The Signaling Role of Subsidies*

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Abstract

This paper investigates the effect of monopoly subsidies on entry deterrence. We consider a potential entrant who observes two signals: the subsidy set by the regulator and the output level produced by the incumbent firm. We show that not only a separating equilibrium can be supported, where information about the incumbent’s costs is conveyed to the entrant, but also a pooling equilibrium, where the actions of regulator and incumbent conceal the monopolist’s type, thus deterring entry. We demonstrate that the regulator strategically designs subsidies to facilitate, or hinder, entry deterrence, depending on which outcome yields the largest social welfare. Furthermore, we compare equilibrium welfare relative to two benchmarks: complete information environments, and standard entry-deterrence games where the regulator is absent.

KEYWORDS: Entry deterrence; Signaling; Monopoly subsidies.

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1 Introduction

Several monopolized industries often benefit from subsidies allowing them to increase their output levels. For instance, Monsanto sells more than 70% of genetically modified seeds in the U.S. and has consistently received subsidies from the USDA.\(^1\) In the context of U.S. commercial airlines, Goldsbee and Syverson (2008) empirically show that some airlines, initially operating as monopolies in the route between two cities, decide to significantly reduce prices in order to prevent entry. Despite their entry-deterring behavior, commercial airlines have recurrently benefited from output-related subsidies.\(^2,3\)

Despite their widespread use, the regulation literature has overlooked the informative content that subsidies provide to potential entrants. In this paper, we demonstrate that subsidy policy can help the incumbent firm conceal information from potential entrants, thus hindering entry and competition under certain conditions. In addition, we show that subsidies can be welfare improving, despite their negative effect on entry. Our results, hence, suggest that the regulator strategically designs subsidies to reveal or conceal information, depending on which outcome yields the largest social welfare.

We examine an entry-deterrence game in which a regulator provides a per-unit subsidy in each period. In particular, we consider settings where the incumbent firm has been recently privatized after being publicly owned and managed for several years, allowing the regulator to accumulate information about the incumbent’s costs.\(^4\) In this context, the potential entrant, being uninformed about the incumbent’s costs, observes two signals to assess market prospects: the incumbent’s output level, as in standard entry-deterrence games, but also the subsidy set by the regulator. As

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\(^1\) In particular, the USDA Federal Crop Insurance Corporation offers significant insurance discounts (about $2 per acre, or $2,000 for a typical 1,000-acre farm) to farmers who plant 75-80% of their crops using Monsanto’s genetically modified seeds. Importantly, these insurance discounts are not offered to farmers using genetically modified seeds from other firms. Similarly, the Korean steel company Posco, which until 1992 had monopoly status for many of its products in the domestic market, also received output-related subsidies. For instance, the Korean government provided Posco with discounted user rates for many government services, such as discounted railroad rate of 40%, port rate of 50%, water-supply rate of 30%, and gas rate of 20%. For more details on the subsidies to Posco, see Amsden (1989) and Park (2003).

\(^2\) As part of the Essential Air Service program, the U.S. Department of Transportation provides output-related subsidies to airlines serving 152 rural communities across the country. These companies frequently maintain a monopolistic position on this type of routes and receive subsidies from this program which, in certain flights, can reach an average of $74 per passenger; see Bailey (2006).

\(^3\) Finally, the industry of electricity production is often regarded as a regional monopoly in many states across the U.S. and firms in this sector have also received generous federal and state subsidies to increase production. According to Slocum (2007), for instance, 92% of U.S. households have no ability to choose an alternative electricity supplier, since the wholesale market of power generation is essentially monopolized. Federal subsidies directly related to electricity production were estimated at $6.7 billion in 2007, or about 41% of total energy subsidies; see EIA (2008).

\(^4\) Several public companies were privatized in the United Kingdom, such as British Steel (privatized in 1988), and British Energy (in 1996). Other examples include Petro-Canada (1991) and Nova Scotia Power (1992) in Canada. In addition, many planned economies have also experienced major privatization processes, such as Russia with LUKOil (1995) and Novolipetsk Steel (1995). Many of these firms still receive generous subsidies, such as LUKOil, the largest oil producer in Russia, which benefited from a large share of the US$100 billion in subsidies directed to fossil-fuel producers in 2009, IEA (2010). For accounts of these privatization processes, see Kay and Thompson (1986) and Waterson (1988). Finally, China has recently started to privatize some public companies, as reported by the OECD (2009) and Gan et al. (2012).
a consequence, we study a new role of subsidies since, in addition to their standard goal to induce efficient output levels, they can be used as a tool to facilitate the transmission of information, thus promoting or deterring entry. The case of Dow Chemicals, a monopolist in the U.S. magnesium industry, provides evidence of entry-deterring practices facilitated by regulation. Regulators accumulated information about Dow’s production during the Korean War, a period in which magnesium production plants were publicly owned and managed. In 1970, the EPA introduced the National Ambient Air Quality Standards (NAAQS), affecting the emission of two pollutants generated in the production of magnesium: carbon monoxide and particulate matter. The following year, however, the state of Texas, where most Dow magnesium plants were located, passed its own Clean Air Act, allowing Dow to ignore some of the emission requirements in the NAAQS. Such state law can, hence, be interpreted as an output subsidy to Dow. Interestingly, this implicit subsidy led Dow to substantially increase its magnesium production during the early 1970s, which successfully deterred the entry of potential competitors, such as Kaiser Aluminum and Norsk Hydro; and delayed the entry of Alcoa until 1976.5

The paper shows the existence of two types of equilibrium outcomes: a separating equilibrium, where information about the incumbent’s costs is fully revealed to the entrant, and a pooling equilibrium, where such information is concealed. In the separating equilibrium, the actions of both informed agents (regulator and incumbent) convey the incumbent’s type to the entrant, i.e., they both choose the same type-dependent strategies as under complete information. Hence, the presence of an additional signal (originating from the regulator) induces players to behave as under complete information contexts, entailing a similar welfare level; a non-distortionary result in the line of models in which the entrant observes signals stemming from two incumbent firms, such as Bagwell and Ramey (1991) and Schultz (1999).

This non-distortionary finding, however, does not imply that the regulator’s presence in a setting of incomplete information is welfare neutral. Instead, the regulator’s ability to induce optimal output levels during both periods produces a positive effect on welfare; ultimately yielding an unambiguously larger welfare than in signaling games where the regulator is absent.

In the pooling equilibrium, in contrast, both regulator and incumbent’s actions conceal information from the entrant (they select type-independent strategies), thus deterring entry. In particular, the high-cost incumbent increases its output—in order to mimic the low-cost incumbent, i.e., it “overproduces”— while the regulator provides the subsidy corresponding to the low-cost incumbent, i.e., he “over-subsidizes.” By setting such a subsidy, the regulator gives rise to negative and positive welfare effects: on one hand, he induces the production of an inefficient output level but, on the other hand, entry is deterred, thus entailing savings in entry costs. As a consequence, the regulator is only willing to “over-subsidize” when the savings in entry costs offset the welfare loss that arises from overproduction. In this setting, regulator’s and incumbent’s preferences are aligned and, hence, the former supports the firm’s entry-deterring practices. In contrast, their preferences about entry are misaligned if suboptimal subsidies generate large welfare losses. In this case, the

5For more details, see Lieberman (1987), Rosenbaum (1998) and Friedrich and Mordike (2006).
regulator prefers to behave as under complete information, thus hindering the incumbent’s ability
to deter entry.

We furthermore show that the pooling equilibrium is more likely to emerge when firms’ costs
are symmetric, i.e., the difference between a high- and low-cost incumbent is small. Specifically,
the welfare loss that arises from the incumbent’s mimicking effort diminishes as costs become
symmetric, thus expanding the set of parameters under which this equilibrium can be sustained.
From a policy perspective, this result suggests that policies that support inefficient firms in their
acquisition of more advanced technologies would actually facilitate the concealment of information
from potential entrants, further promoting entry deterrence.

Our findings, hence, show that regulatory agencies can strategically facilitate or inhibit the
entry-deterring practices of established firms, an element often ignored when designing or evaluating
subsidy programs to monopolized industries. While these programs might entail entry-deterring
consequences, our results demonstrate that their welfare effects might be positive.

Related literature. Our paper contributes to the literature on monopoly regulation where
the social planner has accurate information about the incumbent’s costs, extended by Baron and
Myerson (1982) to contexts where the regulator does not observe the incumbent’s costs, and further
developed by Laffont and Tirole (1986) and Lewis and Sappington (1989).\(^6\) Unlike these articles,
however, we consider a setting where a regulated monopolist faces the threat of entry in the next
period. In the complete information game, we show that monopoly subsidies cannot be used
to deter entry, since the entry decision solely depends on the incumbent’s efficiency level.\(^7\) Under
incomplete information, however, monopoly subsidies can be used to convey or conceal information,
thus affecting entry in the industry.

Our paper also connects to entry-deterrance models where the regulator is absent; see Milgrom
and Roberts (1982), Harrington (1986), and Ridley (2008). Unlike these studies, we analyze firms’
actions within a standard regulatory framework, and investigate the effects of regulation on entry-
deterrence and competition. Since the uninformed entrant observes two signals, our model relates
to the signaling literature that considers industries in which the uninformed party observes several
signals, originating from either one or multiple senders. Milgrom and Roberts (1986), for instance,
analyze an informed firm who uses two signals, price and advertising, to convey the quality of its
product to potential customers.\(^8\) While we also study information transmission with two signals, in
our model they stem from two different informed agents (the regulator and the incumbent), rather
than from the same player. We demonstrate that, in contrast to their results, the presence of two

\(^6\)In particular, Laffont and Tirole (1986) consider the effects of distortive taxation on optimal regulation, showing
the existence of a trade-off between the information rent of the regulated firm and efficiency. Lewis and Sappington
(1989) extend the Baron-Myerson model by having the regulator not only uninformed about a firm’s marginal cost
but also about its fixed costs, the latter being negatively correlated with the former.

\(^7\)In a setting where the regulator strategically selects trade and subsidy policy, Dixit and Kyle (1985) show that
a perfectly-informed regulator can affect the entry decision of foreign firms. Their paper, however, does not analyze
the signaling role of subsidy policy.

\(^8\)Bagwell and Ramey (1990) and Albaek and Overgaard (1994) also examine entry deterrence in a model where
the potential entrant can perfectly observe both the incumbent’s pre-entry pricing strategy and its advertising
expenditures.
informed agents can support the emergence of a pooling equilibrium in which information about
the incumbent’s costs is concealed from the entrant, thus deterring entry.

This paper is, hence, closer to entry-deterrence models in which the uninformed player observes
signals originating from different senders; such as Harrington (1987) and Bagwell and Ramey (1991),
who study the use of limit pricing by two incumbent firms with common private information about
their production costs.\textsuperscript{9,10} Our analysis is specially connected to Schultz’s (1999) study of entry
deterrence in markets where two incumbent firms have opposing interests regarding entry. He
finds that a pooling equilibrium can be supported whereby firms’ signals conceal information about
market demand from the entrant, thus deterring it from the industry. This pooling equilibrium
emerges when the interests of both firms are similar. We likewise show that such equilibrium arises
when the regulator’s and incumbent’s preferences are aligned. Our paper, furthermore, shows that
such equilibrium, despite deterring entry, unambiguously entails a welfare improvement relative to
complete information.\textsuperscript{11} By contrast, if their preferences are misaligned, subsidy policy inhibits the
incumbent’s concealment of information, and entry occurs.\textsuperscript{12}

The next section describes the model under complete information. Section 3 examines the sig-
naling game and sections 4 and 5 analyze the separating and pooling equilibrium, respectively, also
providing welfare comparisons. Section 6 discusses our equilibrium results and policy implications.

2 Complete information

Let us examine an entry game where a monopolist incumbent initially operates and an entrant
must decide whether or not to join the market. In addition, consider a regulator who sets a subsidy
per unit of output in every stage of the game. This section analyzes the case where all players are
informed about the incumbent’s marginal cost, while sections 3-5 examine the case in which the

\textsuperscript{9}Martin (1995) considers an industry where two incumbent firms might have different, rather than common,
production costs, which each of them privately observes. He shows that each incumbent’s incentives to signal its own
production cost to its rival and to the potential entrant allow for the emergence of pooling strategy profiles under
conditions that did not support this type of equilibrium when all firms share the same production cost.

\textsuperscript{10}These models also relate to the literature that examines industries in which two or more firms use two signals,
price and expenditure on uninformative advertising, to convey their product quality to uninformed consumers; as in
Hertzendorf and Overgaard (2001) and Fluet and Garella (2002), which extend Milgrom and Robert’s (1986) model to
two firms. Daughety and Reinganum (2007) further develop such information setting in two respects: first, allowing
for each firm to privately observe the quality of its own product, rather than assuming that both firms can observe
each others’ qualities and, second, considering that consumers can have different preferences for the good that each
firm offers. For other recent models analyzing firms’ signal of their product quality, see Daughety and Reinganum

\textsuperscript{11}Espinola-Arredondo and Munoz-Garcia (2013) examine a similar information setting, whereby a polluting firm
faces the threat of entry. Their paper, however, focuses on how the incumbent’s overproduction decision, in order to
deter entry, generates more pollution, which gives rise to an additional form of inefficiency, absent under complete
information, ultimately calling for a stricter environmental policy.

\textsuperscript{12}In the field of capital-structure decisions, Gertner et al. (1988) analyze an enlarged entry deterrence model
where an informed firm sends a signal about its profitability to two uninformed audiences: the capital and product
market. In particular, Gertner et al. (1988) show that the emergence of the separating or pooling equilibrium in the
capital market critically depends on whether the incumbent is interested in revealing or concealing her type to the
product market. Hence, the occurrence of separating or pooling equilibria is endogenous. Similarly, in our paper, the
emergence of the separating or pooling equilibrium depends on whether the regulator seeks to attract or deter entry,
respectively.
entrant is unable to observe such a cost. We study a two-stage game where, in the first stage, the regulator selects a subsidy \( s_1 \) and the monopolist responds by maximizing its profits,

\[
\max_q (1 - q)q - (c^K_{\text{inc}} - s_1)q
\]

where \( c^K_{\text{inc}} \) denotes the incumbent’s marginal costs for any type \( K = \{H, L\} \), \( 1 > c^H_{\text{inc}} > c^L_{\text{inc}} \geq 0 \), and \( P(q) = 1 - q \) is the inverse market demand. In the second stage, a potential entrant decides whether or not to join. The regulator then revises his subsidy policy \( s_2 \) and, if entry occurs, firms compete as Cournot duopolists, simultaneously selecting production levels \( x_{\text{inc}} \) and \( x_{\text{ent}} \), for the incumbent and entrant, respectively. Otherwise, the incumbent maintains its monopoly power during both periods. In addition, the entrant’s marginal cost, \( c_{\text{ent}} \), coincides with that of the high-cost incumbent, since it lacks experience in the industry.\(^{13}\) The entrant must incur a fixed entry cost \( F > 0 \) which induces entry when the incumbent’s costs are high, but deters it when they are low. Finally, the regulator’s social welfare function in the first period is

\[
SW = \lambda CS(q) + (1 - \lambda) [PS(q) - s_1q]
\]

where \( CS(q) \) (\( PS(q) \)) denotes consumer surplus (producer surplus, respectively), \( s_1q \) is the government’s total expenditure on subsidies, and \( \lambda \) represents the weight that the regulator assigns to consumer surplus. The second term, thus, reduces to \((1 - q)q - c^K_{\text{inc}}q\). For compactness, let us use the normalization \( \gamma \equiv \lambda/(1 - \lambda) \), where \( \gamma > 0 \) represents the relative weight on consumer surplus.\(^{14}\) Social welfare, hence, simplifies to \( \gamma CS(q) + (1 - q)q - c^K_{\text{inc}}q \).\(^{15}\) A similar social welfare function applies to the second-period game. We next describe output and subsidies in the subgame perfect equilibrium of the game.

**Lemma 1.** In the first period, the regulator sets a subsidy \( s^K_1 = \gamma \frac{1-c^K_{\text{inc}}}{2-\gamma}, \) where \( K = \{H, L\} \), and the incumbent responds with an output function \( q^K(s_1) = \frac{1-c^K_{\text{inc}} - s_1}{2} \), which in equilibrium implies \( q^K(s^K_1) = \frac{1-c^K_{\text{inc}}}{2-\gamma} = q^K_{SO} \). Entry only occurs when the incumbent’s costs are high. In the second period, if entry does not ensue (NE), the regulator maintains subsidies at \( s^K_{2,\text{NE}} = s^K_1 \), and the incumbent responds selecting \( x^K_{\text{inc}}(s^K_2) \) which coincides with \( q^K(s^K_1) \). If entry occurs (E), the regulator sets a second-period subsidy \( s^K_{2,E} = \frac{(2\gamma-1)(1-c^K_{\text{inc}})}{2(2-\gamma)} \) and \( s^L_{2,E} = \frac{2\gamma-1+(2-\gamma)c^H_{\text{inc}}-(1+\gamma)c^L_{\text{inc}}}{2(2-\gamma)} \).

\(^{13}\)If the entrant’s costs were, instead, lower than those of the high-cost incumbent, \( c_{\text{ent}} < c^H_{\text{inc}} \), the entrant would be willing to enter under larger parameter conditions, but would not qualitative affect our equilibrium results.

\(^{14}\)For generality, our model allows for \( \gamma < 1 \), which could arise if, for instance, the regulator holds stocks from the recently privatized monopoly (a common observation in certain underdeveloped countries, or highly corrupted regimes) and \( \gamma > 1 \) whereby the regulator assigns a larger weight to consumer than producer surplus.

\(^{15}\)The subsidy in our model is hence financed with non-distortionary taxes, as in Dixit and Kyle (1985). Otherwise, the marginal cost of raising public funds should enter into the regulator’s social welfare function. Section 5.1 discusses that this consideration would shrink the set of parameter values sustaining some of our equilibrium results.
\[
\frac{1 + (1 - \gamma) c_{\text{inc}}^L}{2 - \gamma} > c_{\text{inc}}^H > c_{\text{inc}}^L.
\]

Under monopoly, the regulator seeks to induce the socially optimal output level \( q_{SO}^K \equiv \frac{1 - c_{\text{inc}}^L}{2 - \gamma} \), which is increasing in the weight on consumer surplus, \( \gamma \), and decreasing in the incumbent’s costs, \( c_{\text{inc}}^L \). Therefore, the subsidy that induces this output level is also increasing in \( \gamma \) and decreasing in \( c_{\text{inc}}^L \). Note that when the regulator assigns no weight to consumers, \( \gamma = 0 \), output level \( q_{SO}^K \) coincides with that of an unregulated monopoly, i.e., \( \frac{1 - c_{\text{inc}}^L}{2} \), whereas when \( \gamma = 1 \), the socially optimal output \( q_{SO}^K \) becomes the perfectly competitive output \( 1 - c_{\text{inc}}^L \).

Upon entry, the regulator seeks to induce the same socially optimal output at the aggregate level.\(^{16}\) In this case, however, subsidy \( s_{2, E}^{K, E} \) is not as generous as under monopoly, i.e., \( s_{2, E}^{K, E} < s_{2, NE}^{K, NE} \), since aggregate output under duopoly is closer to the social optimum.\(^{17}\)

Therefore, under complete information subsidy policy cannot be used by the regulator to promote or hinder entry, since the entry decision solely depends on the incumbent’s costs. Under incomplete information, however, we next show that the informative content of subsidies can be used as a tool to deter entry. In particular, one might expect that subsidy policy could be used to attract rather than deter entry. However, our results show that regulator can have incentives to facilitate the incumbent’s entry-deterring strategies when firms’ costs are relatively symmetric (and thus subsidies give rise to small inefficiencies) and when, despite costs being asymmetric, the weight on consumer surplus is sufficiently large.

### 3 Signaling

Let us now analyze the case where the incumbent and regulator are privately informed about the incumbent’s marginal costs. This information context describes settings where the social planner has accumulated information about the incumbent’s cost structure over time, e.g., publicly managed monopolies that were recently privatized.\(^{18}\) The entrant, however, does not observe the incumbent’s cost and, hence, bases its entry decision on the observed first-period output level and subsidy.

\(^{16}\)Therefore, in a context of complete information, the regulator can induce socially optimal output level both with and without entry, thus implying that entry is socially undesirable for any entry cost \( F > 0 \). However, under a setting of incomplete information, we next show that this outcome does not necessarily hold for all entry costs.

\(^{17}\)In addition, the duopoly subsidy when the incumbent’s costs are high, \( s_{2}^{H, E} \), is positive for \( \gamma > 1/2 \), yielding a positive output for both firms. Since we aim at investigating the effects of subsidies on entry patterns, we hereafter focus on positive subsidies, i.e., \( \gamma > 1/2 \). If, in contrast, \( \gamma < 1/2 \), the subsidy would become zero, and firms would produce the standard duopoly output.

\(^{18}\)Assuming that the regulator was, instead, imperfectly informed about the incumbent’s costs would not qualitatively affect our equilibrium results. It could nonetheless induce the regulator to use subsidy policy in order to conceal the incumbent’s type under larger conditions, thus facilitating this firm’s entry-deterring practices. In the context of environmental policy, Espinola-Arredondo and Munoz-Garcia (2014) consider a regulator who, despite being uninformed, has an informational advantage relative to the potential entrant, and sets emission fees to curb pollution. Similarly as in our paper, they show that entry-deterring equilibria can be sustained in which emission fees facilitate the incumbent’s concealment of information from potential competitors. However, such equilibrium emerges under larger conditions when the regulator is poorly informed about the incumbent’s costs than when he perfectly observes its costs.
time structure of this signaling game is as follows.\(^\text{19}\)

1. Nature decides the realization of the incumbent’s marginal costs, either high or low, with probabilities \(p \in (0, 1)\) and \(1 - p\), respectively. Incumbent and regulator privately observe this realization but the entrant does not.

2. The regulator sets a first-period subsidy \(s_1\) and the incumbent responds choosing its first-period output level, \(q(s_1)\).

3. Observing the pair of signals \((s_1, q(s_1))\), the entrant forms beliefs about the incumbent’s marginal costs. Let \(\mu(c_{inc}^H|q(s_1), s_1)\) denote the entrant’s posterior belief about the incumbent’s costs being high.

4. Given these beliefs, the entrant decides whether or not to enter the industry.

5. If entry does not occur, the regulator sets a second-period subsidy, \(s_2^{K,NE}\), and the incumbent responds producing \(x_{inc}^{K,NE}(s_2^{K,NE})\). If, in contrast, entry ensues, the entrant observes the incumbent’s costs and the regulator sets a second-period subsidy \(s_2^{K,E}\). Both firms then compete as Cournot duopolists, producing \(x_{inc}^{K,E}(s_2^{K,E})\) and \(x_{ent}^{K,E}(s_2^{K,E})\).\(^\text{20}\)

Hence, step 5 implies that information is revealed after entry and all agents behave as under complete information. As a consequence, we hereafter focus on the informative role of first-period actions, as described in steps 1-4. For compactness, let \(D_{ent}^{K}\) denote the entrant’s duopoly profits in equilibrium evaluated at subsidy \(s_2^{K,E}\) when the entrant faces a \(K\)-type incumbent. As specified in the previous section, entry is unprofitable when the incumbent’s costs are low, whereas it is profitable when costs are high, i.e., \(D_{ent}^{H} > F > D_{ent}^{L}\), where \(F\) is the fixed entry cost.

### 3.1 Beliefs upon observing two signals

Since the potential entrant observes two signals (subsidy level and output) originating from two different agents, the specification of its beliefs are more intricate than in standard entry-deterrence games. In particular, we assume that beliefs must meet the following consistency requirements.

Consider a separating strategy profile in which the regulator facing a high(low)-cost firm selects \(s_1^H (s_1^L)\) and the incumbent responds with output level \(q^H(s_1^H) (q^L(s_1^L))\), respectively. In this setting, if the entrant observes an equilibrium strategy pair \((s_1^H, q^H(s_1^H))\), it believes that the incumbent’s costs must be high, i.e., \(\mu(c_{inc}^H|q^H(s_1^H), s_1^H) = 1\), and enters; while after \((s_1^L, q^L(s_1^L))\), the

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\(^{19}\)To facilitate the comparison of our results with those of the literature on entry-deterrence games, e.g., Milgrom and Roberts (1982), we consider a similar information setting and time structure, whereby the incumbent’s costs are unobserved by the potential entrant, but the entrant’s can be anticipated by the incumbent given its experience in the industry.

\(^{20}\)Espinola-Arredondo et. al. (2014) show that, in the context of environmental policy, the use of emission fees which cannot be revised across periods still gives rise to equilibrium outcomes in which the regulator facilitates the incumbent’s concealment of information from potential entrants, thus promoting entry-deterring practices. Such entry-deterring equilibrium, nonetheless, emerges under larger conditions when regulation remains unaffected across time than when it can be rapidly revised, as that considered in our paper.
entrant’s beliefs are \( \mu(c^H_{inc} | q^L(s^H_1), s^L_1) = 0 \), and stays out, where \((s^H_1, q^H(s^H_1)) \neq (s^L_1, q^L(s^L_1)) \). Let us now examine off-the-equilibrium beliefs. First, if the regulator chooses an equilibrium subsidy \( s^H_1 \) but the incumbent deviates to an off-the-equilibrium output \( q(s^H_1) \), where \( q(s^H_1) \neq q^H(s^H_1) \), the entrant only relies on the signal of the non-deviating player (the regulator). Following the notion of “unprejudiced beliefs” by Bagwell and Ramey (1991) and Schultz (1999), we assume that the entrant’s beliefs are compatible with the strategy selected by the non-deviating player, and hence \( \mu(c^H_{inc} | q(s^H_1), s^H_1) = 1 \), thus attracting entry.\(^{21}\) Analogously, after strategy pair \((s_1, q^K(s_1))\), in which the regulator now selects the off-the-equilibrium subsidy \( s_1 \), where \( s_1 \neq s^H_1, s^L_1 \), but the incumbent selects equilibrium strategies, the entrant bases its entry decision on the incumbent’s signal alone, i.e., \( \mu(c^H_{inc} | q^H(s_1), s_1) = 1 \) and \( \mu(c^H_{inc} | q^L(s_1), s_1) = 0 \). Second, if the regulator sets an equilibrium subsidy of \( s^H_1 \), but the high-cost incumbent imitates the output function of the low-cost firm, \( q^L(s) \), the entrant observes equilibrium signals corresponding to two different types of incumbents. In this case, the entrant is in the dark: is the deviation originating from the high-cost incumbent, who mimics the output function of the low-cost firm, \( q^L(s_1) \), in order to deter entry? Or, is it coming from a regulator facing a low-cost incumbent, who chooses \( s^H_1 \) in order to attract entry? According to unprejudiced beliefs, the entrant cannot discern the incumbent’s costs with certainty, and thus cannot assign full probability to either type, i.e., \( \mu(c^H_{inc} | q^L(s^H_1), s^H_1) = \mu' \in (0, 1) \). Finally, when both regulator and incumbent select type-independent strategies, the entrant cannot update its beliefs upon observing subsidies and output, and the use of unprejudiced beliefs does not restrict the entrant’s beliefs. Hence, we apply the Cho and Kreps’ (1983) Intuitive Criterion to limit the set of pooling equilibria with reasonable beliefs.\(^{22}\)

The following section focuses on strategy profiles where both regulator and incumbent select type-dependent strategies and, thus, private information is conveyed to the entrant. Because both informed agents choose separating strategies, we refer to this type of profiles as two-sided separating equilibria. (Appendix 2 analyzes profiles where only one agent, either the regulator or the incumbent, chooses a type-dependent strategy, which we refer as one-sided separating equilibria.) Finally, we analyze strategy profiles in which both incumbent and entrant select type-independent strategies, i.e., pooling equilibria, and thus the entrant cannot infer the incumbent’s type.

### 4 Separating equilibrium

The following proposition shows that a separating equilibrium can be sustained where players behave as under complete information.

**Proposition 1.** A two-sided separating equilibrium (TS) can be supported in which the regulator

\(^{21}\)A similar argument is applicable to strategy pair \((s^H_1, q(s^H_1))\), where the entrant solely relies on the regulator’s actions and, hence, \( \mu(c^H_{inc} | q(s^H_1), s^H_1) = 0 \), deterring the entrant from the industry.

\(^{22}\)The application of Cho and Kreps’ (1983) Intuitive Criterion to the setting we analyze, with two signals, follows that in Schultz (1996, 1999). In particular, if the entrant observes an off-the-equilibrium message, and such a message is equilibrium dominated when the incumbent’s costs are high, but undominated when they are low, then the only incumbent who could benefit from sending such an off-the-equilibrium message is the low-cost firm. As a consequence, the entrant should believe that the incumbent’s costs must be low.
lator chooses the complete information type-dependent subsidy pair \((s^H_1, s^L_1)\), and the incumbent responds choosing the complete information type-dependent output pair \((q^H(s_1), q^L(s_1))\). This equilibrium can be sustained when the entrant’s off-the-equilibrium beliefs \(\mu(c^H_{\text{inc}}|q^L(s^H_1), s^H_1) = \mu'\) are sufficiently high, i.e., \(\mu' \geq \mu_1 = \frac{F-D^L_{\text{cost}}}{D^L_{\text{cost}}-D^H_{\text{cost}}}\), for any production costs. If, in contrast, \(\mu' < \mu_1\), this equilibrium exists if firms’ costs are sufficiently asymmetric, i.e., \(\epsilon^H_{\text{inc}} > \epsilon = \frac{\sqrt{3\delta+(2-\gamma)c^L_{\text{inc}}}}{\sqrt{3\delta+2-\gamma}}\).

Let us first examine the case in which off-the-equilibrium beliefs satisfy \(\mu' \geq \mu_1\). Hence, upon observing contradictory signals \((s^H_1, q^L(s^H_1))\), the entrant’s beliefs prescribe that the incumbent’s costs are likely high, thus attracting it to the industry. In this setting, the high-cost firm cannot deter entry by mimicking the output decision of the low-cost incumbent, \(q^L(s^H_1)\). Similarly, the regulator does not deviate from equilibrium strategies, since the TS yields optimal output levels, while deviations would entail inefficiencies, and thus the TS can be sustained for all cost parameters. In contrast, when the entrant’s beliefs are relatively low, \(\mu' < \mu_1\), the entrant responds staying out after observing contradictory signals \((s^H_1, q^L(s^H_1))\). In this context, the high-cost incumbent could successfully deter entry by imitating the low-cost firm, \(q^L(s^H_1)\), but such overproduction effort becomes too costly when \(\epsilon^H_{\text{inc}} > \epsilon\), thus inducing the incumbent to behave as under complete information.\(^{23}\)

The next corollary compares equilibrium welfare relative to two benchmarks: that arising under a complete information setting when the regulator is present, \(W_{RL}^{CI}\), and that in the separating equilibrium of a signaling game where the regulator is absent, \(W_{SE}^{NRL}\), as in Milgrom and Roberts (1982).

**Corollary 1.** Social welfare in the separating equilibrium, \(W_{SE}^{RL}\), coincides with that under complete information, \(W_{CI}^{RL}\), and it is weakly larger than that arising in signaling games where the regulator is absent, \(W_{SE}^{NRL}\).

Since subsidy and output levels under the TS coincide with those in complete information settings, both information contexts yield the same welfare level; a non-distortionary result similar to that in models where the potential entrant observes signals originating from two incumbent firms, such as Bagwell and Ramey (1991) and Schultz (1999). However, unlike signaling models where the regulator is absent, the presence of the regulator guarantees the production of the socially optimal output \(q_{SO}^K\) during both periods, entailing a higher social welfare, i.e., \(W_{SE}^{RL} > W_{SE}^{NRL}\). (Recall that Appendix 2 shows that strategy profiles where only one agent selects a type-dependent strategy cannot be sustained in equilibrium.)

\(^{23}\)Because unprejudiced beliefs allow for \(\mu' \in (0,1)\), this equilibrium result still holds when the entrant’s beliefs upon observing contradictory signals approach zero or one, i.e., the entrant has a “prejudice” in favor of the low- or high-cost incumbent, respectively.
5 Pooling equilibrium

In this section, we examine settings in which both regulator and incumbent choose a type-independent strategy and, therefore, no information is conveyed to the entrant.

**Proposition 2.** A pooling equilibrium can be supported in which the regulator selects a type-independent subsidy \( s_1^L \), the incumbent responds with a type-independent output function \( q^L(s_1) \), and entry does not ensue, if priors satisfy \( p \leq \overline{p} \), and entry costs are high, \( F > F(\gamma) \), where 
\[
F(\gamma) \equiv \frac{(c_{inc}^H - c_{inc}^L)^2}{2(2 - \gamma)}. 
\]
In addition, for admissible entry costs \( D_{ent}^H > F > D_{ent}^L \), \( F > F(\gamma) \) implies that \( \gamma > \overline{\gamma} \), where 
\[
\overline{\gamma} = 2 - \frac{(1 - c_{inc}^H)^2}{2(c_{inc}^H - c_{inc}^L)^2}. 
\]

The high-cost incumbent exerts an overproduction effort in order to mimic the low-cost firm, raising its output function from \( q^H(s_1) \) to \( q^L(s_1) \) as depicted in figure 1. The regulator, in addition, chooses a type-independent subsidy \( s_1^L \), rather than that under complete information \( s_1^H \), i.e., he over-subsidizes. Hence, the entrant cannot infer the incumbent’s type, and stays out of the industry given its low priors.\(^{24}\)

![Diagram of Pooling Equilibrium](image)

**Figure 1.** Pooling equilibrium.

Let us next examine the regulator’s incentives to set subsidy \( s_1^L \). On one hand, setting such a suboptimal subsidy generates a welfare loss, since the induced output \( q^L(s_1^L) \) is larger than the optimal output \( q^H_{SO} \). This welfare loss, however, becomes smaller as the regulator assigns a larger weight on consumer surplus. On the other hand, the subsidy deters entry, thus entailing savings in the entry costs, \( F \), i.e., a welfare gain. Therefore, the regulator is willing to set \( s_1^L \) when the savings in the entry costs are relatively large, i.e., \( F > F(\gamma) \). In this setting, the welfare gains offset the losses, ultimately yielding a larger social welfare than under complete information.

\(^{24}\)Note that if, instead, the regulator chose a subsidy \( s_1^B \) inducing the optimal output for this type of incumbent, \( q^H_{SO} \), the entrant would infer the incumbent’s cost and enter, thus eliminating the high-cost firm’s incentives to overproduce.
In addition, cutoff $F(\gamma)$ is increasing in $\gamma$. Intuitively, when the welfare loss from overproduction is relatively small (high $\gamma$) and the savings in entry costs are sufficiently large (high $F$), the regulator’s preferences for entry deterrence are aligned with the incumbent’s. In this context, the regulator sets $s^L_1$, which facilitates the incumbent’s entry-deterring practices. Otherwise, the regulator assigns a small weight on consumer surplus, $\gamma \leq \bar{\gamma}$, and the welfare loss from overproduction generates large inefficiencies. In this case, the regulator’s and incumbent’s preferences are misaligned, since the former prefers to behave as under complete information, setting a subsidy $s^H_1$, which ultimately attracts entry. The following corollary examines how cost symmetry affects the emergence of the pooling equilibrium (PE).

**Corollary 2 (Cost symmetry).** When firms’ costs are relatively symmetric, i.e., $C_2 > c^{inc}_H$, and therefore, the PE can be supported for all values of $\gamma$. In addition, when $C_1 \geq c^{inc}_H > C_2$, cutoff $\gamma$ satisfies $\gamma \in [1/2, 1]$ and, hence, the PE exists for all $\gamma > \bar{\gamma}$. Finally, when costs are asymmetric, i.e., $c^{inc}_H > C_1$, cutoff $\gamma$ satisfies $\gamma > 1$.

Corollary 2 shows that the PE can be sustained under larger parameter values when firms’ costs are symmetric. Figure 4 illustrates this result. Specifically, the welfare loss that arises from the incumbent’s mimicking effort diminishes as costs become more symmetric, thus expanding the set of parameters under which this equilibrium can be supported. This result helps us evaluate the effects of cost-reducing policies which, for instance, support relatively inefficient firms in their installation of new technologies. In particular, these policies would entail a reduction in the cost asymmetry between firms, ultimately facilitating the emergence of the entry-deterring practices predicted by the PE. Let us next investigate if the presence of two signals restricts the parameter values under which the PE can be sustained.

---

**Fig. 2.** Pooling PBE and cost symmetry.
Corollary 3 (Equilibrium conditions). The set of production costs that support a PE when the regulator is present, \( c^H_{\text{inc}} \leq C_1 \), also sustains this equilibrium when he is absent, \( c^H_{\text{inc}} \leq C_{\text{PE}}^{NR} \), where \( C_1 < C_{\text{PE}}^{NR} \) and \( C_{\text{PE}}^{NR} \equiv \frac{9c^L_{\text{inc}}-3(1-c^L_{\text{inc}})\sqrt{5}}{4} \).

Intuitively, the presence of two informed agents that can convey information, hinders the emergence of pooling equilibria, relative to settings where a single player seeks to conceal information from the entrant. The PE with and without regulator can be sustained only when incumbent’s costs are relatively symmetric, i.e., \( c^H_{\text{inc}} \leq C_1 \) and \( c^H_{\text{inc}} \leq C_{\text{PE}}^{NR} \). However, in models where the regulator is absent, such symmetry condition arises because the high-cost incumbent is only willing to mimic the output decision of the low-cost firm when its overproduction effort is not very costly.\(^{25}\) When regulation is present, in contrast, this condition emerges because of the inefficiencies that the regulator must bear. Specifically, he weighs the welfare loss that subsidy \( s^L_1 \) entails (which increases in the cost asymmetry) against the savings in entry costs, thus leading the regulator to select \( s^L_1 \) only when firms’ costs are relatively symmetric. The following corollary compares the welfare arising in this equilibrium with that when the regulator is absent.\(^{26}\)

Corollary 4 (Welfare comparison). Social welfare in the PE when the regulator is present, \( W_{\text{PE}}^{R,H} \), is larger than that arising in signaling games where the regulator is absent, \( W_{\text{PE}}^{NR,H} \), if \( \gamma \geq \overline{\gamma} \), where \( \overline{\gamma} \equiv \frac{4(c^H_{\text{inc}}-c^L_{\text{inc}})(1-c^L_{\text{inc}})}{2+(c^H_{\text{inc}}-2)c^H_{\text{inc}}+(c^L_{\text{inc}}-2)c^L_{\text{inc}}} \). In addition, (1) when firms’ costs are symmetric, i.e., \( C_4 \geq c^H_{\text{inc}} \), cutoff \( \overline{\gamma} \) satisfies \( \overline{\gamma} < 1/2 \) and, therefore, \( W_{\text{PE}}^{R,H} \geq W_{\text{PE}}^{NR,H} \) holds for all values of \( \gamma \); (2) when costs are \( C_3 \geq c^H_{\text{inc}} > C_4 \), cutoff \( \overline{\gamma} \) satisfies \( \overline{\gamma} \in [1/2,1] \) and, thus, \( W_{\text{PE}}^{R,H} \geq W_{\text{PE}}^{NR,H} \) for all \( \gamma \geq \overline{\gamma} \); and (3) for asymmetric costs, \( c^H_{\text{inc}} > C_3 \), cutoff \( \overline{\gamma} \) satisfies \( \overline{\gamma} > 1 \) and, hence, \( W_{\text{PE}}^{R,H} \leq W_{\text{PE}}^{NR,H} \) holds if \( \gamma < 1 \), where \( C_3 \equiv 3-\sqrt{7}(1-c^L_{\text{inc}})-2c^L_{\text{inc}} \) and \( C_4 \equiv 5-\sqrt{23}(1-c^L_{\text{inc}})-4c^L_{\text{inc}} \).

Under no regulation, the PE prescribes that the high-cost incumbent, despite increasing its production level to \( q^L(0) \) in order to deter entry, still produces below the social optimum, i.e., \( q^L(0) < q^H_{SO} \). This underproduction pattern continues in the second-period game, in which the incumbent produces its monopoly output \( x^H_{\text{inc}}(0) \). In contrast, when regulation is present, the incumbent produces a first-period output \( q^L(s^L_1) \), which exceeds the social optimum \( q^H_{SO} \). In addition, in the second period, the regulator induces an optimal output by setting \( s^H_1 \), i.e., \( x^H_{\text{inc}}(s^H_1) = x^H_{SO} \). Hence, if the weight on consumer surplus is high, the welfare loss arising from overproduction — when the regulator is present — is smaller than that emerging from underproduction — when he is absent — which entails that regulation becomes welfare improving.

\(^{25}\) Appendix 1 provides more details about the parameter conditions (i.e., cost asymmetry between the two types of firms) for which the pooling equilibrium can be sustained when the regulator is absent.

\(^{26}\)Corollary 4 does not compare social welfare in the pooling equilibrium with that under complete information since, for the pooling equilibrium to exist, it must yield a larger welfare level than in complete information.
Corollary 4 also shows that, when firms’ costs are symmetric, the welfare loss arising from overproduction decreases and, hence, welfare is larger with than without regulator for all values of \(\gamma\), i.e., the presence of the regulator is welfare improving. Figure 3 depicts this result. However, when costs become more asymmetric, such welfare loss is more substantial, and the PE entails a smaller welfare with than without regulator. In this context, the absence of regulation can, hence, be welfare superior. This result, however, does not entail that the regulator has incentives to set a zero subsidy to the incumbent when costs lie in region \(C_1 > c_{inc}^H > C_3\). While welfare is larger in the PE when the regulator is absent than in the PE in which he is present, setting a zero subsidy would convey the incumbent’s inefficient costs to the potential entrant, thus attracting entry.\(^{27}\) This decision would thus give rise to two inefficiencies: an underproduction during the first-period game (when the incumbent produces its monopoly output without subsidies) and the fixed entry costs that reduce producer surplus during the second-period game. The regulator would increase social welfare if, instead, sets complete-information subsidies. While this policy also induces entry, it eliminates the first type of inefficiency mentioned above, as it yields socially optimal output levels during the first-period game. However, we know that in this context there is an alternative policy generating an even larger welfare: the subsidies in the PE of the game, which are welfare superior to complete-information strategies. Hence, the regulator has incentives to still behave as prescribed in Proposition 2 and set positive subsidies.

As a consequence, the welfare comparisons in Corollary 4 help predict in which contexts the presence of the regulator is more necessary; namely, when firms are relatively symmetric in costs.

\(^{27}\)Note that the regulator does not have incentives to set a zero subsidy on the low-cost incumbent. Under PE subsidies, welfare is at the optimum, whereas setting zero subsidies in the first period would entail welfare losses emerging from underproduction in that period, and a costly entry in the second period.
Our results, however, cannot be interpreted as suggesting that regulators, after being present in an industry, have incentives to strategically set a zero subsidy, since the information that such action reveals would ultimately yield a lower welfare than that under the PE examined in this section.

6 Discussion and Conclusions

Publicizing the incumbent’s costs. At the beginning of the game, an informed regulator could have incentives to strategically disseminate information about the incumbent’s costs to potential entrants by, for instance, publicizing its costs in different media outlets. This action would transform the information structure of the game, from one where the entrant is uninformed, to a game where all agents are perfectly informed about the incumbent’s costs. Our results nonetheless suggest that the regulator is not always willing to distribute such information. In particular, the regulator is only interested in publicizing information when his weight on consumer surplus is low and/or firms’ costs are relatively asymmetric. Specifically, under these parameter conditions overall social welfare in the complete information game exceeds that in the PE.\textsuperscript{28} Otherwise, the regulator prefers to not publicize such information, thus supporting the incumbent’s concealment of its type from the entrant, as predicted in the PE.\textsuperscript{29} A similar argument can be used to evaluate the welfare consequences of distributing the statements, hearings, etc. of those Senate and House committees which are in charge of designing subsidy policy. Our findings suggest that regulators with interests that are misaligned with those of incumbent firms would try to make this information publicly available, thus hindering firms’ entry-deterring practices. Regulators whose preferences are aligned to the incumbent’s would, in contrast, limit the dissemination of such information.

Regulation. Our results identify a strategic role of monopoly subsidies often overlooked by the literature on monopoly regulation. In particular, a monopoly subsidy—usually considered as a tool to induce the incumbent produce the socially optimal output—provides additional entry-deterrence benefits, thus increasing the extent of the incumbent’s overproduction. The regulator anticipates this behavior when designing his subsidy policy and, in certain cases, he may strategically support the monopolist’s concealment of information, thus deterring entry. Our results do not imply, however, that social welfare decreases by the presence of the regulator. In particular, while the regulator can facilitate the monopolist ability to deter entry under certain cases, we show that his presence can actually be welfare improving.

Entry costs. Our results suggest that the PE is unlikely to arise in industries whose entry costs have experienced significant reductions, arising from technological or political reasons. Only the separating equilibrium would emerge in this case, whereby subsidy policy coincides with that under

\textsuperscript{28}If, in contrast, parameter conditions support the emergence of the separating equilibrium, the regulator is indifferent between distributing information or concealing it, since equilibrium behavior, and welfare, coincides under both information contexts.

\textsuperscript{29}Such context describes a different setting than that in our paper, since the regulator is allowed to strategically choose whether or not to publicize the incumbent’s type before setting $s_1$. Specifically, the sheer decision of the regulator of not distributing any information before the beginning of the game, suggests that he must be facing a high-cost incumbent, since otherwise the regulator would have publicized the incumbent’s type.
complete information, thus allowing the regulator to essentially ignore the information context in which firms operate. In contrast, if entry costs are high, the PE is more likely to arise. In this context, a regulator who ignored the information structure of the game, behaving as under complete information, would yield a suboptimal welfare level, $W_{R,H}^{CI}$, rather than the higher $W_{R,H}^{PE}$.

**Alternative policy tools.** The regulator could alternatively use entry costs, rather than first-period subsidies, in order to promote or hinder entry. In particular, by setting extremely high fixed entry costs, the regulator could hinder entry both when the incumbent’s costs are low and high, and both when the potential entrant is informed or uninformed. In this context, the regulator could still achieve socially optimal output levels from the incumbent by setting the subsidy policy described in Lemma 1 when entry does not ensue. Therefore, the use of such alternative policy would simplify the game to one in which the entrant’s access to information becomes irrelevant, as entry would be deterred both under complete and incomplete information, and in which the incumbent would not need to use its output decisions as an entry-deterring tool. Such a setting, however, is only possible if the regulator can set prohibitive entry costs, i.e., banning entry of new firms. If the regulator is not allowed to significantly increase entry costs (perhaps because of international agreements), and entry is still profitable when the incumbent’s costs are high (as considered in our model), our analysis demonstrates that regulatory agencies can use existing subsidies to incumbent firms (not those offered to the potential entrants) as a policy that conveys or conceal information from potential entrants and ultimately promotes social welfare.

**Distortionary taxation.** Following Dixit and Kyle (1985), we consider that subsidies are raised using non-distortionary taxes. If, instead, production subsidies are raised with distortionary taxes, the social welfare function would include the deadweight loss from taxation, and the parameter values sustaining our equilibrium predictions would be affected. In the separating equilibrium, players behave as under complete information, but welfare levels would be lower than in our model. In contrast, the over-subsidization result predicted by the PE would be ameliorated, since the cost of raising public funds in this context is larger. Hence, this would ultimately shrink the region of parameter values for which the regulator helps the incumbent to conceal its type from the potential entrant.

**Positive externalities.** Our analysis can be easily extended to the regulation of products that generate positive externalities, e.g., hybrid cars and solar panels. In particular, regulation under complete information would internalize the positive effects that these products generate, thus calling for a higher optimal output than in our current study, and thus larger subsidies. Under the PE, however, subsidies to this type of firms would exceed the social optimum. Nonetheless, if such over-subsidization occurs, our results suggest that it would be welfare improving.

**Further research.** The model provides extensions to other fields of economics. In particular, the monopolist’s first-period actions do not generate externalities on other agents’ payoffs. In several settings, however, governments use subsidies in order to promote (reduce) goods that impose positive (negative, respectively) externalities. An extension allowing for the presence of externalities could be modeled, for instance, by introducing the social benefit (or cost) of output in the regulator’s
social welfare function. Another venue of further research would consider contexts where the subsidy set in the first period is inflexible across time, i.e., it cannot be revised at the beginning of the second-period game. This regulatory setting describes countries whose legislative process is rigid, thus not allowing the regulator to rapidly adjust his second-period policy. Such inflexibility could affect the regulator’s willingness to increase subsidies, as prescribed in the pooling equilibrium, since such subsidies would be permanent, thus imposing welfare effects across both periods.

7 Appendix

7.1 Appendix 1 - Separating and Pooling equilibrium without regulation

Separating PBE. In the absence of subsidy policy, the high-cost incumbent selects its complete-information output $q^H$ rather than deviating towards the production level of the low-cost firm, $q^{L,Sep} > q^L$, which deters entry, if

$$M_{inc}^H(q^H) + \delta D_{inc}^H \geq M_{inc}^H(q^{L,Sep}) + \delta \overline{M}_{inc}^H.$$  \hspace{1cm} (C1)

Specifically, note that output $q^L$ and $q^H$ in this context are not a function of subsidy $s_1$ since the regulator is absent. In contrast, the low-cost incumbent selects an output level $q^{L,Sep} > q^L$ in order to reveal his type to the uninformed entrant, if

$$M_{inc}^L(q^{L,Sep}) + \delta \overline{M}_{inc}^L \geq M_{inc}^L(q^L) + \delta D_{inc}^L.$$  \hspace{1cm} (C2)

where $M_{inc}^K(q^H) = \frac{(1-c_{inc}^K)^2}{4}$, which coincides with $\overline{M}_{inc}^K$, and $D_{inc}^K = \frac{(1-2c_{inc}^K+c_{ent})^2}{9}$. Finally, $M_{inc}^H(q^{L,Sep}) = (1 - q^{L,Sep})q^{L,Sep} - c_{inc}^H q^{L,Sep}$. Solving for $q^{L,Sep}$ in C1 and C2, we obtain that $q^{L,Sep} \in [q^A, q^B]$, where $q^A$ ($q^B$) solves C1 (C2, respectively) with equality. Applying the Cho and Kreps’ (1987) Intuitive Criterion, it is straightforward to restrict the separating equilibrium to only the lowest-cost output level $q^A$, for all $p > \frac{F-D_{inc}^L}{D_{ent}^H-D_{ent}^L}$. Solving for $q^A$ in C1, we obtain $q^A \equiv \frac{(1-c_{inc}^H)^2(2-\gamma+\sqrt{3\gamma})}{2(2-\gamma)}$ and $q^A > q^L \equiv \frac{1-c_{inc}^L}{2}$ when costs satisfy $c_{inc}^H < \frac{\sqrt{3\gamma+2-\gamma}}{\sqrt{3\gamma+2-\gamma}}$.

Pooling PBE. In the absence of subsidy policy, the high-cost incumbent is willing to mimic the low-cost firm, i.e., selecting a first-period output $q^L$, in order to deter entry if the following incentive compatibility condition holds

$$M_{inc}^H(q^L) + \delta \overline{M}_{inc}^H \geq M_{inc}^H(q^H) + \delta D_{inc}^H,$$

for the high-cost incumbent, and

$$M_{inc}^L(q^L) + \delta \overline{M}_{inc}^L \geq M_{inc}^L(q^H) + \delta D_{inc}^L.$$

17
for the low-cost firm. The incentive compatibility condition for the low-cost incumbent holds since by selecting $q^L$ it deters entry, and $q^L$ maximizes its first-period profits. The incentive compatibility condition of the high-cost firm, however, does not necessarily hold for all parameter values. In particular, the first-period monopoly profits that this incumbent obtains when selecting $q^L$ are $M_{inc}^H(q^L) = \frac{1}{4} \left( 1 - 2c_{inc}^H + c_{inc}^L \right) \left( 1 - c_{inc}^L \right)$, whereas its monopoly profits when selecting $q^H$ are $M_{inc}^H(q^H) = \frac{1}{4} \left( 1 - c_{inc}^H \right)^2$. Hence, the high-cost incumbent’s incentive compatibility condition holds if

$$\frac{\delta (1 - c_{inc}^H)^2 + (1 - 2c_{inc}^H + c_{inc}^L) (1 - c_{inc}^L)}{4} \geq \frac{(9 + 4\delta)(1 - c_{inc}^H)^2}{36},$$

which implies that $c_{inc}^H$ satisfies $c_{inc}^H < \frac{5\delta - 3(1 - c_{inc}^L)}{5\delta - 9} \sqrt{5\delta - 9} - c_{inc}^L$. (Note that for the parametric examples considered throughout the paper, $\delta = 1$ and $c_{inc}^L = 2/3$, this cutoff becomes $c_{inc}^H < 0.81$.) Finally, the entrant cannot update its beliefs after observing $q^L$, and thus coincide with the prior probability $\mu(c_{inc}^H|q^L) = p$, leading the entrant to stay out given that priors satisfy $p < \frac{F_{D_{ent}} - D_{L_{ent}}}{D_{H_{ent}} - D_{L_{ent}}}$.

### 7.2 Appendix 2 - One-sided separating equilibrium

The following proposition shows that strategy profiles in which only one agent, either the incumbent or the regulator, selects a type-dependent strategy, cannot be sustained as equilibria in our model.

**Proposition A1.** One-sided separating equilibria (OS), in which the entrant observes only one type-dependent strategy, originating from either the incumbent or the regulator, cannot be supported.

Figure A1a describes a strategy profile in which the incumbent selects a type-independent output function, $q(s_1)$, while the regulator chooses a type-dependent subsidy $(s_1^H, s_1^L)$. In this case, subsidies disseminate private information to the entrant. Hence, subsidies nullify the high-cost incumbent’s incentives to select a type-independent output, since it cannot deter entry. Therefore, this firm would increase its first-period profits by deviating to $q^H(s_1^H)$, and strategy profile $(s_1^K, q(s_1^K))$ cannot be supported as a OS.

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30This result is independent on the precise output function that the incumbent chooses. Specifically, if incumbent selects a type-independent output $q^L(s_1)$, the regulator’s type-dependent subsidy would still allow the entrant to infer the incumbent’s costs, either high upon observing $(s_1^H, q^L(s_1^H))$, or low, after $(s_1^K, q^L(s_1^K))$; thereby inhibiting the concealment of information.
Figure A1b depicts the case where the regulator chooses a type-independent subsidy $s'_1$, whereas the incumbent responds with a type-dependent output $(q^H(s'_1), q^L(s'_1))$. The incumbent’s signal now reveals information to the potential entrant, thus inhibiting the regulator’s concealing strategy. The regulator facing a high-cost incumbent, thus, cannot deter entry by selecting $s'_1$, and could increase social welfare by deviating towards $s^H_1$. As a consequence, this type of strategy profile cannot be sustained as a OS either.

**Proof of Proposition A1:** **Type-independent output.** Let us first analyze OS when the incumbent selects a type-independent output function $q(s_1) \neq q^H(s_1), q^L(s_1)$ while the regulator chooses a type-dependent subsidy $(s^H_1, s^L_1)$. In this case, after observing the equilibrium subsidy $s^H_1$ followed by output level $q(s^H_1)$, the entrant believes $\mu(c^H_{inc}|q(s^H_1), s^H_1) = 1$, and enters; while upon observing subsidy $s^L_1$ followed by output level $q(s^L_1)$, the entrant believes $\mu(c^H_{inc}|q(s^L_1), s^L_1) = 0$, and stays out. Hence, if the high-cost incumbent deviates to $q^H(s^H_1)$, entry will still occur, while his first-period profits increase. Therefore, this type of incumbent does not have incentives to behave according to the prescribed strategy. A similar argument applies if the incumbent selects a type-independent output function $q^L(s_1)$. In particular, the entrant’s equilibrium beliefs would be $\mu(c^H_{inc}|q^L(s^H_1), s^H_1) = 1$, and $\mu(c^H_{inc}|q^L(s^L_1), s^L_1) = 0$, thus allowing the entrant to still infer the incumbent’s type even in the presence of contradictory signals $(s^H_1, q^L(s^H_1))$.

**Type-independent subsidy.** Let us next examine the OS in which the regulator chooses a type-independent subsidy $s_1 \neq s^H_1, s^L_1$, while the incumbent selects a type-dependent output function $(q^H(s_1), q^L(s_1))$. In this context, the entrant, upon observing the equilibrium subsidy $s_1$ followed by equilibrium output level $q^H(s_1)$, infers the incumbent’s costs being high, i.e., $\mu(c^H_{inc}|q^H(s_1), s_1) = 1$, and enters. If the entrant observes equilibrium subsidy $s_1$ followed by equilibrium output level $q^L(s_1)$, he also infers the incumbent’s type, i.e., $\mu(c^H_{inc}|q^L(s_1), s_1) = 0$, and stays out. A regulator
facing a low-cost incumbent, however, has incentives to deviate from this strategy profile. Specifically, if he deviates to \( s_1^H \neq s_1 \) and output level is \( q^H(s_1^H) \), entry is still deterred, and first-period output coincides with the social optimum, i.e., \( q^L(s_1^L) = q^L_{SO} \). Hence, this strategy profile cannot be sustained as a OS. A similar argument applies to the case in which the regulator selects a type-independent \( s_1^L \) and the incumbent selects a type-dependent output function \((q^H(s_1^H), q^L(s_1^L))\). In this environment, the regulator facing a high-cost incumbent has incentives to deviate towards \( s_1^H \) since the entrant, upon observing \((s_1^H, q^H(s_1^H))\), can still infer that the incumbent’s type is high, thus entering. Given that such a deviation entails an optimal output, i.e., \( q^H(s_1^H) = q^H_{SO} \) unlike \( q^H(s_1^L) \), this strategy profile cannot be sustained as a OS. ■

7.3 Proof of Lemma 1

No entry. Given a second-period subsidy \( s_2 \), under no entry the \( K \)-type incumbent solves

\[
\max_{x_{inc}} (1 - x_{inc})x_{inc} - (c^K_{inc} - s_2) x_{inc}
\]

which yields an output function \( x_{inc}^{K,NE}(s_2) = \frac{1 - (c^K_{inc} - s_2)}{2} \). The social planner seeks to induce an output level that maximizes the sum of consumer and producer surplus,

\[
\max_{x_{inc}} \gamma CS(x_{inc}) + [(1 - x_{inc})x_{inc} - c^K_{inc}x_{inc}]
\]

where \( CS(x_{inc}) = \frac{1}{2}(x_{inc})^2 \). Taking first-order conditions, we obtain the socially optimal output \( x_{SO}^{K} = \frac{1 - c^K_{inc}}{2\gamma} \). Hence, the subsidy \( s_2 \) that induces the monopolist to produce \( x_{SO}^{K} \) is that solving

\[
\frac{1 - (c^K_{inc} - s_2)}{2} = \frac{1 - c^K_{inc}}{2\gamma}, \; \text{i.e.,} \; s^{K,NE} = \gamma\frac{1 - c^K_{inc}}{2\gamma}, \; \text{where} \; \frac{\partial s^{K,NE}}{\partial c_{inc}} > 0 \; \text{and} \; \frac{\partial s^{K,NE}}{\partial \gamma} < 0. \; \text{(A similar subsidy,} \; s^{K,1}_1 \; \text{, is implemented in the first period, since the incumbent is the unique firm operating in the market.)}

Entry. In the case of entry, the incumbent (entrant) solves

\[
\max_{x_{inc}} (1 - x_{inc} - x_{ent})x_{inc} - (c^K_{inc} - s_2) x_{inc} \quad \text{and} \quad \max_{x_{ent}} (1 - x_{ent} - x_{inc})x_{ent} - (c_{ent} - s_2) x_{ent} - F
\]

respectively, yielding an output function \( x_j^{K,E}(s_2) = \frac{1 - 2c_j + c_j^K + s_2}{3} \) for any firm \( i = \{inc, ent\} \) where \( j \neq i \). The social planner seeks to induce an output level that maximizes the sum of consumer and producer surplus,

\[
\max_X \gamma CS(X) + [(1 - X)X - c^K_{inc}X - F]
\]

where \( X \equiv x_{inc} + x_{ent}, \; CS(X) = \frac{1}{2}(X)^2 \). Note that the last expression considers the incumbent’s marginal costs. This is due to the fact that, in order to allocate the production decision of the socially optimal output, a benevolent social planner would select to produce using the most efficient firm. Specifically, when the incumbent’s costs are low, all socially optimal output would be produced by this firm, whereas when they are high, incumbent and entrant are equally efficient and hence
the socially optimal output can be split among them. Taking first-order conditions, we obtain the aggregate socially optimal output \( X_{SO}^K = \frac{1 - c_{inc}^K}{2 - \gamma} \), which coincides with \( x_{SO}^K \). Finally, in order to find subsidy \( s_{K,E}^2 \) and individual output levels \( x_{inc,SO}^{K,E} \) and \( x_{ent,SO}^{K,E} \), the social planner must simultaneously solve

\[
\dot{x}_{inc,SO}^{K,E} + \dot{x}_{ent,SO}^{K,E} = \frac{1 - c_{inc}^K}{2 - \gamma}
\]

(the sum of incumbent’s and entrant’s output coincides with the socially optimal output \( X_{SO}^K \)) and

\[
x_{inc}^{K,E}(s_2) = \frac{1 - 2c_{inc}^K + c_{ent} + s_2}{3}, \quad \text{and}
\]

\[
x_{ent}^{K,E}(s_2) = \frac{1 - 2c_{ent} + c_{inc}^K + s_2}{3}
\]

Simultaneously solving equations (1)-(3) yields the subsidy \( s_{H,E}^2 = \frac{(2\gamma - 1)(1 - c_{inc}^H)}{2 - \gamma} \) when the incumbent’s costs are high, which is strictly positive if \( \gamma > \frac{1}{2} \). Substituting \( s_{H,E}^2 \) into the output function \( x_i^{K,E}(s_2) \) yields \( x_{inc}^{H,E}(s_2) = x_{ent}^{H,E}(s_2) = \frac{1 - c_{inc}^H}{2 - \gamma} \). When \( \gamma < \frac{1}{2} \), \( s_{H,E}^2 = 0 \) and duopoly output hence becomes \( x_{inc}^{H,E}(0) = x_{ent}^{H,E}(0) = \frac{1 - c_{inc}^H}{3} \).

Simultaneously solving equations (1)-(3) when the incumbent’s costs are low, yields the subsidy \( s_{L,E}^2 = \frac{-2\gamma - (2 - \gamma)c_{inc}^L - (1 - \gamma)c_{inc}^L}{2(2 - \gamma)} \), where \( \frac{\partial s_{L,E}^2}{\partial \gamma} > 0 \) and \( \frac{\partial s_{L,E}^2}{\partial c_{inc}} < 0 \). Hence, the equilibrium output levels given the subsidy \( s_{L,E}^2 \) are

\[
x_{inc}^{L,E}(s_2^L) = \frac{1 + (2 - \gamma)c_{inc}^H - (3 - \gamma)c_{inc}^L}{2(2 - \gamma)} \quad \text{and} \quad x_{ent}^{L,E}(s_2^L) = \frac{1 - (2 - \gamma)c_{inc}^H + (1 - \gamma)c_{inc}^L}{2(2 - \gamma)}
\]

which are positive if, respectively, \( c_{inc}^H > \frac{-1 + (3 - \gamma)c_{inc}^L}{2 - \gamma} \) and \( c_{inc}^H < \frac{1 + (1 - \gamma)c_{inc}^L}{2 - \gamma} \); see figure A1 below.
In addition, the subsidy $s^L_2$ is positive if $c^H_{inc} > \frac{1-2\gamma+(1+\gamma)c^L_{inc}}{2-\gamma}$. [Note that when $\gamma > \frac{1}{2}$, cutoff \( \frac{1-2\gamma+(1+\gamma)c^L_{inc}}{2-\gamma} \) originates at the negative quadrant, and thus the initial condition $c^H_{inc} > c^L_{inc}$ is more restrictive than $c^H_{inc} > \frac{1-2\gamma+(1+\gamma)c^L_{inc}}{2-\gamma}$. Otherwise, the opposite holds]. Hence, combining these three conditions, we need $\frac{1+(1-\gamma)c^L_{inc}}{2-\gamma} > c^H_{inc} > \frac{1-2\gamma+(1+\gamma)c^L_{inc}}{2-\gamma}$ when $\gamma \leq \frac{1}{2}$ and $\frac{1+(1-\gamma)c^L_{inc}}{2-\gamma} > c^H_{inc} > c^L_{inc}$ when $\gamma > \frac{1}{2}$, as depicted in the shaded area of the figure below.

### 7.4 Proof of Proposition 1

After observing the equilibrium subsidy $s^H_1$ followed by output level $q^H(s^H_1)$, the entrant believes $\mu(c^H_{inc}|q^H(s^H_1), s^H_1) = 1$, and enters; while upon observing subsidy $s^L_1$ followed by output level $q^L(s^L_1)$, the entrant believes $\mu(c^H_{inc}|q^L(s^L_1), s^L_1) = 0$, and stays out. If, instead, the entrant observes an off-the-equilibrium subsidy $s_1 \neq s^H_1, s^L_1$ followed by an output level $q^K(s_1)$, where $K = \{H, L\}$, he relies on the signal of the non-deviating player alone, and thus $\mu(c^H_{inc}|q^H(s_1), s_1) = 1$ and $\mu(c^H_{inc}|q^L(s_1), s_1) = 0$. Similarly, if the entrant observes an equilibrium subsidy $s^K_1$ followed by an off-the-equilibrium output level $q(s^K_1) \neq q^H(s^K_1), q^L(s^K_1)$, he relies on the regulator’s subsidy alone in order to infer the incumbent’s type, and his beliefs become $\mu(c^H_{inc}|q(s^H_1), s^H_1) = 1$ and $\mu(c^H_{inc}|q(s^L_1), s^L_1) = 0$. Finally, if the deviating player selects the strategy corresponding to the other type of incumbent, the entrant observes a subsidy $s^H_1$ followed by output $q^L(s^H_1)$. In this context, the entrant cannot infer the incumbent’s type. In particular, he does not know if the deviation
is from the high-cost incumbent, who mimics the output function of the low-cost firm, \( q^L(s_1) \), or if the regulator (facing a low-cost incumbent) is mimicking the subsidy level of the high-cost firm in order to attract entry. These contradictory signals, induce beliefs of \( \mu'(c^H_{\text{inc}}|q^L(s_1^H), s_1^H) = \mu' \). Specifically, the entrant responds entering if and only if

\[
\mu' \times (D^H_{\text{ent}} - F) + (1 - \mu') \times (D^L_{\text{ent}} - F) \geq 0, \text{ or } \mu' \geq \mu_1 \equiv \frac{F - D^L_{\text{ent}}}{D^H_{\text{ent}} - D^L_{\text{ent}}}.
\]

Hence, for the TS to exist, neither the regulator nor the incumbent finds profitable to send these signals. Let us analyze these incentives separately for \( \mu' \geq \mu_1 \) and \( \mu' < \mu_1 \).

**Case 1:** \( \mu' \geq \mu_1 \). The entrant’s beliefs hence do not allow the high-cost incumbent to deter entry. Conditional on entry, and upon observing the equilibrium subsidy \( s_1^H \), the incumbent has no incentives to deviate from \( q^H(s_1^H) \) to \( q^L(s_1^H) \). In this setting, the regulator facing a low-cost incumbent has no incentives to deviate from \( s_1^L \) (which deters entry) to \( s_1^H \) (which attracts entry since \( \mu' \geq \mu_1 \)). In particular, in the first period, subsidy \( s_1^H \) induces an output level \( q^L(s_1^H) < q^L(s_1^L) = q^L_{SO} \), thus generating inefficiencies. In addition, in the second period, the regulator provides subsidy \( s_2^{L,E} \) to the high-cost entrant and the low-cost incumbent, which induces positive output levels from both firms, and a socially optimal production at the aggregate level. However, the first-period inefficiency does not arise if the regulator selects equilibrium subsidy \( s_1^L \). Hence, the regulator does not have incentives to deviate.

**Case 2:** \( \mu' < \mu_1 \). The entrant responds staying out after observing subsidy \( s_1^H \) followed by output \( q^L(s_1^H) \). For the TS to exist, the high-cost incumbent, upon observing the equilibrium subsidy \( s_1^H \), must have no incentives to deviate from \( q^H(s_1^H) \) to \( q^L(s_1^H) \), that is \( M^H_{\text{inc}}(q^H(s_1^H), s_1^H) + \delta D^H_{\text{inc}} \geq M^H_{\text{inc}}(q^L(s_1^H), s_1^H) \). Hence, condition

\[
M^H_{\text{inc}}(q^H(s_1^H), s_1^H) - M^H_{\text{inc}}(q^L(s_1^H), s_1^H) \geq \delta \left[ M^H_{\text{inc}} - D^H_{\text{inc}} \right]
\]

where \( M^H_{\text{inc}}(q^H(s_1^H), s_1^H) = \frac{(1-c^H_{\text{inc}})^2}{(2-\gamma)^2} \) and \( M^H_{\text{inc}}(q^L(s_1), s_1) = \frac{(1-c^H_{\text{inc}})^2}{(2-\gamma)^2} \) for any given subsidy \( s_1 \), \( M^H_{\text{inc}} = \frac{(1-c^H_{\text{inc}})^2}{(2-\gamma)^2} \), where \( M^H_{\text{inc}} \) is evaluated at \( s_2^{H,NE} \), and \( D^H_{\text{inc}} = \frac{(1-c^H_{\text{inc}})^2}{(2-\gamma)^2} \), which is evaluated at \( s_2^{H,E} \) > 0 given that \( \gamma > \frac{1}{3} \). Solving for \( c^H_{\text{inc}} \) yields \( c^H_{\text{inc}} \geq \frac{\sqrt{3\delta+2-2\gamma}c^L_{\text{inc}}}{\sqrt{3\delta+2-2\gamma}} \equiv \bar{c} \). In addition, cutoff \( \bar{c} \) satisfies \( \bar{c} < \frac{1+(1-\gamma)c^L_{\text{inc}}}{2-\gamma} \), since \( \bar{c} \) originates at \( \frac{\sqrt{3}}{2+\sqrt{3}-\gamma} \), which is lower than the origin of \( \frac{1+(1-\gamma)c^L_{\text{inc}}}{2-\gamma} \), for all \( \gamma \in [1/2, 1] \), and reaches \( c^H_{\text{inc}} = 1 \) when \( c^L_{\text{inc}} = 1 \). Hence, condition \( c^H_{\text{inc}} \geq \bar{c} \) holds for all costs in the admissible region \( \frac{1+(1-\gamma)c^L_{\text{inc}}}{2-\gamma} > c^H_{\text{inc}} > c^L_{\text{inc}} \). In this setting, the regulator facing a low-cost incumbent must have no incentives to deviate from \( s_1^L \) (which induces socially optimal output and deters entry) to \( s_1^H \) (which induces an inefficient output level and also deters entry since \( \mu' < \mu_1 \)).
7.5 Proof of Corollary 1

TS equilibrium vs Complete information. Since the type-dependent subsidy \((s_1^H, s_1^L)\) and the type-dependent output function \((q^H(s_1), q^L(s_1))\) in the TS coincides with that under complete information, socially optimal output levels are induced in both information settings, thus yielding the same social welfare.

TS equilibrium vs Signaling model without regulator. The second-period welfare is larger under the TS equilibrium where the regulator is present than in the separating equilibrium where he is absent, since the socially optimal output \(q_{SO}^L\) is being produced. Regarding the first-period welfare, note that the regulator also induces a socially optimal output \(q_{SO}^H = q^H(s_1^H)\) which yields a larger social welfare than that arising when the regulator is absent. Similarly, when the incumbent’s costs are high, the regulator induces socially optimal output levels, \(q_{SO}^H\), yielding a overall larger social welfare than when the regulator is absent. ■

7.6 Proof of Proposition 2

In the pooling strategy profile, the regulator sets a type-independent subsidy \(s_1'\) and the incumbent responds with a type-independent first-period output function \(q(s_1)\) for any subsidy \(s_1\). After observing equilibrium subsidy \(s_1'\) and output level \(q(s_1')\) entrant’s equilibrium beliefs are \(\mu(c^H_{inc}|q(s_1'), s_1') = p\), which coincide with the prior probability distribution. After observing a deviation from the regulator \(s''_1 \neq s_1'\), the entrant’s off-the-equilibrium beliefs cannot be updated using Bayes’ rule, and for simplicity, we assume that \(\mu(c^H_{inc}|q(s''_1), s''_1) = 1\). A similar argument can be made in the case when only the incumbent deviates towards an output \(q'(s_1') \neq q(s_1')\) while the regulator still selects \(s_1'\), i.e., \(\mu(c^H_{inc}|q'(s_1'), s_1') = 1\). The same is true when both informed agents deviate, i.e., \(\mu(c^H_{inc}|q'(s''_1), s''_1) = 1\).

Therefore, after observing an equilibrium subsidy \(s_1'\) and an equilibrium output level \(q(s_1')\), the entrant enters if its expected profit from entering satisfies \(p \times D^H_{ent} + (1 - p) \times D^L_{ent} - F > 0\) or \(p > \frac{F - D^L_{ent}}{D^H_{ent} - D^L_{ent}} = \bar{p}\), where \(\bar{p} \in (0, 1)\) by definition. Hence, if \(p > \bar{p}\) entry occurs; otherwise the entrant stays out. Note that if \(p > \bar{p}\), entry occurs when \(s_1'\) and \(q(s_1')\) are selected, which cannot be optimal for both types of incumbent, inducing them to select type-dependent output \(q^K(s_1')\). But since \(q^H(s_1') \neq q^K(s_1')\) this strategy cannot be a PE. Thus, it must be that \(p \leq \bar{p}\), inducing the entrant to stay out.

Incumbent. Let us check the conditions under which the high-cost incumbent chooses output function \(q(s_1)\). After observing an equilibrium subsidy of \(s_1'\), the high-cost incumbent obtains profits \(M^H_{inc}(q(s_1'), s_1') + \delta \overline{M}^H_{inc}\). If, instead, the incumbent deviates towards an off-the-equilibrium output \(q'(s_1') \neq q(s_1')\), entry ensues and its profits become \(M^H_{inc}(q'(s_1'), s_1') + \delta D^H_{inc}\), which are maximized at \(q'(s_1') = q^H(s_1')\). Hence, the high-cost incumbent selects \(q(s_1')\) if \(M^H_{inc}(q(s_1'), s_1') + \delta \overline{M}^H_{inc} \geq M^H_{inc}(q^H(s_1'), s_1') + \delta D^H_{inc}\), or alternatively

\[
\delta \left[\overline{M}^H_{inc} - D^H_{inc}\right] \geq M^H_{inc}(q^H(s_1'), s_1') - M^H_{inc}(q(s_1'), s_1')
\]  

(C3)
which, using the functional forms in the paper, implies \( q(s'_1) \leq \frac{(1-c^H_{\text{inc}})[2-\gamma+\sqrt{3\gamma}]}{2(2-\gamma)} + \frac{s'_1}{2} \). After observing an off-the-equilibrium subsidy \( s''_1 \neq s'_1 \), entry ensues regardless of the incumbent’s output function, and therefore \( M^H_{\text{inc}}(q(s'_1), s''_1) + \delta D^H_{\text{inc}} \geq M^H_{\text{inc}}(q(H(s''_1), s''_1) + \delta D^H_{\text{inc}} \) cannot hold by definition. Similarly for the low-cost incumbent. If, after observing equilibrium subsidy \( s'_1 \), it selects equilibrium output level \( q(s'_1) \), its profits are \( M^L_{\text{inc}}(q(s'_1), s'_1) + \delta D^L_{\text{inc}} \). However, if it deviates towards \( q'(s'_1) \) entry ensues, obtaining profits \( M^L_{\text{inc}}(q'(s'_1), s'_1) + \delta D^L_{\text{inc}} \) which are maximized at \( q'(s'_1) = q^L(s'_1) \). Hence, the low-cost incumbent chooses \( q(s'_1) \) if \( M^L_{\text{inc}}(q(s'_1), s'_1) + \delta D^L_{\text{inc}} \geq M^L_{\text{inc}}(q^L(s'_1), s'_1) + \delta D^L_{\text{inc}}, \) or alternatively

\[
\delta [\overline{M}^L_{\text{inc}} - D^L_{\text{inc}}] \geq M^L_{\text{inc}}(q^L(s'_1), s'_1) - M^L_{\text{inc}}(q(s'_1), s'_1)
\]

(C4)

After observing an off-the-equilibrium subsidy \( s''_1 \neq s'_1 \), entry ensues regardless of the incumbent’s output function, and therefore, \( q(s''_1) \) is not optimal for the low-cost firm.

**Regulator.** Let us now examine the regulator’s incentives to choose a type-independent subsidy \( s'_1 \). When the incumbent’s costs are high, the regulator obtains \( SW^{H,NE}(s'_1, s''_2^{H,NE}) \) by selecting \( s'_1 \). If, instead, he deviates to any off-the-equilibrium subsidy \( s''_1 \neq s'_1 \), the incumbent selects \( q^H(s''_1) \) and entry ensues. Hence, he obtains \( SW^{H,NE}(s''_1, s''_2^{H,NE}) \), which is maximized at the complete information subsidy \( s''_1 = s^H_1 \). Thus, the regulator chooses \( s'_1 \) if

\[
SW^{H,NE}(s'_1, s''_2^{H,NE}) \geq SW^{H,NE}(s^H_1, s''_2^{H,NE}). \tag{C5a}
\]

which, solving for \( s'_1 \) yields \( s'_1 \geq \frac{1-c^H_{\text{inc}}}{2-\gamma} = s^H_1 \). When the incumbent’s costs are low, the regulator obtains \( SW^{L,NE}(s'_1, s''_2^{L,NE}) \) by selecting the type-independent subsidy \( s'_1 \). If instead, he deviates to \( s''_1 \), the incumbent selects \( q^L(s''_1) \) and entry follows. The regulator’s social welfare is therefore maximized at \( s''_1 = s^L_1 \), yielding \( SW^{L,NE}(s''_1, s''_2^{L,NE}) \). Thus, the regulator chooses \( s'_1 \) if

\[
SW^{L,NE}(s'_1, s''_2^{L,NE}) \geq SW^{L,NE}(s^L_1, s''_2^{L,NE}). \tag{C5b}
\]

which, solving for \( s'_1 \) yields \( s'_1 < \frac{(1-c^L_{\text{inc}})^2-2[(c^L_{\text{inc}}-c^H_{\text{inc}})[1+(\gamma-2)c^L_{\text{inc}}-(\gamma-1)c^H_{\text{inc}}]]^{1/2}}{2-\gamma} = s^L_1 \), where \( s^A_1 > s^L_1 \), since the difference \( s^A_1 - s^L_1 = \frac{2[(c^L_{\text{inc}}-c^H_{\text{inc}})[1+(\gamma-2)c^L_{\text{inc}}-(\gamma-1)c^H_{\text{inc}}]]^{1/2}}{2-\gamma} \) is positive for all \( c^L_{\text{inc}} < \frac{(1+(1-\gamma)c^H_{\text{inc}})}{2-\gamma} \), which holds by definition in the set of admissible costs \( c^L_{\text{inc}} < c^H_{\text{inc}} < \frac{(1+(1-\gamma)c^L_{\text{inc}})}{2-\gamma} \). Therefore, the subsidy \( s''_1 \) in the PE satisfies \( s^A_1 > s^L_1 \geq s'_1 \geq s^H_1 \). Summarizing, any subsidy \( s'_1 \) and output function \( q(s_1) \) simultaneously satisfying conditions C3-C5 constitutes a PE of the signaling game.

**Intuitive Criterion.** We next show that the type-independent output function \( q(s_1) = q^L(s_1) \) survives the Cho and Kreps’ (1987) Intuitive Criterion, and then demonstrate that, given this output function, only the type-independent subsidy \( s'_1 = s^L_1 \) survives this equilibrium refinement.

**Incumbent, case 1a.** Let us first check if the type-independent first-period output function \( q(s_1) = q^L(s_1) \) survives the Cho and Kreps’ (1987) Intuitive Criterion for any \( s_1 \). For simplicity, we first analyze the case where \( q(s_1) < q^H(s_1) < q^L(s_1) \) and then that in which \( q^H(s_1) < q(s_1) < q^L(s_1) \). On one hand, the highest profit that the low-cost incumbent obtains by deviating towards
\[ q'(s_1) \neq q(s_1) \] is \( M_{inc}(q'(s_1), s_1) + \delta M_{inc} \), which exceeds its equilibrium profit \( M_{inc}(q(s_1), s_1) + \delta M_{inc} \) for any \( q'(s_1) \in (q(s_1), q^L(s_1)) \) due to the concavity of \( M_{inc}(q'(s_1), s_1) + \delta M_{inc} \). On the other hand, the high-cost incumbent obtains \( M_{inc}(q(s_1), s_1) + \delta M_{inc} \) in equilibrium. If instead, it deviates towards \( q'(s_1) \neq q(s_1) \), \( M_{inc}(q'(s_1), s_1) + \delta M_{inc} \) is the highest profit that it can obtain, which exceeds its equilibrium profit if \( q'(s_1) \in (q(s_1), q^H(s_1)) \). Hence, beliefs can be restricted to \( \mu(c_{inc}|q'(s_1), s_1) = 0 \) after observing a deviation \( q'(s_1) \in (q^H(s_1), q^L(s_1)) \). (Otherwise, the entrant’s beliefs are unaffected; since either both types of incumbent, or neither, have incentives to deviate.) Therefore, after observing a deviation \( q'(s_1) \in (q^H(s_1), q^L(s_1)) \), the entrant believes that the incumbent’s cost must be low, and does not enter. Under these updated beliefs, the profit obtained by the low-cost incumbent from deviating exceeds its equilibrium profits. Hence, the low-cost incumbent deviates towards \( q'(s_1) \) and the PE where \( q(s_1) < q^H(s_1) < q^L(s_1) \) violates the Intuitive Criterion for any subsidy \( s_1 \).

Let us now examine the case where the equilibrium output function \( q(s_1) \) satisfies \( q^H(s_1) < q(s_1) < q^L(s_1) \). On one hand, the highest profit that the low-cost incumbent can obtain by deviating towards \( q'(s_1) \neq q(s_1) \) is \( M_{inc}(q'(s_1), s_1) + \delta M_{inc} \), which exceeds its equilibrium profit of \( M_{inc}(q(s_1), s_1) + \delta M_{inc} \) for any \( q'(s_1) \in (q(s_1), q^L(s_1)) \). On the other hand, the highest profit that the high-cost incumbent can obtain by deviating towards \( q'(s_1) \neq q(s_1) \) is \( M_{inc}(q'(s_1), s_1) + \delta M_{inc} \), which exceeds its equilibrium profit of \( M_{inc}(q(s_1), s_1) + \delta M_{inc} \) for any \( q'(s_1) \in (q^H(s_1), q(s_1)) \). Therefore, after observing any deviation \( q'(s_1) \in (q(s_1), q^L(s_1)) \), the entrant believes that the incumbent’s costs must be low, and does not enter. Under these updated beliefs, the profit that the low-cost incumbent obtains deviating exceeds its equilibrium profits. Hence, the PE where \( q(s_1) < q^L(s_1) \) also violates the Intuitive Criterion.

**Incumbent, case 1b.** Next let us check if the type-independent first-period output \( q(s_1) > q^L(s_1) \) survives the Cho and Kreps’ (1987) Intuitive Criterion. On one hand, the low-cost incumbent obtains \( M_{inc}(q(s_1), s_1) + \delta M_{inc} \) in equilibrium. By instead deviating towards \( q^L(s_1) \), \( M_{inc}(q^L(s_1), s_1) + \delta M_{inc} \) is the highest profit it can obtain, which exceeds its equilibrium profits. On the other hand, the high-cost incumbent obtains \( M_{inc}(q(s_1), s_1) + \delta M_{inc} \) in equilibrium. By deviating towards \( q^L(s_1) \), \( M_{inc}(q^L(s_1), s_1) + \delta M_{inc} \) is the highest profit it obtains after no entry, which also exceeds its equilibrium profits, given that \( q^H(s_1) < q^L(s_1) < q(s_1) \). Therefore, both types of incumbent have incentives to deviate towards \( q^L(s_1) \) and entrant’s beliefs cannot be updated, i.e., \( \mu(c_{inc}|q^L(s_1), s_1) = p \) inducing no entry. Given these beliefs, both types of incumbent deviate toward \( q^L(s_1) \), obtaining higher profits than in equilibrium. Hence, the PE in which both types select \( q(s_1) > q^L(s_1) \) also violates the Intuitive Criterion.

**Incumbent, case 1c.** Let us now check if the type-independent first-period output \( q(s_1) = q^L(s_1) \) survives the Cho and Kreps’ (1987) Intuitive Criterion. On one hand, \( M_{inc}(q^L(s_1), s_1) + \delta M_{inc} \) is the highest payoff the low-cost incumbent obtains by deviating towards \( q'(s_1) \neq q^L(s_1) \), which lies below its equilibrium profits since \( M_{inc}(q'(s_1), s_1) + \delta M_{inc} \) reaches its maximum at exactly \( q'(s_1) = q^L(s_1) \). Hence, the low-cost incumbent does not have incentives to deviate from the type-independent output function \( q(s_1) = q^L(s_1) \). On the other hand, \( M_{inc}(q'(s_1), s_1) + \delta M_{inc} \)
is the highest payoff the high-cost incumbent can obtain by deviating toward \( q'(s_1) \neq q^L(s_1) \). Therefore, the high-cost incumbent does not have incentives to deviate if \( M^H_{inc}(q^L(s_1), s_1) + \delta M^H_{inc} \geq M^H_{inc}(q'(s_1), s_1) + \delta M^H_{inc} \), which only holds for deviations closer to its first-period profit-maximizing output, i.e., \( q'(s_1) \in [q^H(s_1), q^L(s_1)) \). Hence, the entrant believes with certainty the incumbent is a high type for every deviation in this interval, i.e., \( \mu \left( c^H_{inc} | q'(s_1), s_1 \right) = 1 \), and enters. In contrast, its updated beliefs are unaffected after observing any other deviation. The high-cost incumbent’s profits from deviating towards \( q'(s_1) \) are hence \( M^H_{inc}(q'(s_1), s_1) + \delta D^H_{inc} \), which are lower than its equilibrium profits if

\[
M^H_{inc}(q^L(s_1), s_1) + \delta M^H_{inc} \geq M^H_{inc}(q'(s_1), s_1) + \delta D^H_{inc} \tag{C6}
\]

Note that deviation profits, \( M^H_{inc}(q'(s_1), s_1) + \delta D^H_{inc} \), are maximal at \( q'(s_1) = q^H(s_1) \), yielding profits of \( M^H_{inc}(q^H(s_1), s_1) + \delta D^H_{inc} \). Hence, if \( M^H_{inc}(q^L(s_1), s_1) + \delta M^H_{inc} \geq M^H_{inc}(q^H(s_1), s_1) + \delta D^H_{inc} \), then condition C6 holds for all deviations \( q'(s_1) \in [q^H(s_1), q^L(s_1)) \). Note that the last inequality holds since the equilibrium output function \( q(s_1) = q^L(s_1) \) satisfies condition C3. Therefore, the high-cost incumbent does not have incentives to deviate from \( q^L(s_1) \), and the type-independent output function \( q^L(s_1) \) must be part of a PE surviving the Intuitive Criterion.

**Regulator, case 2a.** Given output function \( q^L(s_1) \) selected by both types of incumbent, let us finally analyze the regulator’s equilibrium subsidy \( s'_1 \). First, consider the case where \( s'_1 < s^L_1 \). For simplicity, we analyze the case where \( s^H_1 < s'_1 < s^L_1 \) and then \( s'_1 < s^H_1 < s^L_1 \). The regulator facing a low-cost incumbent obtains an equilibrium social welfare of \( SW^{L,NE}(s'_1, s^L_1) \). By deviating towards an off-the-equilibrium subsidy of \( s^L_1 \neq s'_1 \), \( SW^{L,NE}(s'_1, s^L_1) \) is the highest payoff that the regulator obtains, since \( s^L_1 \) maximizes social welfare conditional on no entry. On the other hand, the regulator facing a high-cost incumbent obtains an equilibrium social welfare of \( SW^{H,NE}(s'_1, s^H_1) \). By deviating towards an off-the-equilibrium subsidy of \( s^H_1 \neq s'_1 \), \( SW^{H,NE}(s'_1, s^H_1) \) is the highest payoff that the regulator obtains when entry is deterred, which does not exceed his equilibrium welfare since \( SW^{H,NE}(s'_1, s^H_1) < SW^{H,NE}(s'_1, s^H_1) \), given that \( s^H_1 < s'_1 < s^L_1 \). Therefore, after observing a deviation \( s^L_1 \neq s'_1 \), the entrant believes that the incumbent’s cost must be low, and does not enter. Hence, the regulator facing a low-cost incumbent deviates towards \( s^L_1 \) and the PE where the regulator selects the type-independent subsidy \( s'_1 \) where \( s^H_1 < s'_1 < s^L_1 \) violates the Intuitive Criterion.

Second, let us now consider the case where \( s'_1 < s^H_1 < s^L_1 \). On one hand, the regulator facing a low-cost incumbent obtains an equilibrium social welfare of \( SW^{L,NE}(s'_1, s^L_1) \). By deviating towards an off-the-equilibrium subsidy of \( s''_1 \neq s'_1 \), \( SW^{L,NE}(s''_1, s^L_1) \) is the highest payoff that the regulator obtains, which exceeds equilibrium welfare if \( SW^{L,NE}(s''_1, s^L_1) \geq SW^{L,NE}(s'_1, s^L_1) \), which is satisfied for all \( s''_1 \in (s'_1, s^L_1] \) since \( s^L_1 \) maximizes social welfare conditional on no entry. On the other hand, the regulator facing a high-cost incumbent obtains an equilibrium social welfare of \( SW^{H,NE}(s'_1, s^H_1) \). By deviating towards an off-the-equilibrium subsidy of \( s''_1 \neq s'_1 \),
The highest payoff that the regulator obtains, which exceeds equilibrium welfare for all \( s''_1 \in (s'_1, s_H] \). Therefore, after observing a deviation \( s''_1 \in (s^H_1, s'_1] \), the entrant believes that the incumbent’s cost must be low, and does not enter. Under these updated beliefs, the social welfare from deviating to \( s''_1 \in (s^H_1, s'_1] \), exceeds that in equilibrium, \( SW^{L,NE}(s'_1, s''_2^{L,NE}) \). Hence, the regulator facing a low-cost incumbent deviates towards \( s''_1 \) and the PE in which the regulator selects a type-independent subsidy \( s'_1 \), where \( s'_1 < s^H_1 < s'_2 \), also violates the Intuitive Criterion.

**Regulator, case 2b.** Let us now examine the case in which the equilibrium subsidy \( s'_1 \) satisfies \( s'_1 > s''_1 \). On one hand, the regulator facing a low-cost incumbent obtains an equilibrium social welfare of \( SW^{L,NE}(s'_1, s''_2^{L,NE}) \). By deviating towards an off-the-equilibrium subsidy of \( s''_1 \), the highest payoff that the regulator can obtain occurs when entry is deterred, yielding welfare of \( SW^{L,NE}(s''_1, s''_2^{L,NE}) \), which exceeds his equilibrium welfare since \( SW^{L,NE}(s''_1, s''_2^{L,NE}) \geq SW^{L,NE}(s'_1, s''_2^{L,NE}) \). On the other hand, the regulator facing a high-cost incumbent obtains an equilibrium social welfare of \( SW^{H,NE}(s'_1, s''_2^{H,NE}) \). By deviating towards an off-the-equilibrium subsidy of \( s''_1 \), \( SW^{H,NE}(s'_1, s''_2^{H,NE}) \) is the highest payoff that the regulator obtains, which exceeds his equilibrium welfare since \( SW^{H,NE}(s''_1, s''_2^{H,NE}) \geq SW^{H,NE}(s'_1, s''_2^{H,NE}) \), given that \( s''_1 < s''_2 < s'_1 \). Therefore, the regulator has incentives to deviate towards \( s''_1 \) for both types of incumbent and the entrant’s beliefs cannot be updated, i.e., \( \mu(c^{H,inc}q(s'_1), s''_1) = p \) inducing no entry since \( p < \bar{p} \). Given these beliefs, the regulator has incentives to deviate toward \( s''_1 \), obtaining higher social welfare than in equilibrium. Hence, the pooling strategy profile where the regulator selects \( s'_1 > s''_1 \) also violates the Intuitive Criterion.

**Regulator, case 2c.** Let us finally analyze the case where the equilibrium subsidy \( s'_1 \) satisfies \( s'_1 = s''_1 \). On one hand, the regulator facing a low-cost incumbent obtains an equilibrium social welfare of \( SW^{L,NE}(s'_1, s''_2^{L,NE}) \). By deviating towards an off-the-equilibrium subsidy of \( s''_1 \), the highest payoff that the regulator can obtain occurs when entry is deterred, yielding welfare of \( SW^{L,NE}(s''_1, s''_2^{L,NE}) \), which is strictly lower than the equilibrium welfare of \( SW^{L,NE}(s'_1, s''_2^{L,NE}) \). On the other hand, the regulator facing a high-cost incumbent obtains an equilibrium social welfare of \( SW^{H,NE}(s'_1, s''_2^{H,NE}) \). By deviating towards an off-the-equilibrium subsidy of \( s''_1 \), \( SW^{H,NE}(s'_1, s''_2^{H,NE}) \) is the highest payoff that the regulator obtains, which exceeds the equilibrium welfare if \( SW^{H,NE}(s''_1, s''_2^{H,NE}) \geq SW^{H,NE}(s'_1, s''_2^{H,NE}) \), which holds for any deviation \( s''_1 \in (s_H, s'_1] \). Hence, the entrant assigns full probability to the cost being high for every deviation \( s''_1 \in (s_H, s'_1] \), i.e., \( \mu(c^{H,inc}q(s''_1), s''_1) = 1 \), and entry ensues. Given these updated beliefs, the social welfare that the regulator facing a high-cost incumbent obtains when he deviates towards a subsidy of \( s''_1 \) is \( SW^{H,E}(s''_1, s''_2^{H,E}) \), which is lower than his equilibrium welfare if \( SW^{H,E}(s''_1, s''_2^{H,E}) < SW^{H,NE}(s'_1, s''_2^{H,NE}) \). This condition holds since, according to condition C5a, the equilibrium subsidy \( s'_1 \) must satisfy \( SW^{H,E}(s'_1, s''_2^{H,E}) < SW^{H,NE}(s'_1, s''_2^{H,NE}) \). We can hence conclude that \( SW^{H,E}(s''_1, s''_2^{H,E}) < SW^{H,E}(s'_1, s''_2^{H,E}) < SW^{H,NE}(s'_1, s''_2^{H,NE}) \) since \( s''_H \) maximizes \( SW^{H,E}(s'_1, s''_2^{H,E}) \). Therefore, the regulator facing a high-cost incumbent does not have incentives to deviate either, and the PE where the regulator selects \( s''_1 \) survives the Intuitive Criterion.

Hence, at the equilibrium subsidies of \( s''_1 \) and \( s''_2^{H,E} \), the high-cost incumbent overproduces,
selecting a first-period output function \(q^L(s_1)\), if condition C3 holds, which implies that \(c^H_{\text{inc}}\) satisfies \(c^H_{\text{inc}} < C^*\), where 
\[ C^* = \frac{(2-\gamma)(1-(c^H_{\text{inc}})^{\sqrt{2}}+2\gamma(1-c^H_{\text{inc}})^{\sqrt{2}}-3\gamma)}{(2-\gamma)^2-3\gamma}. \]
However, for the set of production costs considered in the paper, \(\widetilde{C} > c^H_{\text{inc}} > c^L_{\text{inc}}\), where 
\[ \widetilde{C} = \frac{1+(1-\gamma)\gamma c^L_{\text{inc}}}{2-\gamma}, \] cut-off \(C^*\) satisfies \(C^* > \widetilde{C}\). In particular, the vertical intercept of cut-off \(C^*\) lies above that of \(\widetilde{C}\), and both cut-offs reach \(c^H_{\text{inc}} = 1\) when \(c_{\text{inc}} = 1\), implying that \(C^* > \widetilde{C}\). Hence, condition \(c^H_{\text{inc}} < C^*\) is not binding in the PE for the admissible set of production costs \(\widetilde{C} > c^H_{\text{inc}} > c^L_{\text{inc}}\).

Finally, under the PE, social welfare \(SW^{H,NE}(s^L_1, s^H_1, s^L_2, s^H_2)\) is
\[ 1 + \delta + \delta(c^H_{\text{inc}})^2 - 2c^H_{\text{inc}}(1 + \delta c^L_{\text{inc}} - (c^L_{\text{inc}}))^2 \]
\[ \frac{2(2-\gamma)}{2(2-\gamma)} - \delta F \]
while when selecting subsidy \(s^H_1\), which is responded with output level \(q^H(s^H)\), thus attracting entry, welfare is \(SW^{H,E}(s^L_1, s^H_1, s^L_2, s^H_2)\), which coincides with that arising under complete information, that is
\[ \frac{(1+\delta)(1-c^H_{\text{inc}})^2}{2(2-\gamma)} - \delta F \]

Therefore, \(SW^{H,NE}(s^L_1, s^H_1, s^L_2, s^H_2) \geq SW^{H,E}(s^L_1, s^H_1, s^L_2, s^H_2)\) when 
\[ F > \frac{(c^H_{\text{inc}} - c^L_{\text{inc}})^2}{2(2-\gamma)} \equiv F(\gamma), \]
which is increasing in \(\gamma\). Hence, among the set of admissible entry costs \(D^H_{\text{ent}} > F > D^L_{\text{ent}}\), the PE can be sustained for all \(F > F(\gamma)\). In particular, cutoff \(F(\gamma)\) lies below \(D^H_{\text{ent}}\) for all \(\gamma > \gamma\), where \(\gamma\) solves \(D^H_{\text{ent}} = F(\gamma)\). Assuming no discounting, cutoff \(\gamma\) is
\[ \gamma = 2 - \frac{(1-c^H_{\text{inc}})^2}{2(c^H_{\text{inc}} - c^L_{\text{inc}})^2}, \]
for all \(c^H_{\text{inc}} \leq C_1\), where
\[ C_1 = \sqrt{2}(1-c^L_{\text{inc}}) + 2c^L_{\text{inc}} - 1. \] Cutoff \(C_1\) originates at \(\sqrt{2} - 1 \simeq 0.41\) and reaches \(c^H_{\text{inc}} = 1\) when \(c^L_{\text{inc}} = 1\). Hence, cutoff \(C_1\) satisfies \(C_1 > \frac{1+(1-\gamma)\gamma c^L_{\text{inc}}}{2-\gamma} \equiv \widetilde{C}\), since \(\widetilde{C}\) originates at \(\frac{1}{2-\gamma}\), which lies above 0.41 for all values of \(\gamma\). In addition, \(\gamma > 1/2\) for all costs \(c^H_{\text{inc}} > \sqrt{3}(1-c^L_{\text{inc}}) + 3c^L_{\text{inc}} - 1 \equiv C_2\). Furthermore, cutoff \(C_2\) satisfies \(C_2 < \widetilde{C}\). Specifically, both \(C_2\) and \(\widetilde{C}\) are linear in \(c^L_{\text{inc}}\) and reach \(c^H_{\text{inc}} = 1\) when \(c^L_{\text{inc}} = 1\). However, cutoff \(C_2\) originates at \(\sqrt{3} - 1 \simeq 0.36\) while \(\widetilde{C}\) originates at \(\frac{1}{2-\gamma}\), which lies above 0.36 for any value of \(\gamma\).

### 7.7 Proof of Corollary 2

From the proof of Proposition 2, we obtain a complete ranking of cutoffs in the \((c^H_{\text{inc}}, c^L_{\text{inc}})\)-space: \(\widetilde{C} > C_1 > C_2\), and the PE exists for all \(c^H_{\text{inc}} \leq C_1\). Hence, we can identify three regions of costs (for a graphical reference, see figure 2 in the text): (1) for \(c^H_{\text{inc}} > C_1\), cutoff \(\gamma\) lies above 1; (2) for costs in the interval \(C_1 \geq c^H_{\text{inc}} > C_2\), cutoff \(\gamma\) satisfies \(\gamma \in [\frac{1}{2}, 1]\) and the PE exists for all \(\gamma > \gamma\); and (3) for costs \(C_2 \leq c^H_{\text{inc}}\), cutoff \(\gamma\) lies below 1/2 and, therefore, the PE can be supported for all values of \(\gamma\).
7.8 Proof of Corollary 3

As shown in Appendix 1, when the regulator is absent the PE can be supported for all costs $c_{inc}^H \leq \frac{5\delta - 3(1-c_{inc}^L)\sqrt{5\delta - 9c_{inc}^L}}{5\delta - 9} = C_{PE}^{NR}$ after considering no discounting, where cutoff $C_{PE}^{NR} > C_1$, since $C_{PE}^{NR}$ originates at $\frac{3\sqrt{5} - 5}{4} \approx 0.427$, while $C_1$ does at 0.41, and reaches $c_{inc}^H = 1$ at $c_{inc}^L = 1$. Hence, the PE can be sustained in the region $c_{inc}^H \leq C_{PE}^{NR}$ without regulation, and in the subregion $c_{inc}^H \leq C_1 < C_{PE}^{NR}$ with regulation. In other words, for all costs under which the PE exists when the regulator is present, this equilibrium can also be sustained when the regulator is absent.

7.9 Proof of Corollary 4

Let us now evaluate social welfare in the PE without regulation,

$$W_{PE}^{NR,K} = \frac{(2 + \gamma)(1 - c_{inc}^H)^2 + \gamma(1 - c_{inc}^L)^2 + 2(1 - c_{inc}^L)(1 - 2c_{inc}^H + c_{inc}^L)}{8}$$

Comparing it with the welfare that arises under a PE when the regulator is present, $SW^{H,NE}(s_1^L, s_2^H,NE)$ (found in the proof of Proposition 2), we obtain that $SW^{H,NE}(s_1^L, s_2^H,NE)$ exceeds $W_{PE}^{NR,K}$ when $\gamma \geq \bar{\gamma}$, where $\bar{\gamma} \equiv \frac{4(c_{inc}^H-c_{inc}^L)(1-c_{inc}^L)}{2+(c_{inc}^H-2)c_{inc}^L+(c_{inc}^L-2)c_{inc}^H}$. In addition, cutoff $\bar{\gamma}$ satisfies $\bar{\gamma} < 1$ for all $c_{inc}^H \leq C_3$, where $C_3 \equiv 3 - \sqrt{\pi(1 - c_{inc}^L) - 2c_{inc}^L}$. Note that cutoff $C_3$ lies below $C_1$, since both cutoffs are linear in $c_{inc}^L$, and reach $c_{inc}^H = 1$ at $c_{inc}^L = 1$, but cutoff $C_3$ originates at $3 - \sqrt{\pi} \approx 0.35$ while $C_1$ starts at 0.41. Cutoff $C_3$ also lies below $C_2$, since the latter originates at 0.36. Therefore, social welfare with regulation is larger than without in the subregion $C_1 \geq C_3 \geq c_{inc}^H$. For completeness, note that cutoff $\bar{\gamma} > 1/2$ for all $c_{inc}^H > C_4$, where $C_4 \equiv 5 - \sqrt{23}(1 - c_{inc}^L) - 4c_{inc}^L$. Comparing this cutoff with those analyzed above, note that $C_4$ also reaches $c_{inc}^H = 1$ at $c_{inc}^L = 1$, but originates at $5 - \sqrt{23} \approx 0.20$, below all other cutoffs. Hence, (1) for costs in $C_1 \geq c_{inc}^H > C_3$, cutoff $\bar{\gamma}$ satisfies $\bar{\gamma} > 1$ and, hence, the PE exists but it yields a lower welfare with than without regulator; (2) for costs in the interval $C_3 \geq c_{inc}^H > C_4$, cutoff $\bar{\gamma}$ satisfies $\bar{\gamma} \in [1/2, 1)$ and, thus, the PE exists and yields a larger welfare with than without regulator for all $\gamma \geq \bar{\gamma}$; and (3) for costs $C_4 \geq c_{inc}^H$, cutoff $\bar{\gamma}$ satisfies $\bar{\gamma} < 1/2$ and, therefore, the PE exists and yields a larger welfare with than without regulator for all values of $\gamma$. ■

References


