Can Incomplete Information Lead to Under-exploitation in the Commons?
TECHNICAL APPENDIX

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1 Appendix 1 - Single crossing property

In the following lemma we describe under which conditions the single-crossing property holds. Let second-period equilibrium costs be denoted by $z^i(x_1, \theta_K)$ and $c^1(x_1, \theta_K)$ after entry and no entry, respectively.

Lemma A. *When entry does not occur, incumbent's profits satisfy the single-crossing property for all parameter values. When entry occurs, the single-crossing property holds if*

$$\frac{\partial z^1(x_1, \theta_L)}{\partial q_2} \frac{\partial q_2^d(x_1, \theta_L)}{\partial x_1} > \frac{\partial z^1(x_1, \theta_H)}{\partial q_2} \frac{\partial q_2^d(x_1, \theta_H)}{\partial x_1}$$

Intuitively, the incumbent's payoff structure satisfies the single-crossing property if an additional unit of first-period appropriation x_1 produces a larger strategic effect when the stock is low than when it is high.

Proof. If entry does not occur, the high-stock incumbent's profits are $M_1^H(x_1) + \delta \overline{M}_1^H(x_1)$, for a given first-period appropriation x_1, and for a given appropriation level $q_1^m(x_1, \theta_H)$ that maximizes profits in the second period of the game. If the high-stock incumbent marginally increases first period appropriation, it experiences an increase in profits of $1 - c_{x_1}(x_1, \theta_H) - \delta c_{x_1}^1(x_1, \theta_H)$, where $c^1(x_1, \theta_H)$ denotes the high-stock incumbent's second-period cost, given that no entry occurs and that the incumbent selects the monopoly profit-maximizing appropriation in the second period. The previous derivative can be alternatively expressed as $MB - MC^m(x_1, \theta_H)$. Similarly for the low-stock incumbent. Hence, under no entry, the single-crossing property holds if $MB -$

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$MC^m(x_1,\theta_H) \geq MB - MC^m(x_1,\theta_L)$, or alternatively, if $MC^m(x_1,\theta_H) \leq MC^m(x_1,\theta_L)$, i.e., the incumbent's marginal costs from raising x_1 are decreasing in the initial stock. Note that this condition implies

$$\delta\left[c^1_{x_1}(x_1,\theta_H) - c^1_{x_1}(x_1,\theta_L)\right] < c_{x_1}(x_1,\theta_L) - c_{x_1}(x_1,\theta_H)$$

where the right-hand side is positive since first-period costs satisfy $c_{x_1\theta} < 0$ by definition. In addition, the left-hand side is negative since second-period costs satisfy $c^1_{x_1\theta} < 0$ by definition. Hence, the single-crossing property holds under all parameter values if no entry follows.

If entry occurs, the high-stock incumbent's profits are $M_1^H(x_1)+\delta D_1^H(x_1)$, for a given first-period appropriation x_1, and for a given appropriation level $q_1^d(x_1,\theta_H)$ that maximizes profits in the second period of the game. If the high-stock incumbent marginally increases first-period appropriation, it experiences an increase in profits given by $1-c_{x_1}(x_1,\theta_H)-\delta\left[\frac{\partial z^1(x_1,\theta_H)}{\partial x_1}+\frac{\partial z^1(x_1,\theta_H)}{\partial q_2}\frac{\partial q_2^d(x_1,\theta_H)}{\partial x_1}\right]$. The previous condition can be alternatively expressed as $MB-MC^d(x_1,\theta_H)$. Similarly for the low-stock incumbent. Hence, under entry, the single-crossing property is satisfied if $MB - MC^d(x_1,\theta_H) \geq MB-MC^d(x_1,\theta_L)$, or alternatively, if $MC^d(x_1,\theta_H) \leq MC^d(x_1,\theta_L)$, i.e., the incumbent's marginal costs from raising x_1 are decreasing in the initial stock. Note that this condition implies

$$\begin{aligned} & c_{x_1}(x_1,\theta_H)+\delta\left[\frac{\partial z^1(x_1,\theta_H)}{\partial x_1}+\frac{\partial z^1(x_1,\theta_H)}{\partial q_2}\frac{\partial q_2^d(x_1,\theta_H)}{\partial x_1}\right] \\ < \quad & c_{x_1}(x_1,\theta_L)-\delta\left[\frac{\partial z^1(x_1,\theta_L)}{\partial x_1}+\frac{\partial z^1(x_1,\theta_L)}{\partial q_2}\frac{\partial q_2^d(x_1,\theta_L)}{\partial x_1}\right] \end{aligned}$$

rearranging,

$$\begin{aligned} & \delta\left[\frac{\partial z^1(x_1,\theta_H)}{\partial x_1}-\frac{\partial z^1(x_1,\theta_L)}{\partial x_1}\right]+\delta\left[\frac{\partial z^1(x_1,\theta_H)}{\partial q_2}\frac{\partial q_2^d(x_1,\theta_H)}{\partial x_1}-\frac{\partial z^1(x_1,\theta_L)}{\partial q_2}\frac{\partial q_2^d(x_1,\theta_L)}{\partial x_1}\right] \\ < \quad & c_{x_1}(x_1,\theta_L)-c_{x_1}(x_1,\theta_H) \end{aligned}$$

where the right-hand side of the inequality is positive since first-period costs satisfy $c_{x_1\theta} < 0$ by definition. Furthermore, the first term in the left-hand side is negative since $z^1_{x_1\theta} < 0$ by definition. Therefore, the single-crossing property holds if

$$\frac{\partial z^1(x_1,\theta_H)}{\partial q_2}\frac{\partial q_2^d(x_1,\theta_H)}{\partial x_1} < \frac{\partial z^1(x_1,\theta_L)}{\partial q_2}\frac{\partial q_2^d(x_1,\theta_L)}{\partial x_1}$$

2 Appendix 2

First, note that $\delta\left[\overline{M}_1^H(x_1^L) - D_1^H(x_1^{H,E})\right]$ from IC_H and $\delta\left[\overline{M}_1^L(x_1^L) - D_1^L(x_1^{L,E})\right]$ from IC_L are both decreasing and convex in x_1^L, since $\overline{M}_1^K(x_1^L)$ is decreasing and convex in x_1^L, i.e., $\frac{\partial \overline{M}_1^K(x_1^L)}{\partial x_1^L} = -\delta c^1_{x_1} > 0$ and $\frac{\partial^2 \overline{M}_1^K(x_1^L)}{\partial {x_1^L}^2} = -\delta c^1_{x_1x_1} > 0$. We next investigate the conditions under which $\delta\left[\overline{M}_1^H(x_1^L) - D_1^H(x_1^{H,E})\right]$ is above $\delta\left[\overline{M}_1^L(x_1^L) - D_1^L(x_1^{L,E})\right]$. For compactness, let $\overline{M}_1(x_1,\theta)$ denote monopoly second-period equilibrium profits as a function of first-period appropriation, x_1, and the initial stock, θ. Similarly, let $D_1(x_1^E(\theta),\theta)$ represent duopoly second-period equilibrium profits as a function of the first-period equilibrium appropriation that maximizes the incumbent's profits given that entry follows, $x_1^E(\theta)$, for a given initial stock θ. We next show that the difference $\overline{M}_1(x_1,\theta)-D_1(x_1^E(\theta),\theta)$ is increasing in θ. In particular, differentiating with respect to θ and using the envelope theorem, we obtain

$$-c_\theta^1(x_1,\theta)+z_{q_2}^1\left[\frac{\partial q_2^d(x_1,\theta)}{\partial x_1}\frac{\partial x_1^E(\theta)}{\partial \theta}+\frac{\partial q_2^d(x_1,\theta)}{\partial \theta}\right]+z_{x_1}^1\frac{\partial x_1^E(\theta)}{\partial \theta}+z_\theta^1(x_1,\theta)$$

which is positive since $\left|c_\theta^1(x_1,\theta)\right| > \left|z_\theta^1(x_1,\theta)\right|$ and the second and third term are positive by definition.

Finally, note that the negative slope of $\delta\left[\overline{M}_1^H(x_1^L)-D_1^H(x_1^{H,E})\right]$ is $-\delta c_{x_1}^1(x_1,\theta_H)$, which is smaller (in absolute value) than that of $\delta\left[\overline{M}_1^L(x_1^L)-D_1^L(x_1^{L,E})\right]$, $-\delta c_{x_1}^1(x_1,\theta_L)$, where $c^1(x_1,\theta_K)$ denotes the incumbent's second-period cost, given that no entry occurs and that the incumbent selects the appropriation level $q_1^m(x_1,\theta_K)$ that maximizes its monopoly second-period profits. Therefore, $\delta\left[\overline{M}_1^H(x_1^L)-D_1^H(x_1^{H,E})\right]$ is flatter than $\delta\left[\overline{M}_1^L(x_1^L)-D_1^L(x_1^{L,E})\right]$, guaranteeing that the former does not cross the latter. ■

3 *Proof of Proposition 1*

First, note that entrant beliefs become $\mu(\theta_H|x_1^H)=1$ after observing the equilibrium appropriation level x_1^H and $\mu(\theta_H|x_1^L)=0$ after observing the equilibrium level x_1^L for any $x_1^L\in[x_1^A,x_1^B]$. If the entrant observes an off-the-equilibrium appropriation level of $x_1\neq x_1^H\neq x_1^L$, then Bayes' rule does not specify a particular posterior off-the-equilibrium belief, i.e., $\mu(\theta_H|x_1)\in[0,1]$, and for simplicity we assume $\mu(\theta_H|x_1)=1$. Given these beliefs, the entrant enters after observing an appropriation level of x_1^H since $D_2^H(x_1^H)>0$, but stays out after observing an appropriation of x_1^L given that $0>D_2^L(0)>D_2^L(x_1^L)$. After observing an off-the-equilibrium level $x_1\neq x_1^H\neq x_1^L$, the entrant enters if and only if its expected profits from entering satisfy

$$\mu(\theta_H|x_1)\times D_2^H(x_1)+(1-\mu(\theta_H|x_1))D_2^L(x_1)>0,\text{ or }\mu(\theta_H|x_1)>\frac{-D_2^L(x_1)}{D_2^H(x_1)-D_2^L(x_1)}\equiv\overline{\mu}(x_1)$$

where $D_2^H(x_1)>0$, implying $D_2^H(x_1)-D_2^L(x_1)>-D_2^L(x_1)$, and since both sides of the inequality are positive, we can conclude that $\overline{\mu}(x_1)\in(0,1)$. In this case, the entrant enters if its off-the-equilibrium beliefs $\mu(\theta_H|x_1)$ satisfy $\mu(\theta_H|x_1)>\overline{\mu}(x_1)$, which holds since $\mu(\theta_H|x_1)=1$.

Let us now examine the high-stock incumbent's incentives. By selecting the equilibrium appropriation level $x_1^{H,E}$, the high-stock incumbent obtains profits of $M_1^H(x_1^{H,E})+\delta D_1^H(x_1^{H,E})$. First, note that $x_1^{H,E}$ maximizes $M_1^H(x_1)+\delta D_1^H(x_1)$. Second, first-period appropriation $x_1^{H,E}$ coincides with the equilibrium level that the high-stock incumbent selects under complete information, yielding the same profits. By deviating towards the low-stock incumbent's equilibrium appropriation, x_1^L, the high-stock incumbent deters entry, yielding profits of $M_1^H(x_1^L)+\delta\overline{M}_1^H(x_1^L)$. Hence, the high-stock incumbent prefers to select an equilibrium first-period appropriation of $x_1^{H,E}$ rather than deviating towards x_1^L if $M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E}) \geq M_1^H(x_1^L) + \delta \overline{M}_1^H(x_1^L)$, or alternatively,

$$M_1^H(x_1^{H,E}) - M_1^H(x_1^L) \geq \delta \left[\overline{M}_1^H(x_1^L) - D_1^H(x_1^{H,E})\right] \qquad (IC_H)$$

If instead the high-stock incumbent deviates towards an off-the-equilibrium level of $x_1 \neq x_1^{H,E} \neq x_1^L$ then entry follows, yielding profits of $M_1^H(x_1) + \delta D_1^H(x_1)$, which cannot exceed equilibrium profits of $M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$.

Let us now turn to the low-stock incumbent. Selecting the equilibrium first-period appropriation level of x_1^L yields $M_1^L(x_1^L)+\delta\overline{M}_1^L(x_1^L)$. By deviating towards the high-stock incumbent's equilibrium appropriation level, $x_1^{H,E}$, the low-stock incumbent attracts entry, obtaining profits of $M_1^L(x_1^{H,E}) + \delta D_1^L(x_1^{H,E})$. Therefore, the low-stock incumbent selects an equilibrium first-period appropriation level of x_1^L rather than deviating towards $x_1^{H,E}$ if

$$M_1^L(x_1^L) + \delta\overline{M}_1^L(x_1^L) \geq M_1^L(x_1^{H,E}) + \delta D_1^L(x_1^{H,E}) \qquad (A.1)$$

If instead the low-stock incumbent deviates towards any off-the-equilibrium level $x_1 \neq x_{1,monop}^H \neq x_1^L$ then entry follows, and therefore the incumbent selects the value of x_1 that maximizes $M_1^L(x_1)+\delta D_1^L(x_1)$. Let $x_1^{L,E}$ denote the solution to this maximization problem, yielding profits of $M_1^L(x_1^{L,E})+\delta D_1^L(x_1^{L,E})$. Hence, the low-stock incumbent chooses its equilibrium appropriation level of x_1^L rather than deviating towards $x_1^{L,E}$ if

$$M_1^L(x_1^L) + \delta\overline{M}_1^L(x_1^L) \geq M_1^L(x_1^{L,E}) + \delta D_1^L(x_1^{L,E}) \qquad (A.2)$$

Note that condition A.2 implies A.1 since $M_1^L(x_1^{L,E}) + \delta D_1^L(x_1^{L,E}) > M_1^L(x_1^{H,E}) + \delta D_1^L(x_1^{H,E})$, given that $x_1^{L,E}$ maximizes the low-stock incumbent's profits (across both periods) given entry, whereas $x_1^{H,E}$ does not. Therefore, condition A.2 becomes the incentive compatibility condition that must be satisfied in order to guarantee that the low-stock incumbent does not deviate from its equilibrium level of x_1^L. Let us denote this incentive compatibility condition as follows

$$M_1^L(x_1^{L,E}) - M_1^L(x_1^L) \leq \delta \left[\overline{M}_1^L(x_1^L) - D_1^L(x_1^{L,E})\right] \qquad (IC_L)$$

4 *Proof of Corollary 1*

In this case the CPR totally regenerates, $\beta = 1$. First, entrant beliefs become $\mu(\theta_H|x_{1,monop}^H) = 1$ after observing the equilibrium appropriation level of $x_{1,monop}^H$ and $\mu(\theta_H|x_1^L) = 0$ after observing the equilibrium level of x_1^L for any $x_1^L \in [\widetilde{x}_1^A, \widetilde{x}_1^B]$. If the entrant observes an off-the-equilibrium level of $x_1 \neq x_{1,monop}^H \neq x_1^L$, then Bayes' rule does not specify a particular posterior off-the-equilibrium belief, i.e., $\mu(\theta_H|x_1) \in [0,1]$, and for simplicity we take $\mu(\theta_H|x_1) = 1$. Given these beliefs, the entrant enters after observing an appropriation level of $x_{1,monop}^H$ since $D_2^H(0) \equiv D_2^H > 0$, but stays out after observing an appropriation of x_1^L given that $D_2^L(0) \equiv D_2^L < 0$. After observing an off-the-equilibrium level $x_1 \neq x^H_{1,monop} \neq x^L_1$, the entrant enters if and only if its expected profits from entering satisfy $\mu(\theta_H|x_1) \geq \frac{-D^L_2}{D^H_2 - D^L_2} \equiv \overline{\mu}$, which holds as shown in Proposition 1.

Let us now examine the high-stock incumbent's incentives. By selecting the equilibrium appropriation level of $x^H_{1,monop}$, the high-stock incumbent obtains profits of $M^H_1(x^H_{1,monop}) + \delta D^H_1$, where $M^H_1(x^H_{1,monop})$ represents the highest monopoly profit that the high-stock incumbent can obtain during the first period, and where second-period profits are independent on x_1 since the resource is totally regenerated ($\beta = 1$). Note that $x^H_{1,monop}$ coincides with the first-period appropriation level that the high-stock incumbent selects in the complete information context. By deviating towards the low-stock incumbent's equilibrium appropriation, x^L_1, the high-stock incumbent deters entry, yielding profits of $M^H_1(x^L_1) + \delta\overline{M}^H_1$. Hence, the high-stock incumbent prefers an equilibrium first-period appropriation level of $x^H_{1,monop}$ rather than deviating towards x^L_1 if $M^H_1(x^H_{1,monop}) + \delta D^H_1 \geq M^H_1(x^L_1) + \delta\overline{M}^H_1$, or alternatively,

$$M^H_1(x^H_{1,monop}) - M^H_1(x^L_1) \geq \delta \left[\overline{M}^H_1 - D^H_1\right] \qquad (IC_H)$$

Note that if the high-stock incumbent deviates towards an off-the-equilibrium level of $x_1 \neq x^H_{1,monop} \neq x^L_1$, entry follows, yielding profits of $M^H_1(x_1) + \delta D^H_1$, which cannot exceed equilibrium profits of $M^H_1(x^H_{1,monop}) + \delta D^H_1$ given that $x^H_{1,monop}$ is the profit-maximizing appropriation level under monopoly.

Let us now analyze the low-stock incumbent. Selecting the equilibrium first-period appropriation level of x^L_1 yields $M^L_1(x^L_1) + \delta\overline{M}^L_1$. By deviating towards the high-stock incumbent's equilibrium appropriation level, $x^H_{1,monop}$, the low-stock incumbent attracts entry, with associated profits of $M^L_1(x^H_{1,monop}) + \delta D^L_1$. Therefore, the low-stock incumbent selects an equilibrium first-period appropriation level of x^L_1 rather than deviating towards $x^H_{1,monop}$ if

$$M^L_1(x^L_1) + \delta\overline{M}^L_1 \geq M^L_1(x^H_{1,monop}) + \delta D^L_1 \qquad (A.4)$$

If instead the low-stock incumbent deviates towards any off-the-equilibrium level $x_1 \neq x^H_{1,monop} \neq x^L_1$, it attracts entry, and therefore the incumbent selects the value of x_1 that maximizes

$$\max_{x_1} M^L_1(x_1) + \delta D^L_1 \quad \text{subject to } x_1 \neq x^H_{1,monop} \neq x^L_1$$

But note that this maximization problem is equivalent to $\max_{x_1} M^L_1(x_1)$, with solution given by the appropriation level that maximizes the first-period monopoly profits, i.e., $x^L_{1,monop}$, yielding profits of $M^L_1(x^L_{1,monop}) + \delta D^L_1$. Hence, the low-stock incumbent chooses its equilibrium appropriation level of x^L_1 rather than deviating towards $x^L_{1,monop}$ if

$$M^L_1(x^L_1) + \delta\overline{M}^L_1 \geq M^L_1(x^L_{1,monop}) + \delta D^L_1 \qquad (A.5)$$

Condition A.5 implies A.4 since $M^L_1(x^L_{1,monop}) + \delta D^L_1 > M^L_1(x^H_{1,monop}) + \delta D^L_1$, given that

$M_1^L(x_{1,monop}^L) > M_1^L(x_{1,monop}^H)$. Therefore, condition A.5 becomes the incentive compatibility condition that must be satisfied in order to guarantee that the low-stock incumbent does not deviate from its equilibrium appropriation of x_1^L. Let us denote this incentive compatibility condition as follows

$$M_1^L(x_{1,monop}^L) - M_1^L(x_1^L) \leq \delta\left[\overline{M}_1^L - \delta D_1^L\right] \qquad (IC_L)$$

Note that cutoffs $\delta\left[\overline{M}_1^H - D_1^H\right]$ from IC_H and $\delta\left[\overline{M}_1^L - \delta D_1^L\right]$ from IC_L are both independent on x_1^L. We next investigate the conditions under which the former cutoff is above the latter. For compactness, let $\overline{M}_1(\theta)$ and $D_1(\theta)$ denote monopoly and duopoly second-period equilibrium profits as a function of the initial stock, θ. Hence, we want to show that the difference $\overline{M}_1(\theta) - D_1(\theta)$ is increasing in θ. In particular, differentiating with respect to θ and using the envelope theorem, we obtain

$$-c_\theta^1(\theta) + z_{q_2}^1 \frac{\partial q_2^d(\theta)}{\partial\theta} + z_\theta^1(\theta)$$

which is positive since $z_{q_2}^1 \frac{\partial q_2^d(\theta)}{\partial\theta} > 0$ and $\left|c_\theta^1(\theta)\right| > \left|z_\theta^1(\theta)\right|$ by definition, guaranteeing that $\delta\left[\overline{M}_1^H - D_1^H\right]$ is above $\delta\left[\overline{M}_1^L - \delta D_1^L\right]$. ■

5 *Proof of Proposition 2*

Case in which $\beta < 1$. Suppose that the low-stock incumbent appropriates $x_1^L = x_1^A$. Let us first check if a deviation towards $x_1 \in (x_1^A, x_1^B]$ is equilibrium dominated for either type of incumbent.

On one hand, the highest profit that the high-stock incumbent can obtain deviating towards $x_1 \in (x_1^A, x_1^B]$ occurs when entry does not ensue. In such case, the high-stock incumbent obtains $M_1^H(x_1) + \delta\overline{M}_1^H(x_1)$. Hence, it deviates only if $M_1^H(x_1) + \delta\overline{M}_1^H(x_1) > M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$. But IC_H guarantees that this inequality cannot hold for any $x_1 \in (x_1^A, x_1^B]$. Hence the high-stock incumbent does not have incentives to deviate from $x_1^{H,E}$ to $x_1 \in (x_1^A, x_1^B]$.

On the other hand, the highest profit that the low-stock incumbent can obtain from deviating towards $x_1 \in (x_1^A, x_1^B]$ occurs when entry does not ensue. In such case, the low-stock incumbent's payoff is $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$ which exceeds its equilibrium profit of $M_1^L(x_1^A) + \delta\overline{M}_1^L(x_1^A)$ since $x_1^A < x_1 \leq x_1^{L,NE}$ and $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$ reaches its maximum at $x_1^{L,NE}$. Therefore, the low-stock incumbent has incentives to deviate from x_1^A to x_1.

Hence, after observing a first-period appropriation of $x_1 \in (x_1^A, x_1^B]$, the entrant concentrates its posterior beliefs on the initial stock being low, i.e., $\mu(\theta_H|x_1) = 0$, and does not enter. Given this updated off-the equilibrium beliefs, the low-stock incumbent appropriates x_1 and deters entry, yielding payoff $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$, which exceeds its equilibrium profit from appropriating x_1^A. Thus, the low-stock incumbent deviates from x_1^A, and the separating equilibrium in which it selects x_1^A violates the Intuitive Criterion. A similar argument is applicable to all separating equilibria in which the low-stock incumbent selects $x_1 \in (x_1^A, x_1^B)$, all of them also violating the Intuitive Criterion.

Finally, let us check that the separating equilibrium in which the low-stock incumbent chooses $x_1^L = x_1^B$ survives the Intuitive Criterion. If the low-stock incumbent deviates towards $x_1 \in (x_1^A, x_1^B)$ the highest profit that it can obtain is $M_1^L(x_1)+\delta\overline{M}_1^L(x_1)$, which is lower than its equilibrium payoff of $M_1^L(x_1^B) + \delta\overline{M}_1^L(x_1^B)$. If, instead, it deviates towards $x_1 > x_1^B$, the highest payoff that it can obtain is $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$, which exceeds its equilibrium payoff for all $x_1 \in (x_1^B, x_1^{L,NE}]$. Hence, the low-stock incumbent has incentives to deviate. Let us now check if the high-stock incumbent also has incentives to deviate towards $x_1 \in (x_1^B, x_1^{L,NE}]$. The highest profit that it can obtain is $M_1^H(x_1) + \delta\overline{M}_1^H(x_1)$, which exceeds its equilibrium profits if $M_1^H(x_1) + \delta\overline{M}_1^H(x_1) > M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$. This condition can be rewritten as $\delta\left[\overline{M}_1^H(x_1) - D_1^H(x_1^{H,E})\right] > M_1^H(x_1^{H,E}) - M_1^H(x_1)$, which is satisfied for all $x_1 > x_1^B$ (see figure 1). Hence, the high-stock incumbent also has incentives to deviate towards $x_1 \in (x_1^B, x_1^{L,NE}]$. This implies that, after observing a deviation x_1, the entrant cannot update his prior beliefs, and chooses to enter if the expected profit from entering satisfies $pD_2^H(x_1) + (1-p)D_2^L(x_1) > 0$ or

$$p > \frac{-D_2^L(x_1)}{D_2^H(x_1) - D_2^L(x_1)} \equiv \overline{p}(x_1)$$

where $D_2^H(x_1) > 0$, implying that $D_2^H(x_1) - D_2^L(x_1) > -D_2^L(x_1)$, and since both sides of the inequality are positive, then $\overline{p}(x_1) > 0$. Hence, if $p > \overline{p}(x_1)$, entry occurs, yielding profits $M_1^L(x_1) + \delta D_1^L(x_1)$ for the low-stock incumbent. Such profits are lower than its equilibrium profits $M_1^L(x_1^B) + \delta D_1^L(x_1^B)$. Indeed, from IC_L we know that $M_1^L(x_1^B)+\delta\overline{M}_1^L(x_1^B) \geq M_1^L(x_1^{L,E})+\delta D_1^L(x_1^{L,E})$. Since, in addition, $M_1^L(x_1^{L,E})+\delta D_1^L(x_1^{L,E}) \geq M_1^L(x_1)+\delta D_1^L(x_1)$ given that $x_1^{L,E}$ is the argmax of $M_1^L(x_1) + \delta D_1^L(x_1)$, then $M_1^L(x_1^B) + \delta\overline{M}_1^L(x_1^B) \geq M_1^L(x_1) + \delta D_1^L(x_1)$ for any deviation x_1, and therefore the low-stock incumbent does not deviate. Regarding the high-stock incumbent, it obtains profits of $M_1^H(x_1) + \delta D_1^H(x_1)$ by deviating towards x_1, which are below its equilibrium profits $M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$ since $x_1^{H,E}$ is the argmax of $M_1^H(x_1)+\delta D_1^H(x_1)$. Hence, the high-stock incumbent does not deviate towards x_1 either, and the separating equilibrium survives the Intuitive Criterion if $p > \overline{p}(x_1)$.

If $p < \overline{p}(x_1)$, then entry does not occur, yielding profits of $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$ for the low-stock incumbent, which exceed its equilibrium profits $M_1^L(x_1^B) + \delta\overline{M}_1^L(x_1^B)$ since $x_1 \in (x_1^B, x_1^{L,NE}]$. Then the separating equilibrium violates the Intuitive Criterion if $p < \overline{p}(x_1)$.

Case in which $\beta = 1$. Suppose that the low-stock incumbent appropriates $x_1^L = \widetilde{x}_1^A$. Let us first check if a deviation towards $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B]$ is equilibrium dominated for either type of incumbent.

On one hand, the highest profit that the high-stock incumbent can obtain deviating towards $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B]$ occurs when entry does not ensue. In such case, the high-stock incumbent obtains $M_1^H(x_1) + \delta\overline{M}_1^H$. These profits, however, are lower than the high-stock incumbent equilibrium profits of $M_1^H(x_{1,monop}^H) + \delta D_1^H$ since from IC_H we know that

$$M_1^H(x_{1,monop}^H) + \delta D_1^H \geq M_1^H(x_1) + \delta\overline{M}_1^H \quad \text{for all } x_1 \in [\widetilde{x}_1^A, \widetilde{x}_1^B]$$

Hence, the high-stock incumbent does not have incentives to deviate. On the other hand, the highest profit that the low-stock incumbent can obtain from deviating towards $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B]$ occurs when entry does not ensue. In such case, the low-stock incumbent's payoff is $M_1^L(x_1)+\delta\overline{M}_1^L$ which exceeds its equilibrium profit of $M_1^L(\widetilde{x}_1^A) + \delta\overline{M}_1^L$ since $x_1 > \widetilde{x}_1^A$. Therefore, the low-stock incumbent has incentives to deviate from $\widetilde{x}_1^A$ to x_1.

Hence, after observing a first period appropriation of $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B]$, the entrant concentrates its posterior beliefs on the initial stock being low, i.e., $\mu(\theta_H|x_1) = 0$, and does not enter. Given this updated off-the equilibrium beliefs, the low-stock incumbent appropriates x_1 and deters entry, yielding payoff $M_1^L(x_1) + \delta\overline{M}_1^L$, which exceeds its equilibrium profit from appropriating $\widetilde{x}_1^A$. Thus, the low-stock incumbent deviates from $\widetilde{x}_1^A$, and the separating equilibrium in which it selects $\widetilde{x}_1^A$ violates the Intuitive Criterion. A similar argument is applicable for all separating equilibria in which the low-stock incumbent selects $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B)$, all of them also violating the Intuitive Criterion.

Finally, let us check that the separating equilibrium in which the low-stock incumbent chooses $\widetilde{x}_1^B$ survives the Intuitive Criterion. If the low-stock incumbent deviates towards $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B)$ the highest profit that it can obtain is $M_1^L(x_1) + \delta\overline{M}_1^L$, which is lower than its equilibrium payoff of $M_1^L(\widetilde{x}_1^B) + \delta\overline{M}_1^L$. If, instead, it deviates towards $x_1 > \widetilde{x}_1^B$, the highest payoff that it can obtain is $M_1^L(x_1) + \delta\overline{M}_1^L$, which exceeds its equilibrium payoff for all $x_1 \in (x_1^B, x_{1,monop}^L]$. Hence, the low-stock incumbent has incentives to deviate. Let us now check if the high-stock incumbent also has incentives to deviate towards $x_1 \in (x_1^B, x_{1,monop}^L]$. The highest profit that it can obtain is $M_1^H(x_1) + \delta\overline{M}_1^H$, which exceeds its equilibrium profits if $M_1^H(x_1) + \delta\overline{M}_1^H > M_1^H(x_{1,monop}^H) + \delta D_1^H$. This condition can be rewritten as $\delta\left[\overline{M}_1^H - D_1^H\right] > M_1^H(x_{1,monop}^H) - M_1^H(x_1)$, which is satisfied for all $x_1 > \widetilde{x}_1^B$ (see figure 2). Hence, the high-stock incumbent also has incentives to deviate towards $x_1 \in (\widetilde{x}_1^B, x_{1,monop}^L]$. This implies that, after observing a deviation x_1, the entrant cannot update its prior beliefs, and chooses to enter if the expected profit from entering satisfies $pD_2^H + (1-p)D_2^L > 0$ or

$$p > \frac{-D_2^L}{D_2^H - D_2^L} \equiv \overline{p}$$

where $D_2^H > 0$, implying that $D_2^H - D_2^L > -D_2^L$, and since both sides of the inequality are positive, then $\overline{p} > 0$. Hence, if $p > \overline{p}$, entry occurs, yielding profits of $M_1^L(x_1) + \delta D_1^L$ for the low-stock incumbent. Such profits are lower than the low-stock incumbent's equilibrium profits of $M_1^L(\widetilde{x}_1^B)+\delta\overline{M}_1^L$. Indeed, from IC_L we know that $M_1^L(\widetilde{x}_1^B)+\delta\overline{M}_1^L \geq M_1^L(x_{1,monop}^L)+\delta D_1^L$. Since, in addition, $M_1^L(x_{1,monop}^L)+\delta D_1^L \geq M_1^L(x_1)+\delta D_1^L$ given that $x_{1,monop}^L$ is the argmax of $M_1^L(x_1)+\delta D_1^L$, then $M_1^L(\widetilde{x}_1^B) + \delta\overline{M}_1^L \geq M_1^L(x_1) + \delta D_1^L$ and therefore the low-stock incumbent does not deviate. Regarding the high-stock incumbent, it obtains profits of $M_1^H(x_1) + \delta D_1^H$ by deviating towards x_1 which are below its equilibrium profits of $M_1^H(x_{1,monop}^H) + \delta D_1^H$ since $x_{1,monop}^H$ is the argmax of $M_1^H(x_1) + \delta D_1^H$. Hence, the high-stock incumbent does not deviate towards x_1 either, and the separating equilibrium survives the Intuitive Criterion if $p > \overline{p}$.

If $p < \overline{p}$, then entry does not occur, yielding profits of $M_1^L(x_1) + \delta\overline{M}_1^L$ for the low-stock incumbent, which exceed its equilibrium profits $M_1^L(\widetilde{x}_1^B) + \delta \overline{M}_1^L$ since $x_1 \in (\widetilde{x}_1^B, x_{1,monop}^L]$. Then the separating equilibrium violates the Intuitive Criterion if $p < \overline{p}$. ■

6 *Proof of Proposition 3*

In a pooling strategy profile where both types of incumbent select x_1, equilibrium beliefs are $\mu(\theta_H|x_1) = p$ and $\mu(\theta_L|x_1) = 1 - p$, which coincide with the prior probability distribution over types. In addition, off-the-equilibrium beliefs cannot be identified using Bayes' rule, and for simplicity let us assume that, after observing $x_1' \neq x_1$, $\mu(\theta_H|x_1') = 1$. As shown in the proof of Proposition 1, these beliefs induce the entrant to enter after observing x_1'. Otherwise the entrant stays out. On the other hand, after observing x_1, the entrant enters if and only if $pD_2^H(x_1) + (1-p)D_2^L(x_1) > 0$ or

$$p > \frac{-D_2^L(x_1)}{D_2^H(x_1) - D_2^L(x_1)} \equiv \overline{p}(x_1)$$

where $D_2^H(x_1) > 0$, implying that $D_2^H(x_1) - D_2^L(x_1) > -D_2^L(x_1)$, and since both sides of the inequality are positive, we can conclude that the entrant enters if $p > \overline{p}(x_1)$, and stays out otherwise. Note that if entry occurs after x_1, this induces every type of incumbent to select $x_1^{K,E}$. But since $x_1^{H,E} \neq x_1^{L,E}$ this strategy profile cannot be a pooling equilibrium. Hence, it must be that $p < \overline{p}(x_1)$ inducing the entrant to stay out. Let us start by checking under which conditions the high-stock incumbent does not deviate from x_1. By selecting x_1, it deters entry obtaining $M_1^H(x_1)+\delta\overline{M}_1^H(x_1)$. By deviating towards $x_1' \neq x_1$ it attracts entry, yielding a payoff of $M_1^H(x_1') + \delta D_1^H(x_1')$, which is maximized at $x_1^{H,E}$. Hence, the high-stock incumbent does not deviate from x_1 if,

$$M_1^H(x_1) + \delta\overline{M}_1^H(x_1) \geq M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$$

or equivalently,

$$M_1^H(x_1^{H,E}) - M_1^H(x_1) \leq \delta\left[\overline{M}_1^H(x_1) - D_1^H(x_1^{H,E})\right] \tag{IC_H}$$

and similarly, for the low-stock incumbent,

$$M_1^L(x_1^{L,E}) - M_1^L(x_1) \leq \delta\left[\overline{M}_1^L(x_1) - D_1^L(x_1^{L,E})\right] \tag{IC_L}$$

Hence, any x_1 simultaneously satisfying IC_H and IC_L constitutes a pooling equilibrium first-period appropriation of the signaling game.

Intuitive Criterion. *Case 1.* Let us analyze if the pooling first-period appropriation $x_1 = x_1^{L,NE}$ survives the Cho and Kreps' (1987) Intuitive Criterion. We first check if such appropriation level is equilibrium dominated for either type of incumbent. On one hand, the low-stock incumbent obtains an equilibrium profit of $M_1^L\left(x_1^{L,NE}\right) + \delta\overline{M}_1^L(x_1^{L,NE})$. By deviating towards $x_1' \neq x_1^{L,NE}$ the highest payoff that it obtains occurs when entry is deterred, yielding payoffs of $M_1^L(x_1') + \delta\overline{M}_1^L(x_1')$, which lie below its equilibrium profits since $M_1^L(x_1') + \delta\overline{M}_1^L(x_1')$ reaches its maximum at exactly $x_1' = x_1^{L,NE}$. Hence, the low-stock incumbent does not have incentives to deviate from the pooling appropriation level $x_1 = x_1^{L,NE}$. On the other hand, the high-stock incumbent obtains an equilibrium profit of $M_1^H\left(x_1^{L,NE}\right) + \delta\overline{M}_1^H(x_1^{L,NE})$. By deviating towards $x_1' \neq x_1^{L,NE}$ the highest payoff that it obtains occurs when entry is deterred, yielding payoffs of $M_1^H(x_1') + \delta\overline{M}_1^H(x_1')$. Therefore, the high-stock incumbent does not have incentives to deviate if $M_1^H\left(x_1^{L,NE}\right) + \delta\overline{M}_1^H(x_1^{L,NE}) \geq M_1^H(x_1') + \delta\overline{M}_1^H(x_1')$, which only holds for $x_1' \in (x_1^{L,NE}, x_1^{H,NE}]$. Hence, the entrant assigns full probability to the stock being high for every deviation $x_1' \in (x_1^{L,NE}, x_1^{H,NE}]$, i.e., $\mu(\theta_H|x_1') = 1$, whereas its updated beliefs are unaffected after observing any other deviation. Thus, after observing $x_1' \in (x_1^{L,NE}, x_1^{H,NE}]$, the entrant believes that such deviation can only come from a high-stock incumbent and enters. The high-stock incumbent's profits from deviating towards x_1' are $M_1^H(x_1') + \delta D_1^H(x_1')$, which are lower than its equilibrium profits if

$$M_1^H\left(x_1^{L,NE}\right) + \delta\overline{M}_1^H(x_1^{L,NE}) \geq M_1^H(x_1') + \delta D_1^H(x_1'). \tag{A.6}$$

Note that deviation profits, $M_1^H(x_1') + \delta D_1^H(x_1')$, are maximal at $x_1' = x_1^{H,E}$, yielding profits of $M_1^H\left(x_1^{H,E}\right) + \delta D_1^H(x_1^{H,E})$. Hence, if $M_1^H\left(x_1^{L,NE}\right) + \delta\overline{M}_1^H(x_1^{L,NE}) \geq M_1^H\left(x_1^{H,E}\right) + \delta D_1^H(x_1^{H,E})$, then condition A.6 holds for all deviations $x_1' \in (x_1^{L,NE}, x_1^{H,NE}]$. Rearranging the last inequality,

$$M_1^H\left(x_1^{H,E}\right) - M_1^H\left(x_1^{L,NE}\right) \leq \delta\left[\overline{M}_1^H(x_1^{L,NE}) - D_1^H(x_1^{H,E})\right]$$

which graphically implies that the height of the $M_1^H\left(x_1^{H,E}\right) - M_1^H\left(x_1^L\right)$ curve evaluated at $x_1^L = x_1^{L,NE}$ is below the height of the $\delta\left[\overline{M}_1^H(x_1^L) - D_1^H(x_1^{H,E})\right]$ curve also evaluated at $x_1^L = x_1^{L,NE}$. This condition is hence satisfied since $x_1^{L,NE} > x_1^B$. Therefore, the high-stock incumbent does not have incentives to deviate either, and the pooling PBE in which $x_1 = x_1^{L,NE}$ survives the Intuitive Criterion.

Case 2. Let us next check if the pooling first-period appropriation level $x_1 > x_1^{L,NE}$ survives the Cho and Kreps' (1987) Intuitive Criterion. On one hand, the low-stock incumbent obtains $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$ in equilibrium. By instead deviating towards $x_1' \neq x_1$, the highest profit that it can obtain is $M_1^L(x_1') + \delta\overline{M}_1^L(x_1')$, which exceeds its equilibrium profit of $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$ if $x_1' \in (x_1^{L,NE}, x_1)$ given the concavity of the $M_1^L(x_1') + \delta\overline{M}_1^L(x_1')$ function with respect to x_1'. On the other hand, the high-stock incumbent obtains $M_1^H(x_1) + \delta\overline{M}_1^H(x_1)$ in equilibrium. By instead deviating towards $x_1' \neq x_1$, the highest profit that it can obtain is $M_1^H(x_1') + \delta\overline{M}_1^H(x_1')$, which exceeds its equilibrium profit of $M_1^H(x_1) + \delta\overline{M}_1^H(x_1)$ if $x_1' \in (x_1^{H,NE}, x_1)$. Hence, beliefs can be restricted to $\mu(\theta_H|x_1') = 0$ after observing a deviation $x_1' \in (x_1^{L,NE}, x_1^{H,NE})$. (Otherwise, entrant's beliefs are unaffected, since either both types of incumbent have incentives to deviate or none of them has.) Therefore, after observing a deviation $x_1' \in (x_1^{L,NE}, x_1^{H,NE})$, the entrant believes that the stock must be low, and chooses not to enter. Under these updated beliefs, the low-stock incumbent's profit exceeds its pooling equilibrium profits. Hence, the low-stock incumbent deviates towards $x_1' \in (x_1^{L,NE}, x_1^{H,NE})$. Therefore, the pooling PBE where $x_1 > x_1^{L,NE}$ violates the Intuitive Criterion.

Case 3. Let us finally check if the pooling first-period appropriation level $x_1 < x_1^{L,NE}$ survives the Cho and Kreps' (1987) Intuitive Criterion. On one hand, the low-stock incumbent obtains $M_1^L(x_1) + \delta \overline{M}_1^L(x_1)$ in equilibrium. By instead deviating towards $x_1^{L,NE}$, the highest profit it can obtain occurs when entry is deterred, yielding profits of $M_1^L\left(x_1^{L,NE}\right) + \delta \overline{M}_1^L(x_1^{L,NE})$, which exceeds its equilibrium profits if $M_1^L\left(x_1^{L,NE}\right) + \delta \overline{M}_1^L(x_1^{L,NE}) \geq M_1^L(x_1) + \delta \overline{M}_1^L(x_1)$, which is true since $x_1 < x_1^{L,NE}$. On the other hand, the high-stock incumbent obtains $M_1^H(x_1) + \delta \overline{M}_1^H(x_1)$ in equilibrium. By instead deviating towards $x_1^{L,NE}$, the highest profit it can obtain occurs after no entry, yielding profits of $M_1^H\left(x_1^{L,NE}\right) + \delta \overline{M}_1^H(x_1^{L,NE})$, which exceeds its equilibrium profits since $M_1^H(x_1) + \delta \overline{M}_1^H(x_1) \leq M_1^H\left(x_1^{L,NE}\right) + \delta \overline{M}_1^H(x_1^{L,NE})$ given that $x_1 < x_1^{L,NE} < x_1^{H,NE}$ and by concavity. Therefore, both types of incumbent have incentives to deviate towards $x_1^{L,NE}$ and entrant's beliefs cannot be updated, i.e., $\mu\left(\theta_H | x_1^{L,NE}\right) = p$, inducing no entry. Given these beliefs, both types of incumbent deviate towards $x_1^{L,NE}$, obtaining higher profits than in equilibrium. Hence, the pooling strategy profile in which both types select $x_1 < x_1^{L,NE}$ violates the Intuitive Criterion. ■