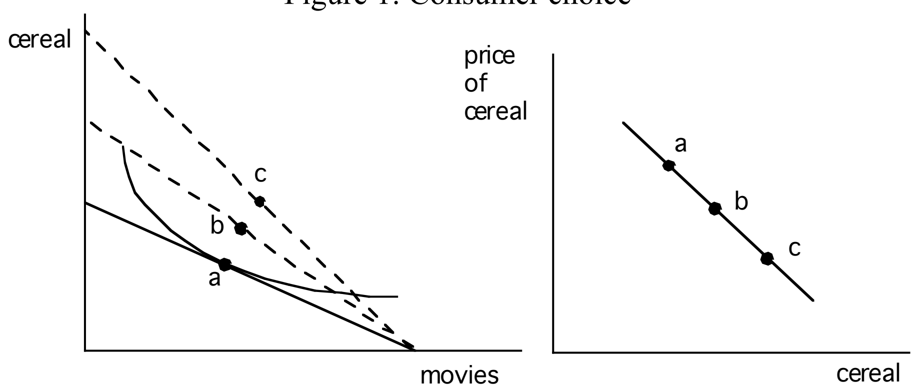
Ch1 A First Look at Money

Money is not like other commodities. For most commodities, we generally assume a consumer has preferences for specific goods and services like pizza, shoes, haircuts, leisure, and movies, and they are confronted with a budget constraint. We imagine the consumer solves a constrained maximization problem and from this we derive the demand function.

We depict a decision between movies and cereal in Figure 1. The consumer has utility for movies and cereal, u(movies, cereal) and a budget constraint. The consumer chooses point a in maximizing utility subject to her budget constraint. This matches up with point a in the right hand frame. As the price of cereal falls, the budget line swivels out, and the consumer re-optimizes choosing bundles b and then c. This allows us to derive the income-constant demand curve for cereal, which relates the price of cereal to the quantity consumed. As we move from a to c the price is falling and utility is increasing in both frames.

Figure 1: Consumer choice



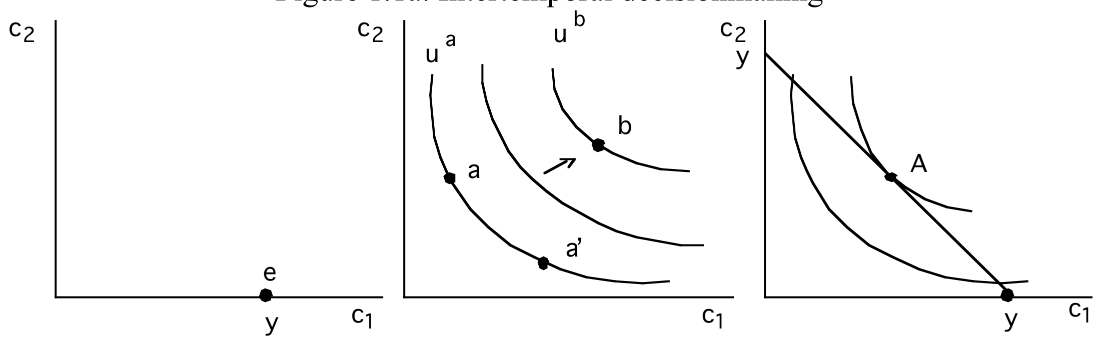
Money is not like cereal or movies. ***Fiat money*** is not backed by anything of value like a stock of gold in Fort Knox. People don't generally hold money because it is printed on nice paper, or they like the design of the bill. It does not have intrinsic value like movies or cereal. People hold money because they believe that someone else will accept it in the future in exchange for real goods and services such as cereal and movies. It acts as a **unit of account, medium of exchange,** and as a **store of value**. As a store of value it is very similar to other assets like stocks, bonds, mutual funds, land, and even simple time deposits. However, as an asset money is also peculiar since it doesn't pay a dividend or interest, or appreciate in value like land. In fact, if there is inflation, money's return is like a time deposit paying negative interest. Even if there is no inflation, fiat money is dominated by other assets. For example, US Treasury bonds are virtually risk-free and pay interest thus dominating money in its rate of return. Why do people hold fiat money?

One reason for holding such an asset is that it facilitates trade that improves welfare. We will study environments where there is trading friction, i.e., costly trading, and see what role money can play. The Samuelson (1958) - Allais overlapping generations model is one such case. Agents are born and are endowed at the beginning of the life cycle and experience a finite lifetime. The environment of the model is such that trading cannot take place and the resulting competitive equilibrium of autarky (no trading) is not optimal. Money, and other institutions like social security, can facilitate trade that improves welfare. Agents may also hold money because of legal restrictions like reserve requirements, minimum denomination restrictions, and other capital controls.[1]

1.1 A simple model of storage

Let's consider the following simple model. The economy lasts forever and time is discrete so t = 1, 2, 3, ... There is one good available, call it the consumption good. It can be consumed or it can be traded for something else. There are N people born in each period. Each lives a life cycle of two periods, young and old. At any point in time there are N young people and N old people. We will assume that each person is endowed with y units of the consumption good in the first period when they are young and none when old. Think of y as **income**. This is depicted in the left hand diagram in the following figure as point e. So at any moment in time Ny units of the good are available to the economy.

Figure 1.1a: Intertemporal decisionmaking



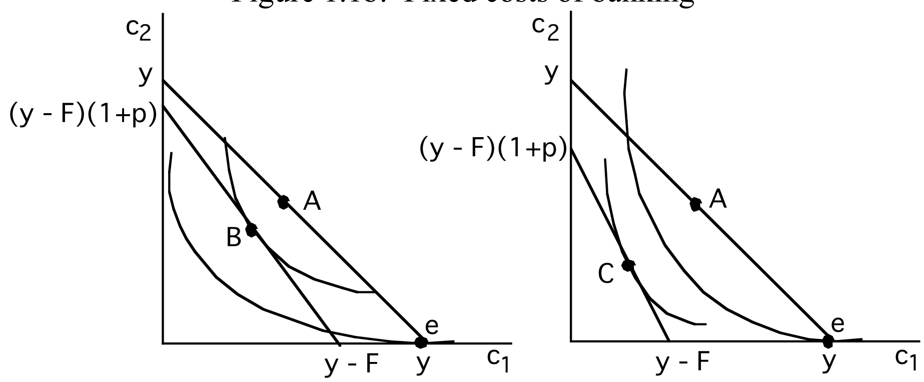
Consumers care about their consumption of the good in both periods of life according to a well defined utility function, $u(c_{1t}, c_{2t+1})$, where c_1 is consumed in the first period at time t and c_{2t+1} is consumed in the second period of life at time t+1 for a person who is born at time t. We will assume that consumers accept tradeoffs between consumption today and consumption in the future and that more is always preferred to less. So the indifference curves slope downward and utility increases in the direction of the arrow. This is depicted in the middle frame. Points a and a' involve different amounts of consumption in the two periods but are considered psychologically equivalent in the sense of producing the same amount of happiness or satisfaction. Bundle b is on a higher indifference curve and so involves a higher level of utility, $u^b > u^a$. In fact, since any point on indifference curve u^b is equivalent to b and any point on the u^a curve is equivalent to a, any point on u^b is preferred to any point on u^a.

A consumer would like to move to the northeast in each diagram as much as possible because more is preferred to less. However, they are constrained by their endowment and the trading possibilities open to them. Suppose there is a "storage" technology available whereby if k units of the good are stored in the first period of life, $(1+\rho)k$ units become available in the second period where $\rho > -1$. Suppose for simplicity that $\rho = 0$ so a unit of the good stored yields one unit of the good next period. The

[1] See Wallace (1983,1984)

consumer has two budget constraints, $y = c_1 + k$ and $(1+\rho)k = c_2$. If $k > 0$, solve the second constraint for k, $k = c_2/(1+\rho)$, and substitute into the first constraint, $y = c_1 + c_2/(1+\rho)$. This is the consumer's wealth constraint; the present value of the endowment is equal to the present value of consumption, where $1/(1+\rho)$ is a "discount" factor. This is graphed as the straight line through point A in the right hand panel of the figure for $\rho = 0$. The consumer can choose any bundle that is affordable in a lifetime sense and chooses A since this bundle maximizes utility. The consumer can do no better choosing another bundle. We can interpret storage as a bank. The agent takes k units of the "good" and stores it at the bank. Next period the agent goes to the bank and withdraws k units and consumes it. We can generalize this analysis a bit. Suppose storage at the bank leads to a return of $\rho > 0$ but there is a fixed cost the bank charges for storage F paid when the good is dropped off. The consumer's constraints become $y - F - c_1 - k = 0$ and $(1+\rho)k = c_2$.

Figure 1.1b: Fixed costs of banking



Point A matches up in all the diagrams. We cannot attain point A if there is a fixed cost to storing the good at the bank. Instead, we can attain the bundle at point B.[2] Notice that if the fixed cost of storing at the bank is small, the indifference curve through point B is above the indifference curve through the endowment point e. The individual is better off storing the good and consuming at B than staying at e. This is depicted in the diagram on the left. However, if the fixed cost of storing the good at the bank is too high, agents will not store the good but simply consume their own endowment. Notice on the right that the indifference curve through the endowment is above the indifference curve through point C. The agent can store the good at the bank and consume at C or not make any transactions and consume at point e. If the fixed cost of trading is too high, the agent won't trade. This is known as **autarky**. One major role of financial assets is to lower the fixed costs associated with trading.

1.2 A simple life cycle model

When the indifference curves have their usual shape in this intertemporal context we are implicitly assuming that a consumer prefers a mixed bundle in the middle of the first quadrant to a corner on one of the axes. We assumed the consumer was endowed in the

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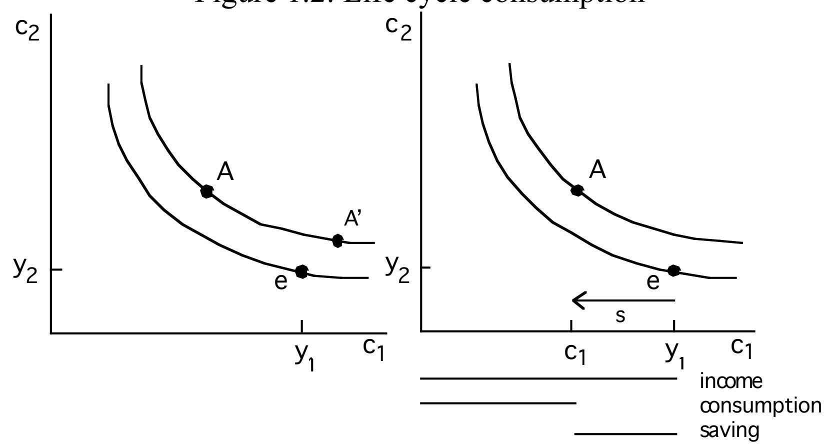
[2] The budget constraints are $y - F = c_1 + k$ and $(1+\rho)k = c_2$. Solve the second for k and substitute into the first to get, $y - F = c_1 + c_2/(1+\rho)$. If $c_2 = 0$, we have $c_1 = y - F$ so the horizontal intercept is the point $(y - F, 0)$. If $c_1 = 0$, we have $c_2 = (y - F)(1+\rho)$ so the vertical intercept is $(0, (y - F)(1+\rho))$.

first period. Suppose instead that the individual is endowed with y_1 in the first period and y_2 in the second period. In figure 1.2a the individual is endowed at point e where $y_1 > y_2$. However, he would prefer to consume at point A. Notice that A' is preferred to e since more of both goods are consumed at A'. Point A and A' are equivalent so A is preferred to e as well.

The consumer receives a stream of income (y_1, y_2), which differs from the preferred consumption bundle (c_1, c_2) in general, i.e., $(y_1, y_2) \neq (c_1, c_2)$. The consumer would like to consume "smoothly" over the life cycle such as at a point like A. However, income may not match up well with the intended consumption. The consumer can smooth her income across periods by acquiring financial assets. She can borrow in period when she would like to consume more and save in periods when her income is higher than her consumption. In Figure 1.2 $y_1 > c_1$, so the consumer can save the difference and acquire assets. Next period she can use the assets plus the asset income to consume more than her income, $c_2 > y_2$. (The arrow distance in the figure denotes saving.)

Figure 1.2: Life cycle consumption



There is some evidence that a large number of consumers conform to the predictions of the life cycle model. Campbell and Mankiw (1993) found that more than half of their sample conformed to the permanent income life cycle hypothesis.

Exercise: How does the budget line respond when y_1 increases? y_2 increases? r increases? How does the consumer respond to these changes? This is an example of comparative statics.

1.3 A first look at money

Let's be more specific about the general structure of the model. At time t = 1 there are N initial old consumers and N initial young consumers. Only the young are endowed with the good; each has y units. There is no storage, i.e., $\rho = -1$. The agent is endowed at a point like e in Figure 1.1a. Since a balanced consumption bundle (point A), where some of each good is consumed, is preferred to being at an extreme bundle (e), where only one good is consumed, the agent would like to trade with someone. However, there is no one to trade with! The old don't have any of the good and the young agents are all the same, i.e., they have the same preferences and endowments. This is an example of **trading friction**. No trades occur and the agent ends up simply consuming the endowment.

Now suppose the government prints up M pieces of paper and gives them to the initial old generation. Each initial old agent receives M/N = m pieces of paper. The paper has no value in and of itself; it is **intrinsically worthless**. It is not backed by anything, e.g., stock of gold. This is also known as **fiat money**. The initial old can now use the paper to buy some of the consumption good from the young at time t = 1. The initial young acquire the paper by trading some of their endowment of the good for it. At the end of the period the initial old leave and the initial young become old and a new young generation is born. The initial young who are now old at t = 2 now have money they can use to buy some of the good from the new young generation at t = 2. They can move to a point like A in Figure 1.1a. The initial old are unambiguously better off since they get to consume some of the good. The initial young are also unambiguously better off since they move from the endowment point e in Figure 1.1a to a point like A. The process continues as time progresses; each period the young give up some of their endowment in exchange for the paper and then in the second period of life use the paper to buy some of the good from the new young generation. Fiat money is being used to make exchanges and it also acts as a store of value. Trading is facilitated and everyone appears to be better off.

The representative young consumer's budget constraints in the first generation are $p_1(y - c_1) - m = 0$ and $p_2c_2 = m$, where p_1 is the price level of the consumption good (dollars per unit of the good) in period one and p_2 is the price level in period 2. The endowment and consumption, y, c_1, and c_2, are denominated in real units, i.e., units of the consumption good. The prices p_1 and p_2 are in units of currency per unit of the good, e.g., dollars per unit of consumption. So the product p_1c_1 is $(\$/c_1)c_1 = \$$, and similarly for p_2c_2. Prices convert consumption units to units of the currency. Notice that m is the number of dollars the agent has acquired so m/p is dollars divided by ($/good) or units of the good. It follows that m/p is ***real money balances*** or how much of the consumption good one can buy with m dollars when the price level is p.

For the initial young generation m/p_1 is real money balances denominated in units of the consumption good in the first period and m/p_2 is real money balances in the second period. If there is a burst of inflation between periods for some reason, $p_2 > p_1$ and real money balances deteriorate in value from m/p_1 to m/p_2, i.e., money loses value with inflation. Substitute the second budget constraint into the first and rearrange, $y = c_1 + (p_2/p_1)c_2$. This is the initial young agent's wealth constraint. In a sense the ratio p_2/p_1 plays the same role as the discount factor $1/(1+\rho)$ in our model of storage. Also notice that $p_2/p_1 - 1$ is the inflation rate.

Next period at t = 2 the initial young agents are now old and each has m/p_2 real money balances that they can use to buy the consumption good from the second-period young. The second-period young are endowed with the good and they trade some of it to acquire the money if they believe that the third-period young are willing to acquire it from them next period at t = 3. The third-period young are willing to acquire the paper if they think the fourth-period young are willing to trade for it at t = 4, and so on into the infinite future. Money has value in this setting if agents today believe that agents in the future are willing to accept it in exchange for the good. Each new young generation coming along is also better off using the money because they can move from their endowment point e to an interior point like A and achieve a higher level of utility. Use of money in this environment can make everyone better off. This was Samuelson's (1958) main point; fiat money can overcome trading friction.

1.4 Stationary monetary equilibria

We can think of the ratio of p_1/p_2 as the return to holding money. Is it possible to figure out what the ratio p_1/p_2 is in the first period, and more generally, the ratio p_t/p_{t+1} at time t? Yes, under certain circumstances. This is based on the agent's expectations and the agent's actions based on those expectations as well as the expectations of the group of agents and the actions that are based on those beliefs. Suppose that everyone knows that every generation coming along in the future has the same number of people with the same endowment. There is no source of randomness. What expectation would an individual in that environment have? A reasonable expectation might be that the price level would be stable so that $p_1 = p_2$ and $p_1/p_2 = 1$. Each agent then chooses his or her economic behavior accordingly and in the aggregate if everyone has that expectation and acts accordingly, then that will bring about an equilibrium where $p_1 = p_2$ and $p_1/p_2 = 1$. In other words, in this simple setting, if agents alive now expect agents in the future to accept the money, then the money today will have value, and if agents now think the value will be constant in the future, then it will be constant today. What is going on is that there is a strong link between expectations of future events, actions taken today, and the resulting equilibrium.

To make this more precise, we will seek what is known as a stationary monetary equilibrium. This is where real per capita money balances settle down to a constant, $M_t/N_tp_t = \Phi$, a constant, where M_t is the stock of money at time t, N_t is the population of young people acquiring the money at time t, and p_t is the price level at time t. If real per capita money balances are equal to a constant Φ at time t, in a stationary monetary equilibrium they are also equal to Φ at time t+1, $M_{t+1}/N_{t+1}p_{t+1} = \Phi$. This implies

$$M_{t+1}/N_{t+1}p_{t+1} = M_t/N_tp_t.$$

Rearrange to get

$$p_t/p_{t+1} = (N_{t+1}/N_t)(M_t/ M_{t+1}).$$

If the population of young agents grows at rate 1+n, then population evolves according to $N_{t+1} = (1+n)N_t$. If the money supply grows at rate 1+z, then $M_{t+1} = (1+z)M_t$. Substituting this information into the price equation,

$$\mathbf{p_t/p_{t+1} = (1+n)/(1+z).}$$

If agents know that money is growing at a given rate and the population is growing at a given rate, and endowments over time are stable, then it seems reasonable for them to formulate the expectation that the return to holding money is (1+n)/(1+z). They will then act on that expectation. In the aggregate, if everyone has the same expectations, i.e., uses the same economic theory to forecast, then their actions, e.g., life cycle consumption and the acquisition of assets, in the aggregate will bring about an equilibrium where $p_t/p_{t+1} = (1+n)/(1+z)$.

Earlier, we assumed the supply of money was fixed so z = 0 and the population was constant so n = 0. Therefore,

$$p_t/p_{t+1} = 1.$$

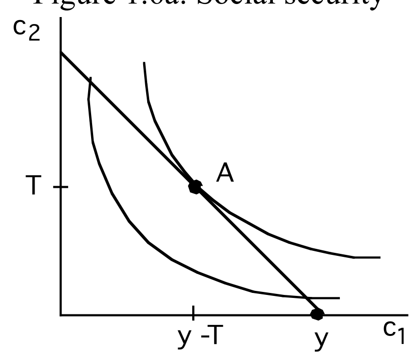
This is the rate of return to holding money in an environment where y, M, and N are fixed. The young consumer's wealth constraint is $y = c_1 + c_2$. The slope of the budget constraint is - 1. Figure 1.1a depicts their choices.

Money facilitates trade across generations and allows each generation to attain a higher indifference curve by making trades available that would not otherwise have been possible. Welfare improves as a result.

1.5 Social security

Suppose the government imposes a tax on the young and transfers the proceeds to the old each period. Let T be the tax. The initial old receive T units of the good as a transfer from the government and are necessarily better off as a result. The initial young pay a tax when young but also receive a transfer of T when old. Their budget constraints are $y - T - c_1 = 0$ and $T - c_2 = 0$. Solve the second equation for T and substitute into the first equation to get $y = c_1 + c_2$. This is the same wealth constraint as in section 1.1 when $\rho = 0$. The initial young are able to move from their endowment point e on the horizontal axis to a point like A in Figure 1.1a and are also better off. Since this is also true of all future generations, social security can also help agents overcome trading frictions. Again, a point made by Samuelson in his seminal paper.

Figure 1.6a: Social security



Geometrically, we tax the young agent T units of the good in the first period. This moves us to the left along the horizontal axis. The government transfers this immediately to the old agents. Next period we repeat the operation taxing each young person T and immediately transferring it to the old. So when the young agent becomes old he receives T and this moves him vertically T units to point A. The social security budget balances each period.

Exercise: How would this tax-transfer scheme work if the population were growing at rate 1+n?

1.6 Rate of return dominance

One of the interesting aspects of fiat money is that it is dominated in its rate of return by other assets. If true, agents will not willingly hold money unless there is another reason for them to do so, or something in the economic environment that requires them to do so. We will study several justifications for this later on.

Let's go back to our earlier model with storage. Suppose the young agent can acquire money and store the good at a bank at a zero fixed cost. Her constraints are $y - c_1 = m/p + k$ and $(1+\rho)k + (p/p')(m/p) = c_2$, where p is the price level now and p' is the price level next period. Assume $k > 0$, solve the second equation for k, $k = [c_2 - (p/p')(m/p)]/(1+\rho)$, and substitute into the first constraint,

$$y = c_1 + [c_2 - (p/p')(m/p)]/(1+\rho) + m/p,$$

and rearrange,

$$y = c_1 + c_2/(1+\rho) + [(1+\rho) - (p/p')](m/p)/(1+\rho).$$

If we observe m/p > and $k > 0$, then $\mathbf{1+\rho = p/p'}$. This is a **no-arbitrage** condition in this model. If $1+\rho > p/p'$, the agent will not hold money and only store the good. In that case, the wealth constraint becomes

$$y = c_1 + c_2/(1+\rho),$$

which is the same as in section 1.1.

There is one problem with the no-arbitrage condition when it is applied to money. Typically, there are private assets like bank accounts, e.g., time deposits, that pay interest so $1+\rho > 1$. The no-arbitrage condition requires $1+\rho = p/p'$. However, we generally observe inflation so $p/p' < 1$. If this were true, agents wouldn't hold the money in this model. Yet, we observe it being used. This combination of facts, $1+\rho > 1$, $p/p' < 1$, and $k > 0$ and $m > 0$, yields a contradiction in this particular model and it provides us with a challenge to come up with models that can be used to explain it. One possibility involves the government restricting the assets private institutions can issue. Many governments impose restrictions on the ability of the private sector to undertake financial intermediation, i.e., converting one asset into another.

1.7 Example

Suppose utility is given by $u = \text{Log}(c_1) + \beta\text{Log}(c_2)$ and agents are wiling to accept fiat money in exchange for the good. The constraints are $y - c_1 - m/p = 0$ and $(p/p')(m/p) = c_2$. Substitute into the utility function. The agent's decision problem is to choose m/p, real cash balances, to $\max\{\text{Log}(y - m/p) + \beta\text{Log}((p/p')(m/p))\}$. The first order condition is

$$- 1/(y - m/p) + \beta(p/p')/(p/p')(m/p) = 0.$$

Solving,

$$m/p = \beta y/(1+\beta).$$

This is the money demand function. The demand for cash balances in this simple setting is proportional to first period income y. An increase in income will increase the demand for cash.

What does the stationary monetary equilibrium look like? Under the definition we look for a value of money that settles down to a constant so that

$$M_{t+1}/p_{t+1}N_{t+1} = M_t/p_tN_t \qquad \text{(SME)}$$

for all time periods. If we rearrange equation (SME), we obtain $\mathbf{p_t/p_{t+1} = (M_t/M_{t+1})(N_{t+1}/N_t)}$. This is the implicit rate of return to holding fiat money. With inflation, $p_t/p_{t+1} < 1$ and with deflation, $p_t/p_{t+1} > 1$. Notice that N_{t+1}/N_t is the growth rate of the population, 1+n. The greater the growth rate of population the more young people there are coming along and the greater the demand for fiat money. This increases the value of the money raising its rate of return. M_{t+1}/M_t is the rate of growth of the money supply, 1+z. The greater the growth of supply the lower the return. Substituting, the return to money is given by

$$p_t/p_{t+1} = 1 + r_m = (1+n)/(1+z).$$

With a fixed stock of money and a constant population, $p/p' = 1+r_m = 1$, so $r_m = 0$.

Exercise: Suppose the representative consumer is endowed (y_1, y_2), with $y_1 > y_2$, and y_2 is positive but close to zero. The budget constraints are $y_1 - c_1 - m/p = 0$ and $y_2 + (1+r_m)(m/p) = c_2$, where $p/p' = 1+r_m$. Utility is $u = Log(c_1) + \beta Log(c_2)$. Pose and solve the maximization problem.

From the exercise, the demand for money is $m/p = [\beta y_1 - y_2/(1+r_m)]/(1+\beta)$. In a stationary monetary equilibrium we know $1+r_m = (1+n)/(1+z)$ and this is equal to one when $n = z = 0$. Substitute $1+r_m = 1$ to get $m/p = [\beta y_1 - y_2]/(1+\beta)$. Money demand is increasing in current income and decreasing in future income.

1.8 The quantity theory of money

The famous quantity theory of money relates prices to the stock of money. Copernicus in 1517 and later Bodin in 1568 noticed that prices increased when gold and silver was brought back from the New World and used as money. The theory was refined by a variety of authors notably, John Stuart Mill and Irving Fisher. Mill first wrote down the famous ***equation of exchange***, which states that **MV = PT**, where M is a measure of the money stock, e.g., M2, V is velocity or the turnover of a dollar in a given period, P is the general price level, and T is a measure of real economic transactions.[3]

Classical economists thought that velocity was determined by institutional arrangements involving how many times a worker was paid per month, that they were paid in cash, and the denominations used to pay workers, for example. They also thought that the real economy was not much affected by changes in the stock of money so T was also given. It follows from the equation of exchange that $V\Delta M = T\Delta P$, since T and V were thought to be fixed. Divide the equation of exchange into the last equation to get the main implication of the theory, $\Delta M/M = \Delta P/P$, where $\Delta P/P$ is the inflation rate. Prices are proportional to the stock of money so an increase in the stock of money feeds directly through to inflation. A ten percent increase in the money supply would lead to a ten percent increase in prices.

Our theory of the last section is a version of the quantity theory. Let m/p represent the real value of money. Then

$$m = p[\beta y_1 - y_2]/(1+\beta),$$

where $V = 1$ and $T = [\beta y_1 - y_2]/(1+\beta)$. Since y_1 and y_2 are given, $\Delta m = \{[\beta y_1 - y_2]/(1+\beta)\}\Delta p$, which implies $\Delta m/m = \Delta p/p$. The quantity theory's equation of exchange holds in our model.

1.9 Helicopter drops of money

Milton Friedman used a famous thought experiment to argue that the main implication of the quantity theory made sense. Suppose helicopters fly over a city and drop money out the door. People gather below and collect the money. Suppose this doubles the stock of money in the city. What would a reasonable person predict about prices? Most likely they would double and there would probably be no real effects on production or consumption.

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[3] More formally, $MV = \Sigma_i p_i q_i$, where Σ_i means take the sum over all i so that $\Sigma_i p_i q_i = p_1 q_1 + p_2 q_2 + ... + p_n q_n$, where p_j is the price of good j, q_j is the quantity of good j that is transacted and $\Sigma_i p_i q_i$ is the value of all transactions in the economy in a given time period.

Several comments are in order. You might think this is a fanciful scenario. However, if the government imposes taxes on people and prints money to finance government spending, a tax cut financed by a one-time increase in the supply of money is a lot like a helicopter drop of cash.[4] We might be tempted to predict that such a change in tax policy will simply cause prices to rise without having any real effects.

Second, helicopter drops of cash, or tax cuts financed by an increase in the stock of money are typically not what people think of when they refer to "monetary policy" per se. "Monetary policy" usually refers to the central bank altering interest rates, or open market operations.

Third, we will see there is a complicated relationship between monetary policy the government's fiscal policy, and its budget constraint. The government's budget constraint and its fiscal policy can constrain what it can do in terms of monetary policy. When the government is committed to running deficits, for example, it may be very difficult, if not impossible, for it to chart a separate course for monetary policy to pursue.

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[4] George McGovern, the Democratic Presidential candidate in 1972, proposed giving every man, woman, and child $1000. If financed through money creation, this would be a helicopter drop of money. Senator McGovern was a history professor before becoming a congressman and later a senator from South Dakota.