/\phi$. A larger ratio of borrowers to lenders increases the net demand for credit and raises the interest rate. β is the discount parameter, as before. A higher discount factor makes the future more valuable and loans increase while borrowing falls. Both actions lead to a lower interest rate. The new element is the ratio of the borrower's income to the lender's income. If the average lender's income increases, the supply of loans will increase and this will reduce the interest rate. If the borrower's future income increases, it ill want to borrow more and this will cause the interest rate to rise.

Suppose we interpret the lender as a bank. A large drop in its assets means the bank has less money to lend. This is like a large decrease in y_L. Our model predicts that this will lead to a decrease in the supply of credit and upward pressure on the interest rate. Indeed, if there is a large drop in the bank's earnings, this can cause credit to dry up. If credit dries up, borrower's are much worse off.

Notice how the individuals are affected by a change in the interest rate. Suppose credit dries up for some reason and the interest rate rises. Lenders who are still making loans receive higher interest and are better off. Borrowers who are still in the market and able to get a loan pay higher interest and are worse off. It follows that the interest of lenders and borrowers are diametrically opposed to one another vis-a-vis the interest rate. Therefore, a government policy that affects the interest rate will affect the two groups in very different ways.

2.8 Moral hazard and adverse selection and banks

In a world of uncertainty two problems almost always arise when people have different information about a given activity, moral hazard and adverse selection. Moral hazard exists when the availability of insurance alters behavior so as to bring a bad state about and adverse selection is a situation where more of the bad risks desire insurance than the good risks.

When individuals in a market have different information, problems can arise. Consider the car insurance market. A good state is when you drive without an accident and a bad state is when you have an accident. Insurance companies offer policies to protect against financial difficulties and personal injury when you have an accident. You pay premiums to the company over time and the company pays to have your car fixed when you have an accident. Obviously, the more careful you are, the lower the probability you will have an accident and the company will have to pay out. You know more about your own driving ability than the company and so information is said to be asymmetric between the buyer and the seller. To overcome this the company obtains information about you and your driving record so it can charge the appropriate premium, e.g., grades, age, general driving area, type of car, home ownership, and so on. It may also obtain other information that is correlated with driving some of which may not be obvious. For example, people who own their own homes tend to have fewer accidents, and people with good credit ratings tend to have fewer accidents. Auto insurance companies will charge such people lower rates, ceteris paribus. When you get a speeding ticket or have an accident, this provides new information to the company and they adjust your premium as a result.

In the car insurance example knowing that you are covered by insurance may cause you to drive less carefully raising the probability of an accident so moral hazard may exist. Second, more of the bad risks would prefer to buy the insurance than the good risks so adverse selection probably exists. These problems make it more difficult for a company to provide the insurance and in some cases, e.g., unemployment insurance, it may be impossible to provide and the market collapses.

How does this affect banking? Repayment of a loan is uncertain. Economic conditions are not known with perfect certainty and a business plan that looked good on paper may not lead to profitable performance. Who is most likely to seek a bank loan, a business that is thriving on its own, or a business that may be in trouble and desperately seeking additional financing to stave off bankruptcy? Probably the latter, so adverse selection exists. Once the loan contract has been signed, what ensures that the loan is actually used as the business stated in its plan to the bank? It is impossible for a bank to perfectly monitor how a loan is used so moral hazard probably also exists. So both moral hazard and adverse selection exist in banking and banks have traditionally responded by obtaining information on the ability of the borrower to repay the loan, e.g., income, wealth, demand that collateral be put up, e.g., house or car, in the event that repayment by the borrower does not occur. In other cases, e.g., student loans, banks may require that the borrower get someone to co-sign the loan and repay it in the event that the borrower does not.

In the recent subprime lending debacle many finance companies were not collecting information on the borrowers, or only collecting minimal information, but signing contracts anyway. In other cases, borrowers were lying and finance companies were not checking. If the borrower couldn't make the payments the company would foreclose and seize the house as collateral. The contracts were aimed at people who were traditionally not able to get bank financing for a mortgage, i.e., people with low income. The contracts started with a low rate and contained a clause that allowed the company to increase the rate after a certain amount of time had elapsed. When rates began rising many homeowners couldn't pay and finance companies started foreclosing. Since the contracts were aimed at low income people, many of these foreclosures occurred in poor neighborhoods. Since many of these properties were coming on the market at the same time the supply was too great and house prices started falling. And since they were concentrated in poor neighborhoods, the potential buyers were also poor risks and finance companies started pulling back.

The problem was exacerbated as the housing bubble burst. Many banks and finance companies took huge write-offs as a result. For example, in early November 2007, Citigroup, one of the largest banks in the US, announced it would write-off $8 - 11 billion because of the subprime loan problem. Another large bank, Bank of America, announced in April, 2008 that it was writing off close to $2 billion due to losses in debt underwriting and other financial operations. And, of course, it also caused the collapse of Bear Stearns, the large investment bank, which was bought out by JPMorgan Chase.

2.9 A reinterpretation of borrowing

Recall our model of borrowers and lenders. The lenders are essentially making loans or saving and the loan contract with a borrower is the means of saving. So if we let s_L be the lender's savings, $s_L = L$. Borrowers are undertaking ***negative saving*** when they borrow. So the "saving" of the borrower is $s_B = - B = - c_{1B}$ (since $B = c_{1B}$). The equilibrium is $\phi L = (1 - \phi)B$, or $\phi L - (1 - \phi)B = 0$, which is also equivalent to $\phi s_L + (1 - \phi)s_B = 0$, i.e., the sum of saving across all savers, some of whom may be borrowing, must equal zero in equilibrium.

With the log utility function of example 2.2b, $s_L = \beta y/(1+\beta)$ for the lender and $s_B = - Ry/(1+\beta) < 0$, where $R = 1/(1+r)$. The equilibrium condition becomes

$$\phi\beta y/(1+\beta) - (1 - \phi)Ry/(1+\beta) = 0.$$

This can be solved for $1+r = (1 - \phi)/\phi\beta$, which is exactly what we got before. Defining the borrower's saving as $s_B = - B$ will be convenient later on.

2.10 Money

Let's consider another asset, fiat money. Fiat money is money that is without intrinsic value in the sense that it's only value is that someone will accept it as a means of payment in the future. It is not backed by anything and it is not valued because it is printed on brightly colored paper, or has some other value like gold that can be used in jewelry. As it turns out, it is difficult to get money into a model with agents who optimize when there are other assets in the model that pay a positive rate of return.

Consider our simple borrowing lending set up. There are some borrowers and some lenders, and the economy lasts for two periods. Lenders are only endowed with some of the one good in the first period and borrowers are only endowed in the second period. There is no other asset and no storage possibilities. Lenders would like to consume in the second period but can't unless they can make a trade. Borrowers would like to consume some of the good in the first period but can't unless they can make a trade. Will money allow them to achieve a mutually beneficial trade? Agents will only hold money if they think someone else will accept it later on in exchange for some of the good. If they think no one will accept the money in the future, they won't hold the money now since it is intrinsically worthless. Suppose the government prints up some pieces of paper and calls them money. We will ignore counterfeiting so we will assume it is impossible to counterfeit the money. The money is intrinsically worthless so people don't value it because it is printed on pretty paper or backed by anything.

Suppose the government gives the money to the lenders at the beginning of the first period. What will the lenders do with it? Suppose they give the money to the borrowers. What will the lenders get in exchange for it? Nothing, since the borrowers have nothing to give in the first period. The lenders could trade amongst themselves. However, this won't help the lenders increase their consumption in the second period. So giving the money to the lenders doesn't help the situation.

Suppose the government gives the money to the borrowers at the beginning of the first period instead. The borrowers could give the money to the lenders to buy some of the good in the first period and the lenders could give up some of their endowment in the first period to get the money. The lenders then have money that they can use to buy some of the good from the borrowers in the second period.

What happens when the second period arrives? The borrowers have their endowment of the good and the lenders have the money. The lenders want to exchange the money for some of the good. Will the borrowers want to make such a trade; will they accept the money? Probably not. Why? Once the borrowers give up some of their endowment for the money in the second period in exchange for the money, what will they then do with the money? Nothing, since there are no other trades to make. Remember the economy only lasts two periods. Knowing this, the borrowers won't accept the money in the second period. However, the lenders can figure this out in the first period and won't trade with the borrowers in the first period. ***Money won't have value in this economic environment.*** Not to put too fine a point on it, but lenders won't exchange some of their endowment for money in the first period because they are smart enough to figure out that the borrowers won't accept the money in exchange for the good in the second period. The borrowers won't accept the money since they will have no one to exchange it with.

It doesn't help if the economy lasts T periods either. Suppose it does. What will happen in period T? Borrowers will not give up some of their endowment in period T since there is no trade to make in period T+1, and the lenders in period T-1 understand this. Lenders will be unwilling to accept the money at T-1 and the money won't be used to make exchanges. This differs dramatically from the last chapter. In the models considered in that chapter, the economy continued beyond the lifetime of a single agent. New agents coming along wanted to acquire the money since they expected agents to accept it in the future and the economy lasted forever; there was no last period. This is a critical difference.

The purpose of considering this example is to illustrate the kind of thinking agents undertake when acquiring assets. Will an asset have value in the future? Who will want to hold the asset? How much value will the asset have? Is it's value expected to increase, e.g., tech stocks in 1998, or fall, e.g., houses in 2008? What happens if the value of the asset changes? Will agents still be willing to hold it?

2.11 Money in the utility function

One popular method of "getting money into the model" is to assume agents obtain utility from real cash balances. For example, $u(c_1, c_2, m/p)$. This might be justified by arguing that money facilitates exchange and reduces shopping time. Lucas and Stokey (1984) made this assumption and then analyzed how the system works.

One problem with this is immediate. Why fiat money? Why not some other asset that dominates money in its rate of return like Treasury bonds? Why not put the individual's stock of T-bills in the utility function? Or, why not some private asset like a share in a mutual fund of blue chip stocks? These are difficult to counterfeit and do not have intrinsic value and yet appreciate in value and so dominate money. Of course, another problem is that no one will accept money in the second period of a two period economy regardless of whether it reduces shopping time in the first period. We will forego making this assumption in much of what follows.