# Chapter 7

Cost and Cost Minimization

## Types of Costs

- **Explicit Costs**: Costs that involve a direct monetary outlay.
- **Implicit Costs**: Costs that do not involve outlays of cash.
  - Example: money that an airline can get by renting, rather than actually using, its own plane.
- **Opportunity Costs**: The value of the best alternative that is forgone when another alternative is chosen

## Example:

- Kaiser Aluminum had two plants, one in Tacoma and another in Spokane in 2000.
  - It had initially signed a long-term electricity contract at a price of $23 megawatt/hour.
  - But the price of electricity was $1000 per megawatt/hour in 2001.
  - What did Kaiser Aluminum do? Shut down the smelters (at least a few days) and sell the electricity in the open market. (other firms, like Terra Industries, producing power, did the same).
- Hence, the opportunity cost of a megawatt/hour in 2001 was not $23, but $1000.

## Types of Costs

- **Sunk Costs (unrecoverable)**: Costs that have already been incurred and cannot be recovered.
  - Example: The rental a firm pays for the building it uses, if the lease contract prohibits subletting.
- **Non-sunk Costs (recoverable)**: Costs that are incurred only if a particular decision is made.

***Example***: Building a factory ($ 5 million)
- Before it is built: All is non-sunk
- After it is built: A portion might be sunk (unrecoverable)

## Falling into the "Sunk Cost Fallacy" – Application 7.3

- **Consider the following condition A:**
  - **Condition A** – you paid $10.95 to see a movie (or Pay TV.)
    - After 5 minutes, you are bored and the movie seems pretty bad
    - How much time do you keep watching the movie?
      - 0 min, 10, 20, 30, until the end of the movie.

**Experiment with/without A:**

Senior citizens: Same amount of time with/without A

College Students: More time with A than without, so they fell into "sunk cost fallacy" treating the $10.95 as a non-sunk cost, while it was already sunk

## Cost Minimization

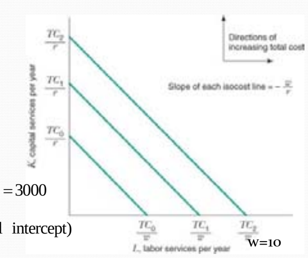
- **Long Run**: The period of time that is long enough for the firm to vary the quantities of all its inputs as much as it desires.
- **Short Run**: The period of time in which at least one of the firm's quantities cannot be changed.
  - *Example*:
    - 1) Restaurant: L is variable, K is fixed
    - 2) Scientific lab: L is fixed, K is variable

## Short-run costs - Cheat sheet

1. Variable and nonsunk:
   - ΔQ➔Δ costs ⟶ Variable
   - But if Q = 0, then costs = 0 ⟶ Nonsunk
   - Example: labor and raw materials
2. Fixed and nonsunk:
   - ΔQ➔ no change in costs. ⟶ Fixed
   - But if Q = 0 then costs = 0 ⟶ Nonsunk
   - Example: Heating
3. Fixed and sunk:
   - ΔQ➔ no change in costs ⟶ Fixed
   - But if Q = 0 then costs>0 ⟶ Sunk
   - Example: mortgage payment
     Lease that cannot be sublet

## Cost Minimization – 2 ingredients: Isocost and Isoquant.

- **Isocost line:** The set of combinations of labor and capital that yield the same total cost for the firm
- TC=wL + rK
  where
  w: price of labor (wage)
  r: price of capital (interest rate)




Example: $TC_0 = 1000$ $TC_1 = 2000$ $TC_2 = 3000$

w=10
r=20

Then, $\frac{TC_0}{r} = \frac{1000}{20} = 50$ (vertical intercept)

$\frac{TC_0}{w} = \frac{1000}{10} = 100$ (horizontal intercept)

- **More on the Isocost line…**

TC = wL + rK

Since K is the vertical axis, we solve for K to obtain TC-wL=rK, or,

$$\frac{TC}{r} - \frac{w}{r}L = K$$

where $\frac{TC}{r}$ denotes the vertical intercept of the Isocost line, and

$-\frac{w}{r}$ denotes the slope of the isocost line.

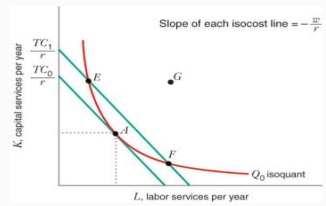
Example (cont.) $TC_0 = 1000$, $w = 10$, $r = 20$ implies an isocost line of

$$K = \frac{1000}{20} - \frac{10}{20}r = 50 - .5r$$

# Cost Minimization

- We want to minimize TC reaching a given output (isoquant).
- This is graphically represented by pushing the isocost line downwards until it reaches the isoquant representing the output we must be producing, $Q_o$.

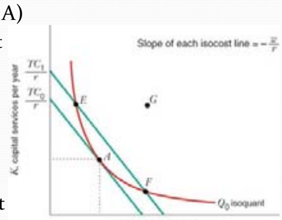
Points E and F also produce output $Q_0$, but at a higher cost $TC_1$




# Cost Minimization

- To find the tangency point (point A)
  - Slope of isoquant=slope of isocost line $-MRTS_{L,K} = \frac{-w}{r}$

$$\frac{MP_L}{MP_K} = \frac{w}{r} \to \frac{MP_L}{w} = \frac{MP_K}{r}$$




  - Additional output per dollar spent on labor = additional output per dollar spent on capital

**At Point E:** Slope of isoquant < Slope of isocost

$$-\frac{MP_L}{MP_K} < -\frac{w}{r} \to \frac{MP_L}{MP_k} > \frac{w}{r} \Rightarrow \frac{MP_L}{w} > \frac{MP_k}{r}$$

(Hence, increasing labor is still optimal)

# Cost-minimization Problem

Reach a given output $\bar{q} = f(L,K)$, where $\bar{q} = Q_0$

Min wL+rK ← minimize isocost line.
L,K
Subject to $\bar{q} = f(L,K)$ ← isoquant

$$\ell(L,K;\lambda) = wL + rK + \lambda\left[\bar{q} - \ell(L,K)\right]$$

F.O.C.s

$$\frac{\partial \ell}{\partial L} = w - \lambda\frac{\partial f}{\partial L} = 0 \to \lambda = \frac{w}{MP_L}$$

$$\frac{\partial \ell}{\partial K} = r - \lambda\frac{\partial f}{\partial K} = 0 \to \lambda = \frac{r}{MP_K}$$

$$\Big\} \frac{w}{MP_L} = \frac{r}{MP_K} \to \frac{w}{r} = \frac{MP_L}{MP_K}$$

Tangency between the Isocost line and the Isoquant!

$$\frac{\partial \ell}{\partial \lambda} = \bar{q} - f(L,K) = 0 \to \bar{q} = f(L,K)$$

# Example

Production function $Q = 50\sqrt{L \cdot K}$

Hence, $MP_L = 25\sqrt{\frac{K}{L}}$ and $MP_k = 25\sqrt{\frac{L}{K}}$

Input prices are $w = \$5$ and $r = \$20$.

a) What is the cost-minimizing combination of L and K that reaches an output Of $Q_0 = 1000$ units?

Tangency: $\frac{MP_L}{MP_K} = \frac{w}{r} \rightarrow \frac{25\sqrt{\frac{K}{L}}}{25\sqrt{\frac{L}{K}}} = \frac{\left(\frac{K}{L}\right)^{1/2}}{\left(\frac{L}{K}\right)^{1/2}} = \frac{K^{1/2} \cdot K^{1/2}}{L^{1/2} \cdot L^{1/2}} =$

$= \frac{K}{L} = \frac{5}{20}$ ← $\frac{w}{r}$

$= \frac{K}{L} = \frac{5}{20} \rightarrow 20K = 5L$

Not Done Yet! 4k=L

We also know that the cost-minimizing combination of L and K must lie on the isoquant $Q_0 = 1000$, that is, $1000 = 50\sqrt{L \cdot K} \rightarrow 20 = \sqrt{L \cdot K} \rightarrow 400 = L \cdot K$

$\Rightarrow \frac{400}{K} = L$

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We now have a system of two equations with two unknowns:

$$\left.\begin{array}{l} 4K = L \\ \frac{400}{K} = L \end{array}\right\} \rightarrow 4K = \frac{400}{K} \rightarrow K^2 = 100 \rightarrow K^* = 10$$

If we plug K*=10 into L=4K, we obtain the optimal value of L,

$$L = 4K = 4 \cdot 10 = 40 \rightarrow L^* = 40$$

Hence, the cost-minimizing combination of inputs is:

$$K^* = 10 \quad and \quad L^* = 40$$

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# Query #1

A firm has a Cobb-Douglas production function for its inputs of capital and labor. The firm is currently paying $10 per labor hour and $5 per machine hour. The firm is currently at an efficient production level, employing an equal number of machines and workers. What can we infer about the marginal productivities of capital and labor at this point?

a) $MP_K = MP_L$
b) $MP_K = 2MP_L$
c) $MP_L = 2MP_K$
d) $MP_L = .5MP_K$

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# Query #1 - Answer

- Answer C
- The cost-minimizing condition:
  - $-MRTS_{L,K} = -MP_L / MP_K$
  - $MP_L / MP_K = (w/r)$
- Input prices: w = $10 and r = $5
- So, the tangency condition for cost-minimization entails
  - $MP_L / MP_K = (\$10/\$5)$
- Cross multiplying, we obtain
  - $5(MP_L) = 10(MP_K)$
- Simplifying,
  - $MP_L = 2MP_K$
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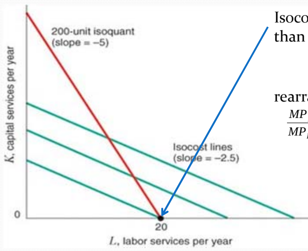
# Corner Point Problem

- Here, the optimal solution doesn't have a tangency between an isocost line and an isoquant curve.
- Corner solutions arise when inputs are <u>perfect substitutes</u>, i.e., Q=aL+bK




Isocost line is flatter than the Isoquant:

$$-\frac{MP_L}{MP_K} < -\frac{w}{r}$$

rearranging:

$$\frac{MP_L}{MP_K} > \frac{w}{r} \Rightarrow \frac{MP_L}{w} > \frac{MP_K}{r}$$

which implies the firm wants to use labor alone

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- **<u>Example of Corner solutions</u>:**

$Production\ Function: Q = 10L + 2K$

$Where\ MP_L = 10\ and\ MP_k = 2$

$Price\ of\ labor: w = 5\ per\ unit$

$Price\ of\ capital: r = 2\ per\ unit$

$Firm\ wish\ to\ produce: Q = 200\ units$

- Using the previous figure we observed that the optimal combination is a corner solution.
- Why? Because $\frac{MP_L}{MP_K} > \frac{w}{r}$ , that is $\frac{10}{2} > \frac{5}{2}$

Alternatively, note that $\frac{MP_L}{w} = \frac{10}{5} = 2 > \frac{MP_K}{r} = \frac{2}{2} = 1$

So that the marginal product per dollar of labor exceeds the marginal product per dollar of capital (2>1) , then the firm will substitute labor per capital until it uses no capital (K=0). In the horizontal axis of the above figure.

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- Then, the quantity of labor must satisfy Q= 10L + 2K, where we know K = 0 then…
  - reaching the isoquant Q=200 units implies 200=10L+2x0, or 200=10L

$$200 = 10L \Rightarrow L = \frac{200}{10} \Rightarrow L = 20$$

- Summarizing, the firm uses L = 20 workers and K = 0 units of capital. (corner point)

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# Query #2

Suppose in a particular production process that capital and labor are perfect substitutes so that three units of labor are equivalent to one unit of capital.

If the price of capital is $4 per unit and the price of labor is $1 per unit, the firm should

a) employ capital only.
b) employ labor only.
c) use three times as much capital as labor.
d) use three times as much labor as capital.

# Query #2 - Answer

- Answer B
- In this particular case, we have a corner point solution.
- The price of labor is $1/Unit, while that of capital is $4/ Unit.
- In addition, we are informed that three units of labor, 3L, are equivalent to one unit of capital, i.e., 3L=K
- Because these inputs are perfect substitutes, this firm can minimize its cost by spending $3/Unit on labor rather than spending $4 / Unit on capital.
  - (Remember that 3 Units of Labor was equal to 1 Unit of Capital)
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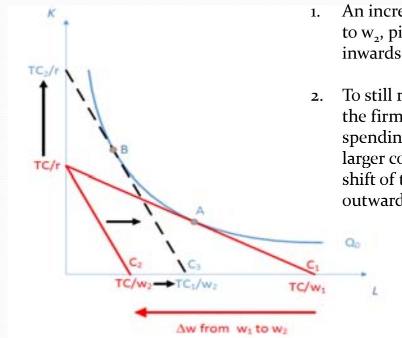
# Comparative statics: An increase in wages $\Delta w$

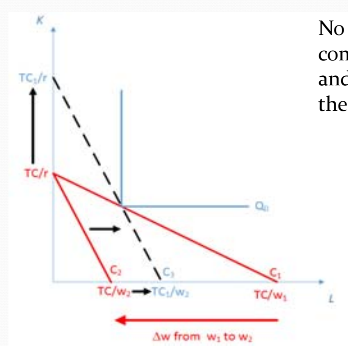



1. An increase in wages from $w_1$ to $w_2$, pivots the isocost line inwards, from $C_1$ to $C_2$.
2. To still reach isoquant $Q_o$, the firm cannot keep spending $TC_o$, it must incur a larger cost $TC_1>TC_o$. (Parallel shift of the isocost line outwards, from $C_2$ to $C_3$).

# An Increase in Wages

- An increase in the price of labor ($\Delta$w) produces an inward pivoting of the isocost line (steeper isocost line)
- But the firm must still reach Q=100 units!
- They'd better incur larger TC! (shift isocost outward)
  - *Comparing A and B:* Then the cost-minimizing amount of labor must go down (🡻 L) and the cost-minimizing quantity of capital must go up (🡹K), from point A to B.

# A increase in w, when the Cost-minimizing pair was at a kink $(L^*, K^*)$




No change in the combination of L and K before/after the $\Delta$w

## Query #3

Suppose capital and labor are perfect complements for a particular production process. If the price of labor increases, holding the price of capital and the level of output constant, the firm should

a) use more capital and less labor.
b) use more labor and less capital.
c) use the same amounts of capital and labor.
d) eliminate all use of labor.

## Query #3 - Answer

- Answer C
- This firm has a fixed-proportions production function, *Q=min{aK,bL}*.
  - Hence, inputs are used in specific ratios, and
  - An increase in the price of labor does not cause the firm to substitute capital for labor.
- If the price of capital *and* the level of output are held constant, the firm would continue to use the same amount of both labor *and* capital.
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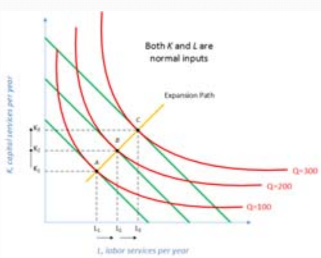
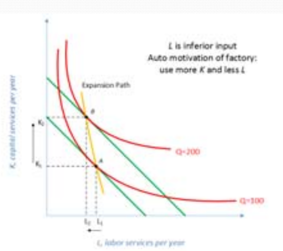
## Comparative Statics- (2) change in "reachable" output




ΔQ➔Δ in L and K

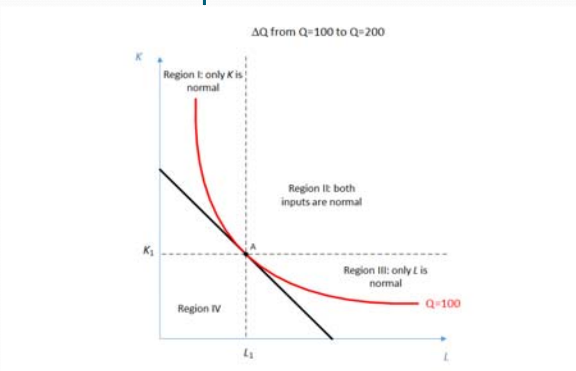



ΔQ ➔ ΔK, but ↓L (inferior input)

## Comparative Statics- (2) change in "reachable" output

- Expansion Path: A line that connects the cost-minimizing input combinations of (L,K) as the quantity of output, Q increases, holding input prices constant.
- ***Normal input***: An input whose cost-minimizing quantity increases as the firm produces more output.
  - The firm's expansion path will have a positive slope.
- ***Inferior input***: An input whose cost-minimizing quantity decreases as the firm produces more output.
  - The firm's expansion path will have a negative slope.

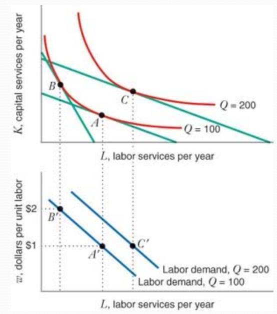
## Can both inputs be inferior? NO!




## Labor Demand Curve

- Labor Demand Curve : A curve that shows how the firm's cost minimizing quantity of labor varies with the price of labor.

1) ΔW from w=$1 (at A) to w=$2 (at B), labor usage decreases. This is depicted in A and B, respectively, in the top figure, and A' and B' in the bottom figure of labor demand for Q=100.

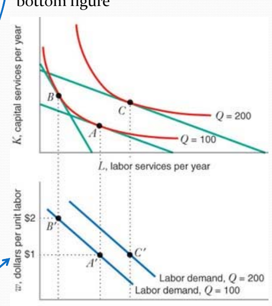



## Labor Demand

2) When we increase output from Q= 100 & Q=200,

- If $L_{dem}$ shifts outward, then L is a normal input (as depicted in the figure).
- If $L_{dem}$ shifts inwards, then L is an inferior input.

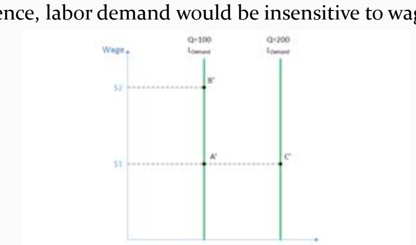
Cost-minimizing input combination varies from A to C in the top figure, which implies a shift from A' to C' in the bottom figure




Price of labor in A' and C' is the same, we only change output from Q=100 to Q=200

## Can Labor demand be vertical?

•Yes,
•When both inputs are used in fixed proportions, we saw that wage changes don't affect the cost-minimizing input combination.
(Remember the figure of right-angled isoquants?).
•Hence, labor demand would be insensitive to wages:



## Finding the Labor Demand Algebraically

•Consider a Cobb-Douglas production function $Q = 50\sqrt{LK}$
•From the tangency condition between isoquant and isocost, we obtain

$$\left(\frac{MP_l}{MP_K}\right) = \left(\frac{w}{r}\right)$$

$$or \ \frac{MP_L}{MP_K} = \frac{\frac{\partial Q}{\partial L}}{\frac{\partial Q}{\partial K}} = \frac{(0.5)(50)\sqrt{LK}K}{(0.5)(50)\sqrt{LK}L} = \frac{K}{L}$$

$$Hence \ , \frac{MP_L}{MP_K} = \frac{w}{r} \ implies$$

$$\frac{K}{L} = \frac{w}{r} or \ \ solving \ \ \ for \ \ L, \ \ L = \frac{r}{w}K$$

## Finding Labor Demand Algebraically

- Plugging the above expression, $L = \frac{r}{w}K$ , into the production function $Q = 50\sqrt{LK}$ we obtain

$$Q = 50\sqrt{\underbrace{\frac{r}{w}K}_{L}K} \Rightarrow \frac{Q}{50} = \sqrt{\left(\frac{r}{w}K^2\right)} \Rightarrow \frac{Q}{50} = \sqrt{\left(\frac{r}{w}\right)}K \Rightarrow K = \frac{Q}{50}\sqrt{\left(\frac{w}{r}\right)}$$

We just found the demand curve for capital, i.e. "capital demand."

## Finding Labor Demand Algebraically

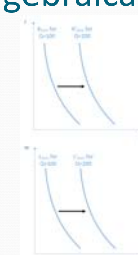
- Plugging the above result, $K = \frac{Q}{50}\sqrt{\left(\frac{w}{r}\right)}$ into $L = \frac{r}{w}K$

$$L = \frac{r}{w}\underbrace{\frac{Q}{50}\sqrt{\left(\frac{w}{r}\right)}}_{K} \Rightarrow L = \frac{r}{\sqrt{r}}\frac{\sqrt{w}}{w}\frac{Q}{50} \Rightarrow L = \frac{Q}{50}\left(\sqrt{\frac{r}{w}}\right)$$

which describes the demand curve for capital, i.e., the "labor demand."

## Finding Labor Demand Algebraically

- Note that:
  1) Capital demand, $K = \frac{Q}{50}\sqrt{\frac{w}{r}}$ , is...
     - Decreasing in r
     - Increasing in w
     - Increasing in Q
  2) Labor demand, $L = \frac{Q}{50}\sqrt{\frac{r}{w}}$ , is...
     - Increasing in r
     - Decreasing in w
     - Increasing in Q



- Since an $\Delta$Q produces an increase in the demand of both K and L, both inputs are normal (not inferior).

- Price Elasticity of Demand for Labor: The percentage change in the cost-minimizing quantity of labor with respect to a 1 percent change in the price of labor.

$$\varepsilon_{L,w} = \frac{\frac{\Delta L}{L} * 100\ \%}{\frac{\Delta w}{w} * 100\ \%} = \frac{\frac{\Delta L}{L}}{\frac{\Delta w}{w}} = \frac{\Delta L}{\Delta w}\frac{w}{L}$$

$\Delta 1\%$ in wages $\Rightarrow$ $\nabla$ in the firm's labor demand of $\varepsilon_{L,W}\%$

- Hence, it depends on the slope of the demand curve for labor.

$\frac{\Delta L}{\Delta W}\left( or\ \frac{\partial L}{\partial K}\right)$ measures such slope

The ***price elasticity of the demand for labor*** depends on the elasticity of substitution, $\sigma$ between two inputs (K and L):

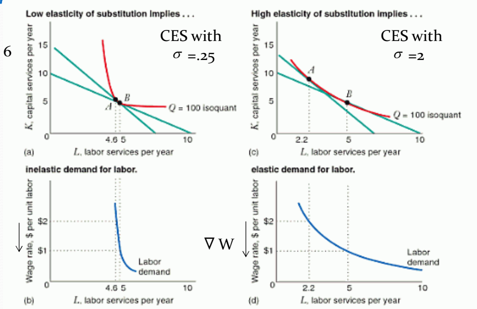
from Ch. 6

CES with $\sigma$ =.25

CES with $\sigma$ =2

$\nabla$ W

$\nabla$ W




A change in w has almost no effect on L

The same change in w induces a great change in L

- In both cases w drops from \$2 to \$1 (a 50% drop), but...

$\frac{5-4.6}{4.6} = 8\%$ Increase in labor demand in figure (a), and labor only increases from 4.6 to 5.

$\frac{5-2.2}{2.2} = 127\%$ Increase in labor demand in figure (b), and labor increases a lot: from 2.2 to 5.

Similarly for the price elasticity of the demand for capital:

$$\varepsilon_{k,r} = \frac{\frac{\Delta K}{K}*100\%}{\frac{\Delta r}{r}*100\%} = \frac{\frac{\Delta K}{K}}{\frac{\Delta r}{r}} = \frac{\Delta K}{\Delta r}\frac{r}{K}$$

***Interpretation***:

$\Delta 1\%$ in interest rates (r) $\Rightarrow \nabla$ in the firm's labor demand for capital of a of $\varepsilon_{K,r}\%$

It depends on the slope of the demand curve for capital,

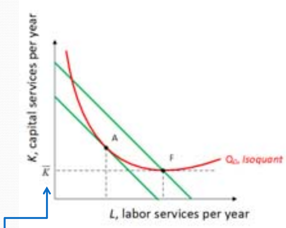
$$\frac{\Delta K}{\Delta r} = \frac{\partial K}{\partial r}$$

## Price elasticities of input demand for manufacturing industries in Alabama

| | Capital | Production Labor | Nonproduction Labor | Electricity |
|---|---|---|---|---|
| Textiles | -0.41 | -0.50 | -1.04 | -0.11 |
| Paper | -0.29 | -0.62 | -0.97 | -0.16 |
| Chemicals | 0.12 | -0.075 | -0.69 | -0.25 |
| Metals | -0.91 | -0.41 | -0.44 | -0.69 |

- Consider the textile industry (first row):
  - The -0.50 in the second cell implies that a 1% increase in the wage rate for production workers only entails a 0.5% decrease in the demand for labor of the typical textile firm in Alabama. (Labor demand is rather insensitive to labor).
- All but one of the price elasticities of input demand are between 0 and -1, suggesting that industries do not aggressively reduce their demand of the input whose price became relatively more expensive.

## Cost Minimization in the Short Run




- In the long-run the firm modifies L and K in order to reach $Q_0$.
  - Solution: Point *A*
- In the Short run K is fixed at $K = \overline{K}$
  - If the firm must reach output level of $Q_0$, it must use F, incurring a larger cost, i.e., a higher isocost.

## Cost Minimization in the Short Run

- Example: consider the Cobb-Douglas production function $Q = 50\sqrt{LK}$
- If K is fixed at $\overline{K}$ in the short run, then the cost-minimizing L is found by solving for L,

$$Q^2 = \left(50\sqrt{L\overline{K}}\right)^2 \rightarrow Q^2 = 2{,}500L\overline{K} \rightarrow L = \frac{Q^2}{2{,}500\overline{K}}$$

This is the demand for labor in the short run, where K is fixed.

*Extra practice:* Learning-by-Doing exercise 7.6 (3 inputs).

We go over this exercise next.

## Three inputs – Learning by Doing 7.6

Consider the Cobb-Douglas production function $Q = \sqrt{L} + \sqrt{K} + \sqrt{M}$, where L denotes labor, K capital, and M raw materials.

Hence, the marginal products are:

$$MP_L = \frac{1}{2\sqrt{L}} \quad MP_k = \frac{1}{2\sqrt{K}} \quad MP_M = \frac{1}{2\sqrt{M}}$$

Assume that input prices are w=1, r=1, m=1

a) If the firm wants to produce Q=12, what is the cost-minimizing input combination L*, K*, M*?

$$\frac{MP_L}{MP_K} = \frac{w}{r} \rightarrow \frac{1/2\sqrt{L}}{1/2\sqrt{K}} = \frac{1}{1} \rightarrow \frac{K}{L} = 1 \rightarrow K = L$$

$$\frac{MP_L}{MP_M} = \frac{w}{m} \rightarrow \frac{1/2\sqrt{L}}{1/2\sqrt{M}} = \frac{1}{1} \rightarrow \frac{M}{L} = 1 \rightarrow M = L$$

K=L=M

Using K=L=M in the production function yields:

$$12 = \sqrt{L} + \sqrt{L} + \sqrt{L} \rightarrow 12 = 3\sqrt{L} \rightarrow \frac{12}{3} = \sqrt{L} \rightarrow 4 = \sqrt{L} \rightarrow 16 = L$$

K=L M=L

Therefore, L=16, which entails that K=16 and M=16

b) If capital is fixed at 4 units, i.e., $\bar{K} = 4$ units, what is the cost-minimizing input combination (L*,M*)?

$$\frac{MP_L}{MP_M} = \frac{w}{m} \rightarrow \frac{1/2\sqrt{L}}{1/2\sqrt{M}} = \frac{1}{1} \rightarrow \frac{M}{L} = 1 \rightarrow M = L$$

Plugging that information into the production function, we obtain:

$$12 = \sqrt{L} + \sqrt{4} + \sqrt{L} \rightarrow 10 = 2\sqrt{L} \rightarrow \frac{12}{2} = \sqrt{L} \rightarrow 5 = \sqrt{L} \rightarrow 25 = L$$

K=4 (fixed) M=L

Hence, since L=25, then M=25, while the fixed amount of capital remains $\bar{K} = 4$.

c) What if now we fix the amount of capital at $\bar{K} = 4$, and the amount of labor at $\bar{L} = 9$ workers?

$$12 = \sqrt{9} + \sqrt{4} + \sqrt{M} \rightarrow 12 - 3 - 2 = \sqrt{M} \rightarrow 7 = \sqrt{M} \rightarrow 49 = M$$

$\bar{L} = 9$ $\bar{K} = 4$

Hence, M=49, while the two other fixed inputs remain at $\bar{K} = 4$ and $\bar{L} = 9$.

# Summary of the Cost-Minimization Problem with 3 Inputs

| | Labor, L | Capital, K | Materials, M | Minimized Total Cost |
|---|---|---|---|---|
| Long-run cost minimization for Q=12 | 16 | 16 | 16 | $48 |
| Short-run cost minimization for Q=12 when K=4 | 25 | 4 | 25 | $54 |
| Short-run cost minimization for Q=12 when K=4 and L=9 | 9 | 4 | 49 | $62 |