Location and Welfare Effects of Spatial Price Discrimination Under Non-Uniform Distributions and Endogenous Market Boundaries

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Abstract

We analyze a two-stage sequential-move game of location and pricing to identify firm location, output, and welfare. We consider two pricing regimes (mill pricing and spatial price discrimination) and, unlike previous literature, allow in each of them for a non-uniform population density, non-constant location costs, and endogenous market boundaries. Under constant location costs, our results show the firm locates at the city center under both mill and discriminatory pricing, and that output is larger under spatial price discrimination. Welfare comparisons are, however, ambiguous. Under non-constant location costs, we find the optimal location can move from the city center, and does not coincide across pricing regimes. We also find that output and welfare are higher (lower) under mill than under discriminatory pricing when transportation rates are low (high, respectively).

KEYWORDS: Monopoly Spatial price discrimination; Non-uniform distribution; Location choice; Social welfare; Mill pricing; Non-constant location costs.

JEL CLASSIFICATION: D42; D60; L12; L50; R32.

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1 Introduction

In some industries, such as cement and ready-mixed concrete, spatial price discrimination is possible because firms are geographically differentiated and transportation is costly (Vogel, 2011; Miller and Osborne, 2014). For example, spatial price discrimination has a long history in the cement industry, where producers privately negotiate contracts with their customers (Miller and Osborne, 2014). This enables producers to price discriminate among its customers without running afoul of the antitrust laws. While such spatial price discrimination yields larger profit, its welfare effects have long been the subject of intense debate. Despite being forbidden in many countries, like U.S. under the Robinson-Patman Act (1936) and China under the Antimonopoly Law (2007), many analysts argue that banning spatial price discrimination may harm social welfare; see Greenhut and Ohta (1972) and Holahan (1975). Hence, further investigating the welfare effects of spatial price discrimination in different settings is of interest for both analysts and policymakers.

In this paper, we analyze the monopolist's location decisions and compare the resulting output and social welfare under two pricing regimes: spatial price discrimination and mill pricing (no discrimination)[1]. Previous studies considered two simplifying assumptions: (1) firm's location was given; (2) consumers' location was uniform; and (3) while some studies relaxed (1) by allowing for firm's location to be endogenous, they assumed that location costs were constant, thus suggesting that the firm incurs the same location cost regardless of its distance from the city center. We separately relax these assumptions, considering a model of endogenous firm location (relaxing 1); in which consumers are not necessarily uniformly distributed (relaxing 2); and whereby location costs are not necessarily constant (relaxing 3). Such a general model allows us to show that output and welfare results are significantly affected by the above assumptions often considered in the literature.

We build our model on the work of Holahan (1975) and Cheung and Wang (1995). Nevertheless, the present paper contributes to the above papers by allowing for: i) non-uniform population distribution; ii) endogenous market boundaries; and iii) endogenous location choice. We present and solve a two-stage sequential-move game of location and pricing in our spatial price discrimination framework. In the game, firm's location is chosen in the first stage and prices are chosen in the second stage. We solve the equilibrium under spatial price

[1]Under mill pricing, the firm charges a consumer a delivery price that is equal to the sum of a constant mill price and the actual transportation cost, while in uniform pricing the firm charges the same delivery price to consumers in all locations.

discrimination and mill pricing by backward induction and Mathematical Program with Equilibrium Constraints, and provide Monte Carlo simulations for those expressions without explicit functional solutions. We then analyze the effects of spatial price discrimination under both constant and non-constant location costs along the market line.

Our results find that when location costs are constant, the firm locates at the city center both under mill and discriminatory pricing since the largest number of customers concentrate at the city center. The market area (i.e., customers served), profit and output are larger under spatial price discrimination than under mill pricing; whereas, the welfare may be larger or lower under discriminatory pricing.

However, when the location costs are non-constant, we find that the optimal location under mill pricing is different than under price discrimination. The optimal location under mill pricing is closer to the city center than that under spatial price discrimination when transportation rates are low; otherwise, the optimal location under mill pricing is further to the city center. Spatial price discrimination, hence, yields unambiguously a larger market radius and higher profits. Output and welfare under this pricing regime, however, depend on transportation rate. For low level of transportation rates, welfare (output) under mill pricing is higher than that under spatial price discrimination; while for high level of transportation rates the opposite results apply. Transportation rates affect location effect and market expanding effect. For high level of transportation rates, both location effect and market expanding effect are of first-order, which enables discriminatory pricing generate greater output and improve welfare relative to mill pricing.

Related literature. Our paper is related to the literature on monopoly spatial price discrimination. Using a linear-market model with uniform distributed consumers and linear demand curves, Greenhut and Ohta (1972) and Holahan (1975) argue that, when the market area is a variable, spatial price discrimination results in firms producing larger output, serving larger market areas, and promoting greater social welfare than under a mill price policy. But their findings apply to the assumption of uniform population density. Beckmann (1976) relaxes this assumption but assume an exogenous market area. He shows that spatial price discrimination yields lower welfare levels than mill pricing; a result that holds for all customer distributions. Some research suggests that both assumption of fixed market boundary and assumption of uniform population distribution should be relaxed (Claycombe, 1996). Yet there is little evidence examining the effects of spatial price discrimination in the case of endogenous market boundaries and non-uniform population density. Our goal in this paper is to understand equilibrium behavior in such settings.

A common assumption on the previous literature is that the monopolist's location is predetermined and coincides across different pricing policies. But since the firm may choose different locations under different pricing policies, the conclusions on the effects of spatial price discrimination may be altered (Beckmann and Thisse, 1987; Cheung and Wang, 1995; Tan, 2001). This point is verified by Hwang and Mai (1990), who treat the location as an endogenous variable. They show that optimal locations under mill pricing and discriminatory pricing are different and that the welfare effect of spatial price discrimination is indeterminate. Cheung and Wang (1995) extend the analysis to non-uniform demands and show that when the monopolist serves a fixed market area at a predetermined location, compared with mill pricing, spatial price discrimination results in the same level of total output, higher profit, lower consumer surplus and lower total welfare. They also demonstrate that when location is chosen endogenously, output falls, and consumer surplus and total welfare may rise or fall under discriminatory pricing. However, the above studies assume a fixed market segment. Since market areas served under different pricing regimes depend on the price policy itself (Greenhut and Ohta, 1972; Holahan, 1975; Ohta and Wako, 1988), we will incorporate the location decision into models of monopolist's spatial price discrimination with endogenous market areas.

Almost all the monopoly spatial price discrimination literature assumes a constant location cost over the market space (Greenhut and Ohta, 1972; Holahan, 1975; Hwang and Mai, 1990; Cheung and Wang, 1995). Assuming the location cost is constant along the market line, Cheung and Wang (1995) show that if the intercept of the inverse demand curve is constant, the firm locates at the median point of the population distribution under mill pricing and locate at the mean point of the population distribution under discriminatory pricing. However, location costs (e.g., building rental and land price) may be lower as the firm locates further from the city center. Hence, the firm faces a trade-off since locating closer to the city center helps it serve a larger number of customers but entails a larger location cost. Our results show how this trade-off affects the firms location and, as a consequence, its output and equilibrium welfare.

This paper contributes to the monopoly spatial price discrimination literature in two ways. First, although several studies analyze output and welfare effects of spatial price discrimination, few of them simultaneously consider non-uniform population density, endogenous market boundaries and endogenous plant location choice. Our setting hence is closer to real market conditions. Second, to our knowledge, this is the first study considering non-constant location cost in the analysis of monopolist's spatial price discrimination. While most studies assume that firm's location costs are constant (i.e., firm incurs the same costs, regardless of their distance from the city center), we allow them to decrease as the firm locates further away from the city center, and show how output and welfare change.

The next section describes the model of our analysis. Section 3 analyzes the pricing decisions in the second stage of the game. The location decisions in the first stage of the game and equilibrium results are presented in Section 4. Section 5 concludes.

2 Model

Consider a setting where a monopolist produces a homogenous good and sells the product to consumers distributed along the market line as shown in Figure 1. The total number of consumers in the market is n. Following Claycombe (1996), we assume the population density is closely approximated using the normal distribution. Let the city center be at point 0. Then the population density at any point x is

$$\phi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

where σ denotes the standard deviation of the population distribution. However, our model and methodology are not limited to normal density function.

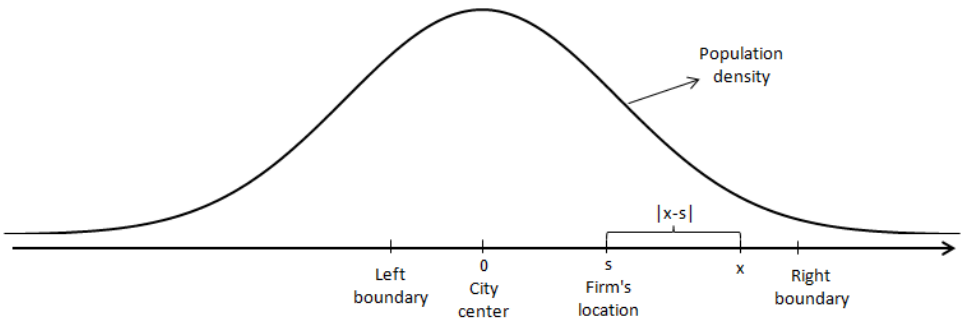

Figure 1: The market line

In this paper, we assume that the monopolist produces at a single location s on the market line. The marginal cost of production is assumed constant, and, without loss of generality, can be normalized to zero. We consider two types of location costs: constant and non-constant. When non-constant, location costs are increasing in the market size, and in the firm's proximity to the city center, where the largest number of customers concentrate.[2] In particular, location costs are represented by

$$F(s,n) = \frac{An}{\sqrt{2\pi}\sigma_F} e^{-\frac{s^2}{2\sigma_F^2}} \tag{2}$$

where $A > 0$ and σ_F is the standard deviation of location cost distribution.[3] If, in contrast, location cost is constant, we consider $F(s,n) = An$, which is a special case of (2) when $\sigma_F \to \infty$.

Assume each consumer has the same identical and linear demand curve.[4] So each consumer at location x has a demand function taking the following form:

$$q_x = a - b(p + t|x - s|) \tag{3}$$

where $a, b > 0$, q_x is the quantity demanded by consumer at location x, p is the good's price, t represents transportation cost per unit of distance, and hence $t|x - s|$ denotes transportation cost facing customers at point x. The monopolist can employ either mill pricing or discriminatory pricing. Under mill (discriminatory) pricing, besides transportation cost, the firm charges the same (different) product price p_m (p_d) to each individual regardless of (depending on, respectively) his location.

Following models of spatial competition with endogenous location and prices, we assume a sequential-move game, with location chosen in the first stage and prices chosen in the second stage (Hwang and Mai, 1990; Braid, 2008). In the following analysis, we employ backward induction to solve the equilibrium.

[2] As argued by Berliant and Konishi (2000), the setup costs in a marketplace depend on location and are proportional to the number of consumers in the market, n.

[3] Our model and methodology are not limited to this location cost functional form.

[4] This assumption conforms to the work by Greenhut and Ohta (1972), Beckmann (1976), Holahan (1975), Guo and Lai (2014), Chen and Hwang (2014), and Andree (2013).

3 Second stage: Pricing decisions

3.1 Mill pricing

Under mill pricing, the monopolist charges each consumer a delivery price, which is equal to a constant mill price p_m plus the transportation cost $t|x-s|$. With equation (1) and (3), we can derive the monopolist's net revenue above location cost at location x

$$NR_m(p_m, x) = n\phi(x)p_m q_x = n\phi(x)p_m[a - b(p_m + t|x-s|)] \tag{4}$$

The market boundaries for the monopolist are points at a distance where the net revenue above location cost is equal to zero. Hence, letting $NR_m(p_m, x) = 0$ and solving for x, we can find market boundaries

$$R_m = s \pm \frac{a - bp_m}{bt} \tag{5}$$

which means that, when depicted over the market line (Figure 1), the firm's left boundary is $LR_m = s - \frac{a-bp_m}{bt}$ and the right boundary is $RR_m = s + \frac{a-bp_m}{bt}$ under mill pricing.

Given the above boundaries, the monopolist's profit is

$$\pi_m = \int_{s-\frac{a-bp_m}{bt}}^{s+\frac{a-bp_m}{bt}} n\phi(x)p_m[a - b(p_m + t|x-s|)]dx - F(s,n) \tag{6}$$

Taking first order condition with respect to the monopolist's mill pricing, p_m, yields[5]

$$\frac{\partial \pi_m}{\partial p_m} = \int_{s-\frac{a-bp_m}{bt}}^{s+\frac{a-bp_m}{bt}} n\phi(x)[a - 2bp_m - bt|x-s|]dx = 0 \tag{7}$$

Given that the density function $\phi(x)$ follows a normal distribution, we cannot solve for p_m^* in equation (7) analytically. It can only be analyzed numerically, as further developed in section 4. Let p_m^* solve equation (7), where $p_m^* \in (0, \frac{a}{2b})$.[6] We can derive the monopolist's aggregate output under mill pricing

[5]Note that the second-order condition for a maximum is satisfied since $\frac{\partial^2 \pi_m}{\partial p_m^2} = -2b\int_{s-\frac{a-bp_m}{bt}}^{s+\frac{a-bp_m}{bt}} n\phi(x)dx < 0$.

[6]We know $\frac{\partial \pi_m}{\partial p_m}$ is decreasing in p_m. This condition, together with the fact that $\frac{\partial \pi_m}{\partial p_m}|_{p_m=0} = \int_{s+a/(bt)}^{s-a/(bt)} n\phi(x)[a - bt|x-s|]dx > 0$ and $\frac{\partial \pi_m}{\partial p_m}|_{p_m=a/(2b)} = -\int_{s+a/(2bt)}^{s-a/(2bt)} n\phi(x)bt|x-s|dx < 0$, implies that, using the mean value theorem, the optimal price p_m^*, which is determined by $\frac{\partial \pi_m}{\partial p_m} = 0$, must be unique and at an interior point of the interval $(0, \frac{a}{2b})$.

$$Q_m = \int_{s-\frac{a-bp_m^*}{bt}}^{s+\frac{a-bp_m^*}{bt}} n\phi(x)[a - b(p_m^* + t|x-s|)]dx \tag{8}$$

which yields profit of

$$\Pi_m = p_m^* Q_m - F(s,n) \tag{9}$$

consumer's surplus of

$$CS_m = \int_{s-\frac{a-bp_m^*}{bt}}^{s+\frac{a-bp_m^*}{bt}} n\phi(x)\frac{[a - b(p_m^* + t|x-s|)]^2}{2b}dx \tag{10}$$

and the social welfare

$$W_m = \Pi_m + CS_m \tag{11}$$

3.2 Discriminatory pricing

Under discriminatory pricing, the monopolist is allowed to charge different prices p_d for the good to consumers at different locations. For the market at point x, the monopolist's net revenue above location cost is

$$NR_d(p_d, x) = n\phi(x)p_d[a - b(p_d + t|x-s|)] \tag{12}$$

Under discriminatory pricing, the monopolist maximizes the net revenue above the location cost at each location.[7] The first order condition is[8]

$$\frac{\partial NR_d(p_d, x)}{\partial p_d} = n\phi(x)(a - 2bp_d - bt|x-s|) = 0 \tag{13}$$

By solving for p_d in equation (13), we can find the price under discriminatory pricing

$$p_d(x) = \frac{a - bt|x-s|}{2b} \tag{14}$$

which is a function of the location of customer x, as opposed to the mill price in expression (7) which was constant for all x. Substituting equation (14) into (12), the net operating revenue as a function of x becomes

[7]Similar arguments are made in the work by Holahan (1975) and Cheung and Wang (1995).

[8]Note that the second-order condition for a maximum is satisfied since $\frac{\partial^2 NR_d(p_d,x)}{\partial p_d^2} = -2bn\phi(x) < 0$.

$$NR_d(x) = \frac{n\phi(x)(a - bt|x-s|)^2}{4b} \tag{15}$$

Let $NR_d(x) = 0$ in order to obtain the boundaries under discriminatory pricing

$$R_d = s \pm \frac{a}{bt} \tag{16}$$

which means that under discriminatory pricing, the monopolist's left boundary is $lR_d = s - \frac{a}{bt}$ and the right boundary is $rR_d = s + \frac{a}{bt}$.

Using the discriminatory price in (14) and the boundaries in (16), we can derive the monopolist's aggregate output under discriminatory pricing

$$Q_d = \int_{s-\frac{a}{bt}}^{s+\frac{a}{bt}} \frac{n\phi(x)(a - bt|x-s|)}{2} dx \tag{17}$$

its profit

$$\Pi_d = \int_{s-\frac{a}{bt}}^{s+\frac{a}{bt}} \frac{n\phi(x)(a - bt|x-s|)^2}{4b} dx - F(s,n) \tag{18}$$

consumers' surplus

$$CS_d = \int_{s-\frac{a}{bt}}^{s+\frac{a}{bt}} \frac{n\phi(x)(a - bt|x-s|)^2}{8b} dx \tag{19}$$

and social welfare

$$W_d = \Pi_d + CS_d \tag{20}$$

4 First stage: location decisions

In this section, the monopolist chooses the plant location. We consider that location cost $F(s,n)$ depends on the firm's location, s, and the size of the market, n, i.e., $F(s,n) = \frac{An}{\sqrt{2\pi}\sigma_F} e^{-\frac{s^2}{2\sigma_F^2}}$. When $\sigma_F \to \infty$, the location cost satisfied $F(s,n) = An$, which implies that it is constant for any location s. Since the population distribution and the location cost are both symmetric with respect to the city center (point 0 in Figure 1), we only need to analyze the case where $s \geq 0$. Analogous results apply when $s \leq 0$.

4.1 Equilibrium results

Mill pricing. Under mill pricing, the monopolist chooses a location to maximize the equilibrium profit shown in (9). Taking first order conditions with respect to location s yields

$$btnp_m^* \left[\int_{s-\frac{a-bp_m^*}{bt}}^{s} \phi(x)dx - \int_{s}^{s+\frac{a-bp_m^*}{bt}} \phi(x)dx \right] = \frac{Ans}{\sqrt{2\pi}\sigma_F^3} e^{-\frac{s^2}{2\sigma_F^2}} \tag{21}$$

The right-hand side of (21) represents the marginal cost that the monopolist bears when locating its plant closer to the city center (since land prices become more expensive as $s \to 0$). The left-hand side, in contrast, indicates the marginal revenue of locating closer to the city center (where a larger mass of customer live). At the optimal location, marginal costs and revenues under mill pricing cancel each other, i.e., $MRL_m(s) = MCL(s)$.

Discriminatory pricing. Under discriminatory pricing, the monopolist chooses a location to maximize the equilibrium profit shown in (18). Taking first order conditions with respect to location s, we find

$$\int_{s-\frac{a}{bt}}^{s} tn\phi(x)\frac{a-bt|x-s|}{2}dx - \int_{s}^{s+\frac{a}{bt}} tn\phi(x)\frac{a-bt|x-s|}{2}dx = \frac{Ans}{\sqrt{2\pi}\sigma_F^3} e^{-\frac{s^2}{2\sigma_F^2}} \tag{22}$$

The right-hand side of (22) coincides with that of (21), intuitively representing that the monopolist's marginal cost of locating closer to the city center is unaffected by the pricing regime that the firm practices. The marginal revenue (on the left-hand side) is, however, different from that under mill pricing.[9] Similarly as under mill pricing, the monopolist stops approaching the city center when marginal costs and revenues under discriminatory pricing offset each other, i.e., $MCL_d(s) = MCL(s)$.

In terms of location costs and population distribution, there are four cases: i) Case 1: constant location cost and uniformly distributed customers; ii) Case 2: non-constant location costs but uniformly distributed customers; iii) Case 3: constant location cost and normally distributed customers; and iv) Case 4: non-constant location costs and normally distributed customers. The following paragraphs discuss each case.

Case 1: Constant location cost and uniformly distributed customers. As $\sigma_F \to \infty$, location costs become constant throughout the market line, i.e., $F(s,n) = An$ for all location s. In addition, as $\sigma \to \infty$, the population density becomes the uniform distribution. In this context,

[9] A direct ranking of the two marginal revenues is unfeasible at this general stage of the model; but several numerical simulations are provided at the end of the section.

a continuum of equilibria emerges for both pricing regimes. That is, equilibrium locations $s^*_{m,1}$ and $s^*_{d,1}$ can be any points on the market line, where subscript 1 denotes Case 1, since profits coincide at all points. In addition, spatial price discrimination leads to more markets being served, and generates a higher level of output and greater social welfare than mill pricing; see Holahan (1975) and Greenhut and Ohta (1972). We next present this result. (All proofs are relegated to the Appendix.)

LEMMA 1: *Given uniform population density and constant location cost, price discrimination yields a larger output and welfare than mill pricing.*

Case 2: Non-constant location costs and uniformly distributed customers. Assume the location costs are distinct at different locations and that the population density is uniform, i.e., $\sigma_F < \infty$ but $\sigma \to \infty$. Since customers are uniformly distributed in this setting, there are no benefits of locating at the city center, i.e., marginal revenues are zero under both pricing regimes. Marginal costs of location are, however, increasing as the firm approaches the city center. Hence, the firm is driven away from the city center, and chooses the location with the lowest cost, $s^* \to \infty$. This result applies under both pricing regimes, i.e., $s^*_{m,2} = s^*_{d,2} = s^*$, where subscript 2 denotes Case 2.

LEMMA 2: *Given non-constant location costs and a uniform population distribution, the firm serves a larger market area and yields a higher level of output and social welfare under spatial price discrimination than under mill pricing.*

Case 3: Constant location cost and normally distributed customers. As described above, when $\sigma_F \to \infty$, location costs become constant throughout the line, i.e., $F(s,n) = An$ for all location s. In such a setting, the marginal costs of locating closer to the city center are zero, both under mill and discriminatory pricing, driving the monopoly to locate as close to the city center as possible in order to benefit from a larger number of customers. This argument applies to both mill and discriminatory pricing, as stated in the following proposition.

PROPOSITION 1. *When location costs are constant, the firm locates at the city center, both under mill and discriminatory pricing, i.e.,* $s^*_m = s^*_d = 0$.

In particular, constant location costs entail that, under mill (discriminatory) pricing, the monopolist locates at the median of the population (demand) distribution, thus leading the same number of customers (aggregate demand) to the left- and right-hand side of its location. This is a common result in the literature of spatial discrimination when location costs are constant; see Greenhut and Ohta (1972) and Holahan (1975).

Since the monopolist's location coincides under both pricing regimes, the market area, profit and total output are larger under spatial price discrimination than under mill pricing. The welfare, however, may be higher or lower under discriminatory pricing (see Appendix D for more details).

Our result on social welfare encompasses but goes beyond the result of Cheung and Wang (1995), who show that spatial price discrimination results in lower total welfare under the assumption of fixed market area and non-uniform demands when the production site is given. In our study, total welfare in the market interval $[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]$, which is served under both mill and discriminatory pricing schedules, is lower under discriminatory pricing than under mill pricing. But in our model, due to the assumption of endogenous market boundaries, a discriminatory pricing monopoly serves a larger market area, thus increasing welfare.

Case 4: Non-constant location costs and normally distributed customers. In this case, the firm faces two opposing forces in its location decision: on one hand, the firm prefers to locate away from the city center since its location costs are cheaper. On the other hand, it prefers a central location in order to capture a larger amount of customers (normal distribution). As a result of this trade-off, the firm does not locate at the city center (as it did in Case 3) nor at the extreme of the line (as far away from the center as possible, as it did in Case 2), but somewhere in between these two polar locations. Analytical solutions for optimal locations are, however, unfeasible because of nonlinearities in equations (21) and (22), but numerical simulations are provided in section 4.2.

Table 1 summarizes firm's optimal location as a function of location costs (in rows) and population distribution (in column).

Table 1: Summary of firm's optimal locations under different locations costs and population distribution

<table>
<tr><td></td><td colspan="3">Population distribution</td></tr>
<tr><td rowspan="3">Location costs</td><td></td><td>Uniform</td><td>Normal</td></tr>
<tr><td>Constant</td><td>s_m^* and s_d^* can be any points on the market line</td><td>$s_m^* = s_d^* = 0$, that is, the city center</td></tr>
<tr><td>Non-constant</td><td>$s_m^* = s_d^* = s^*$, where s^* minimizes the location costs. If location costs follow normal distribution,then $s^* \to \infty$</td><td>$s_m^*, s_d^* \in \mathbb{R}$
(See numerical simulation)</td></tr>
</table>

In order to illustrate the firm's incentives when choosing its location, Figure 2 depicts the marginal cost and revenues of location, $MCL(s)$, $MRL_m(s)$ and $MRL_d(s)$. For simplicity, we set $a = b = t = \sigma = \sigma_F = 1$ and $A = 0.15$. More details about the simulation can be found in Appendix E. Marginal revenues under both pricing regimes are increasing in s first and then decreasing, converging to 0 when $s \to \infty$. For our parameter values, when $0 < s < 1.92$ ($0 < s < 0.85$), the marginal cost of locating closer to the city center $MCL(s)$ lies above the marginal revenue $MRL_m(s)$ ($MRL_d(s)$), while for $s > 1.92$ ($s > 0.85$), $MCL(s)$ lies below the $MRL_m(s)$ ($MRL_d(s)$, respectively). $MRL_d(s)$ lies above $MRL_m(s)$ for all locations.

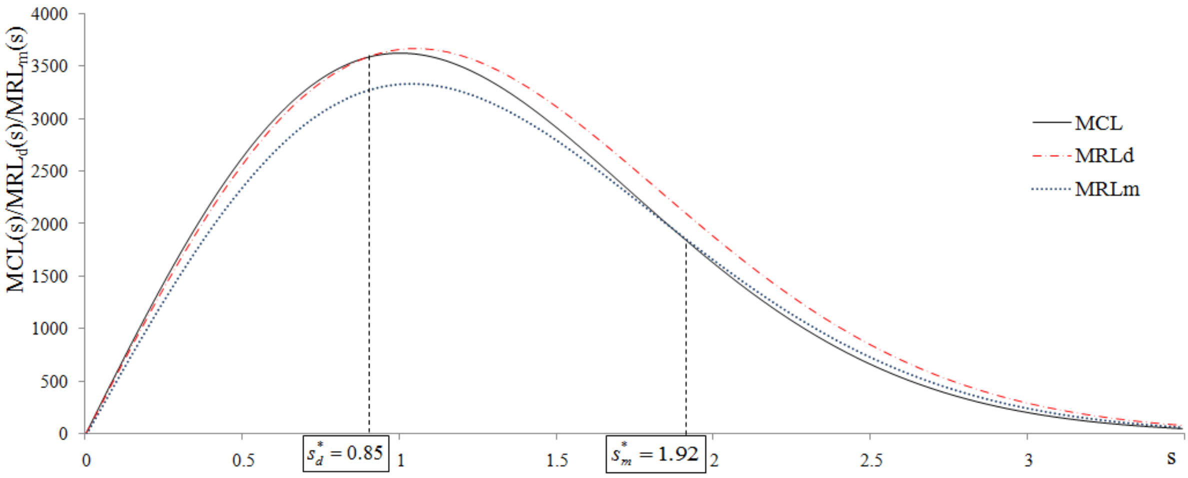

Figure 2: Simulated $MCL(s)$, $MRL_m(s)$ and $MRL_m(s)$

As an illustration, we can evaluate the MCL, MRL_m and MRL_d functions at the special cases discussed above. As in Case 1, if $\sigma \to \infty$ and $\sigma_F \to \infty$, location costs are constant and the population distribution becomes uniform. In this setting, $MCL = MRL_m = MRL_d = 0$ for $s \in [0,\infty)$, thus yielding a continuum of optimal locations s_m^* and s_d^*. If $\sigma \to \infty$, location costs are non-constant and the population density is uniform, yielding $MRL_m = MRL_d = 0$. In addition, MCL lies above both MRL_m and MRL_d, leading the firm to choose a plant location as far away from the city center as possible under both pricing systems (as in Case 2). In contrast, when $\sigma_F \to \infty$, locations costs are constant and the population density is normal, yielding $MCL = 0$ for all $s \in [0,\infty)$ while the marginal revenue curves MRL_m and MRL_d lie both above MCL. As a result, the monopolist chooses the city center as optimal locations under both mill and discriminatory pricing (as in Case 3).

4.2 Numerical simulation

From our previous analysis, we can see that we can get analytical solution for Case 1 to 3. However, given normal density function (1), location cost function (2), and endogenous market boundaries, the first-order conditions for optimal locations in Case 4 cannot be solved analytically. We next resort to numerical simulation, similar to other studies on monopoly spatial price discrimination such as Claycombe (1996) and Tan (2001). In particular, consider parameters $a = b = \sigma = \sigma_F = 1$ and $A = 0.15$, and market size n of 100,000.[10] Appendix E provides a sequential description of our simulation.

Tables 2 and 3 report the simulated results, which comprise equilibrium prices, locations, market radius, profits, outputs, consumers' surplus, and social welfare. Simulation results are given for transportation rates, t, between 0.10 and 1.1 with an increment of 0.05.

Prices. From Table 2, we can see that the mill price decreases as the transportation rate t increases. Under discriminatory pricing, the price policy (expression 14) also indicates that the good's price is decreasing in the transportation rate. Intuitively, when the transportation rate increases, the firm can absorb some transportation cost to sustain sales.

Market radius. Equations 5 and 16 indicate that the widths of the market area under both mill and discriminatory pricing increase as the transportation rate t decreases. This point is also confirmed in Table 2, since the firm can deliver the products to a more distant area with lower transportation rates. Table 2 also shows the market radius is larger under discriminatory pricing than under mill pricing; as shown in Greenhut and Ohta (1972) and Holahan (1975).

This result can be explained by the delivery price $DP = p + t|x - s|$. Let r represent each consumer's distance from the production site. Using p^*_m and $p_d(x)$, we can write the expression for DP under mill pricing and spatial price discrimination as

$$DP_m = p^*_m + tr \tag{23}$$

$$DP_d = \frac{a}{2b} + \frac{tr}{2} \tag{24}$$

[10]Venkatesh and Kamakura (2003) generate a population of 90,000 consumers in their simulation to study bundling strategies and pricing patterns under a monopoly. We use 100,000 consumers in our simulation so that the population sample is closer to a normal distribution. In addition, when n is large enough and the population approximates normal distribution, the value of n does not affect performance comparisons between mill pricing and discriminatory pricing. For example, the sign of $Q_d - Q_m$ is not affected by the value of n.

Table 2: Simulated equilibrium prices, market radius, optimal locations and profits under mill and discriminatory pricing – non-constant location costs and normal population density

t	p_m^*	Radius$_m$	Radius$_d$	π_m	π_d	s_m^*	s_d^*
0.10	0.4451	5.5493	10.0000	15608.9610	15752.0834	0.6792	0.6974
0.15	0.4331	3.7791	6.6667	13436.9589	13692.5677	0.3611	0.4317
0.20	0.4211	2.8946	5.0000	11682.6421	12038.6295	0.0168	0.1010
0.25	0.4046	2.3818	4.0000	10092.5253	10605.0387	0.0072	0.0133
0.30	0.3909	2.0302	3.3333	8652.1889	9297.3083	0.0070	0.0104
0.35	0.3803	1.7705	2.8571	7372.2304	8113.8006	0.0060	0.0080
0.40	0.3721	1.5699	2.5000	6246.0833	7047.7597	0.0073	0.0088
0.45	0.3656	1.4097	2.2222	5263.6036	6096.3630	0.0088	0.0096
0.50	0.3606	1.2788	2.0000	4404.8691	5247.5841	0.0094	0.0100
0.55	0.3566	1.1699	1.8182	3650.2082	4488.5363	0.0121	0.0119
0.60	0.3533	1.0778	1.6667	2988.8412	3813.6167	0.0158	0.0145
0.65	0.3507	0.9989	1.5385	2405.3920	3210.6041	0.0188	0.0162
0.70	0.3488	0.9303	1.4286	1881.9337	2670.0914	0.0293	0.0196
0.75	0.3468	0.8709	1.3333	1426.2834	2183.6689	0.0450	0.0266
0.80	0.3441	0.8199	1.2500	972.5622	1744.9690	0.2057	0.0371
0.85	0.3427	0.7733	1.1765	634.3469	1348.6521	0.3886	0.0514
0.90	0.3340	0.7400	1.1111	348.8360	988.8451	1.0192	0.0869
0.95	0.3246	0.7110	1.0526	174.0311	666.6155	1.5004	0.2564
1.00	0.3154	0.6846	1.0000	82.6227	403.1798	1.9215	0.8454
1.05	0.3077	0.6594	0.9524	35.9230	228.1564	2.2796	1.3126
1.10	0.3015	0.6350	0.9091	14.8039	121.3127	2.5719	1.6938

The delivery price schedules DP_m and DP_d are graphed in Figure 3. As depicted in Figure 3, the two schedules are linear and positively sloped in the consumer's distance to the monopolist, r. DP_d is, however, flatter than DP_m. No matter which pricing regime the firm adopts, customers at a distance $r \in [0, \frac{a-bp_m^*}{bt}]$ are served by the firm, while customers at a distance $r \in (\frac{a-bp_m^*}{bt}, \frac{a}{bt}]$ are only served under discriminatory pricing.[11]

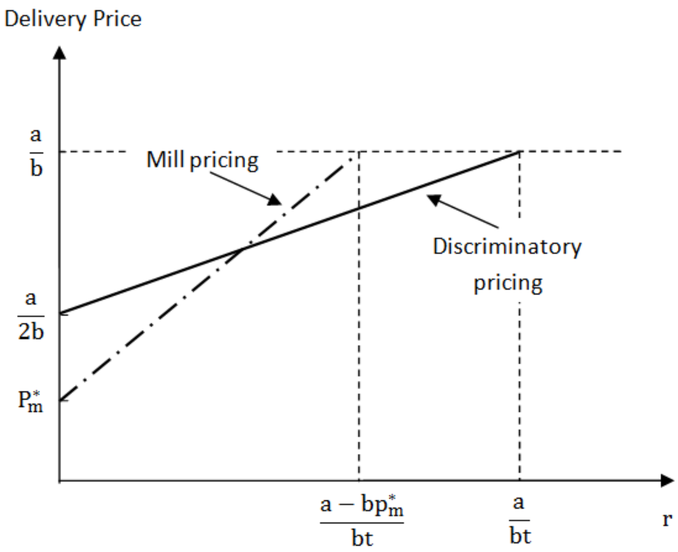

Figure 3: Delivery price schedules: mill versus discriminatory pricing

Profits. As displayed in Table 2, higher transportation rates result in lower profits under both mill and discriminatory pricing. This result is intuitive because more of the revenues are used to pay for transportation cost, which in turn lower profits. Profits in Table 2 show that spatial price discrimination is more profitable than mill pricing at all feasible values of t. So if there is no regulation, the monopolist would choose discriminatory pricing policy.

Location. Table 2 shows that for values of transportation rates at or below 0.35, the firm locates closer to the city center as transportation rate increases under both pricing systems. Since we use simulations with 100,000 draws, at low levels of transportation rate, the firm can capture all the consumers in the sample even if it locates at a place away from the city center. By doing this, the firm can take advantage of lower location costs. For higher level of transportation rates, market radius decreases. In order to capture all the consumers in

[11]For the parameter values considered in our simulation, customers at a distance $r \in [0, \frac{1-p_m^*}{t}]$ are served under both pricing regimes, and customers $r \in (\frac{1-p_m^*}{t}, 1]$ are only served under discriminatory pricing. In figure 3, while intercepts a/b and $a/2b$ become 1 and 1/2 in our parametric example, all remaining intercepts are still functions of transportation rate t and mill price p_m^* (and we could not obtain an analytical expression for such a price, hence we need to rely on numerical simulations).

the sample, the monopolist needs to locate closer to the city center, relative to lower level of transportation rates.

However, for values of transportation rates at or above 0.35, under both pricing regimes higher transportation rates drive the firm's location away from the city center. As transportation costs increase, the marginal revenue of approaching city center decreases as the firm lowers its margin to absorb some transportation cost. Thus, as t increases, the optimal locations under both mill and discriminatory pricing move away from the city center so that the first order conditions (expression 21 and 22) hold at the new equilibrium locations.

Comparing the optimal locations under each pricing regime, we find that the optimal location under mill pricing is different from that under price discrimination. For our parameter values, for $t \leq 0.5$, $s_m^* < s_d^*$, while for $t \geq 0.55$, $s_m^* > s_d^*$.[12] This finding is in line with Hwang and Mai (1990) and Cheung and Wang (1995).

Output. Table 3 shows that output under both pricing schedules decrease in transportation rate. Quantities demanded decrease in delivery prices, which are increasing in the transportation rate (see expressions 23 and 24). Furthermore, as previously discussed, the market area served decreases in t. Hence, Q_m and Q_d decrease as t increases, as seen in Table 3.

We also find that $Q_d < Q_m$ for lower level of transportation rates ($t \leq 0.2$) but $Q_d > Q_m$ for higher level of transportation rates ($t \geq 0.25$). Holahan (1975) argues that spatial price discrimination has a market expanding effect, namely larger market area being served. As transportation rates change, total output is affected by both a change in firm's location and a change in market radius. As the firm locates closer to the city center, the firm sells more products in areas with higher density, which is location effect. For lower level of transportation rates, the location effect would dominate the expanding effect. While for higher level of transportation rates, both location and market expanding effects are of first-order. As we can see from Table 2, optimal locations under mill pricing are closer to the city center than those under discriminatory pricing when transportation rates are small. This makes total output under mill pricing larger than that under spatial price discrimination. However, when transportation rates are high, the optimal location under discriminatory pricing is closer to the city center than that under mill pricing. In addition, the market radius is larger than that under mill pricing. Therefore, spatial price discrimination yields larger output in

[12] When $t \approx 0.538$, $s_m^* \approx s_d^*$.

this case.

Table 3: Simulated outputs, consumers' surpluses, and social welfare under mill and discriminatory pricing – non-constant location costs and normal population density

t	Q_m	Q_d	CS_m	CS_d	W_m	W_d
0.10	44506.92	44436.44	10197.11	9955.07	25806.07	25707.15
0.15	43290.16	43003.14	9869.39	9383.54	23306.35	23076.11
0.20	41948.72	41927.41	9515.35	8975.65	21197.99	21014.28
0.25	39739.72	40024.15	8964.44	8293.71	19056.96	18898.75
0.30	37438.08	38033.52	8421.15	7640.25	17073.34	16937.55
0.35	35116.08	36058.57	7883.55	7048.66	15255.78	15162.46
0.40	32870.71	34119.29	7368.76	6515.56	13614.84	13563.33
0.45	30761.19	32252.15	6887.84	6039.84	12151.44	12136.20
0.50	28807.95	30476.53	6443.84	5615.37	10848.71	10862.96
0.55	27017.24	28801.99	6038.61	5235.65	9688.82	9724.18
0.60	25390.71	27246.98	5671.47	4897.91	8660.31	8711.52
0.65	23911.07	25807.61	5337.18	4596.12	7742.58	7806.72
0.70	22530.75	24475.73	5025.35	4325.23	6907.29	6995.32
0.75	21312.81	23242.22	4753.08	4080.35	6179.36	6264.02
0.80	19131.77	22097.05	4261.28	3857.69	5233.84	5602.66
0.85	17552.15	21027.00	3908.65	3653.40	4541.00	5002.05
0.90	11686.03	19948.49	2596.70	3450.81	2945.54	4439.65
0.95	6577.27	18196.91	1455.49	3128.26	1629.52	3794.88
1.00	3316.06	13339.52	730.74	2249.41	813.37	2652.59
1.05	1607.33	8435.92	352.51	1386.84	388.43	1615.00
1.10	803.57	4893.26	175.60	785.11	190.41	906.42

Consumer surplus and social welfare. Table 3 also shows that consumer surplus and social welfare decrease in the transportation rate. Intuitively, when transportation rate increases, the firm can partially absorb the higher transportation cost and pass some of such cost to consumers (Martin, 2008; Görg et al., 2010; Baldwin and Harrigan, 2011). This causes a loss in both producer and consumer surplus.

For values of t at or below 0.85, consumer surplus satisfies $CS_m > CS_d$, while the opposite ranking applies for higher values of t. Social welfare follows the same pattern, where $W_m \geq W_d$ for $t \leq 0.45$, but $W_m < W_d$ otherwise.

5 Conclusions

Using a monopoly spatial model with normal population distribution of consumers, identical linear demand for each consumer and endogenous market boundaries, the paper analyzes the effects of spatial price discrimination on the firm's location choices, output and social welfare under both constant and non-constant locations costs along the market line. The main conclusions are the following.

First, when the location costs are constant over the market line, the monopolist locates at the median of the population distribution under both mill and discriminatory pricing. We also find that the market area, profit and total output are larger under spatial price discrimination than under mill pricing. Intuitively, under spatial price discrimination, the firm can attract distant consumers by lowering prices, which in turn helps it serve a larger market. Given that firm's optimal locations are the same under both mill and discriminatory pricing, this implies that, under discriminatory pricing, the monopoly does not only serve the market area under mill pricing (hereafter overlapping area) but also a larger market area. Therefore, discriminatory pricing generates a larger total output. The result also indicates the welfare may be larger or lower under spatial price discrimination. If welfare loss in the nearby market region exceeds welfare gain from serving a larger market, spatial price discrimination yields a lower welfare than mill pricing; otherwise, welfare is higher.

Second, when the location costs are non-constant along the market line, the monopolist locates at different places under mill pricing and spatial price discrimination. Relative to spatial price discrimination, when transportation rate is insignificant, the firm locates closer to the city center under mill pricing, but locates further away from the city center when such a cost increases. We also find that spatial price discrimination serves a larger market area and generates higher profit than mill pricing. However, compared with mill pricing, output and social welfare may rise or fall under spatial price discrimination. Output is higher (lower) under mill pricing than that under discriminatory pricing when transportation rate low (high, respectively). Social welfare follows the same pattern. As transportation rate increases, both location effect and market expanding effect of discriminatory pricing dominate, which in turn makes spatial price discrimination welfare improving.

Given that firm may choose different locations under different pricing regimes, the welfare effect of spatial price discrimination may also change with the firm's location (Hwang and Mai, 1990; Cheung and Wang, 1995). Our results on the firm's location choices suggest that the monopolist's optimal locations under the assumption of constant location cost are different from those under the assumption of non-constant location cost. Thus, it is necessary for future studies on monopoly spatial price discrimination to relax the assumption of constant location cost.

Spatial price discrimination is banned in many countries, because the authorities consider that price discrimination is detrimental to social welfare. However, our findings suggest that spatial price discrimination may raise social welfare. For instance, our results suggest that, in industries with high transportation rate (such as ready-mixed concrete and cement), allowing spatial price discrimination can actually improve social welfare.[13] Thus, a blanket prohibition of price discrimination is not socially desirable. As Cheung and Wang (1995) note, selective regulatory policy based on detailed analysis and accurate information would be preferred.

Our model can be extended in two directions. First, a monopolistic industry structure is given in our model, which could be relaxed by allowing for competition. Second, we assume that the monopolist produces at a single location. This assumption may be reasonable when the location setup cost is high, but could be relaxed if these costs are low, thus allowing for multiple plant locations.

Appendix

A Proof of Lemma 1

Consider a uniform population density, $\phi(x) = v$ per unit length. In Case 1, location costs are constant for all s, i.e., $F(s,n) = An$.

Second stage: Pricing decisions. *Mill pricing.* In this case, the first order condition for optimal price under mill pricing (expression (7)) becomes $2\int_0^{s+\frac{a-bp_m}{bt}} nv[a-2bp_m - bt|x-s|]dx = 0$. Solving for p_m^*, we obtain $p_m^* = \frac{a}{3b}$. Thus, under mill pricing, we obtain the market boundaries $R_m = s \pm \frac{2a}{3bt}$, aggregate output $Q_m = \frac{4nva^2}{9bt}$, profit $\Pi_m = \frac{4nva^3}{27b^2t} - An$, and social welfare $W_d = \frac{nva^3}{4b^2t} - An$.

Discriminatory pricing. Given $\phi(x) = v$ and $F(s,n) = An$, we find market boundaries $R_d = s \pm \frac{a}{bt}$, aggregate output $Q_d = \frac{nva^2}{bt}$, profit $\Pi_d = \frac{nva^3}{6b^2t} - An$, and social welfare $W_d = \frac{nva^3}{4b^2t} - An$.

First stage: location decisions. Under mill pricing, profit function is $\Pi_m = \frac{4nva^3}{27b^2t} - An$, which is independent on s.

[13]While Miller and Osborne (2014) find that banning price discrimination would increase consumer surplus, they do not evaluate profit losses, and thus cannot conclude whether social welfare increases or decreases. Our results, hence, help identify under which contexts price discrimination has welfare improving effects.

Under discriminatory pricing, profit function is $\Pi_d = \frac{nva^3}{6b^2t} - An$, which is also independent on s.

Hence, both first order conditions hold for all s, indicating the monopolist obtains the same profits at any location. The market radius, output, and social welfare are not affected by s, either. Finally, we can easy show that $Q_d > Q_m$ and $W_d > W_m$.

B Proof of Lemma 2

Second stage: Pricing decisions *Mill pricing.* Under a uniformly distributed population density $\phi(x) = v$, the first order condition for optimal price under mill pricing (expression (7)) becomes

$$2\int_0^{s+\frac{a-bp_m}{bt}} nv[a-2bp_m - bt|x-s|]dx = 0$$

Solving for p_m^*, we obtain $p_m^* = \frac{a}{3b}$. Thus, under mill pricing, the market boundaries (expression (5)), aggregate output (expression (8)), profit (expression (9)) and social welfare (expression (11)) become

$$R_m = s \pm \frac{2a}{3bt}$$
$$Q_m = \frac{4nva^2}{9bt}$$
$$\Pi_m = \frac{4nva^3}{27b^2t} - F(s,n)$$
$$W_m = \frac{20nva^3}{81b^2t} - F(s,n)$$

Discriminatory pricing. Still under a uniformly distributed population, the expressions for market boundaries, aggregate output, profit, and social welfare (equations (16)-(20)) become

$$R_d = s \pm \frac{a}{bt}$$
$$Q_d = \frac{nva^2}{bt}$$
$$\Pi_d = \frac{nva^3}{6b^2t} - F(s,n)$$
$$W_d = \frac{nva^3}{4b^2t} - F(s,n)$$

First stage: location decisions. Under mill pricing, profit function is $\Pi_m = \frac{4nva^2}{27b^2t} - F(s,n)$.

Taking first order condition with respect to s, we obtain

$$\frac{\partial \Pi_m}{\partial s} = -\frac{\partial F(s,n)}{\partial s} = 0$$

Under discriminatory pricing, profit function is $\Pi_d = \frac{nva^2}{6b^2 t} - F(s,n)$. Taking first order condition with respect to s, we find

$$\frac{\partial \Pi_d}{\partial s} = -\frac{\partial F(s,n)}{\partial s} = 0$$

Hence, both first order conditions indicate that the monopolist will locate at the location where the location cost is minimum, which implies $s_d^* = s_m^* = s^*$.

C Proof of Proposition 1

Under mill pricing, the profit function is shown in expression (9). Taking derivative with respect to firm's location, we get

$$\frac{\partial \Pi_m}{\partial s} = btnp_m^* \left[\int_s^{s+\frac{a-bp_m^*}{bt}} \phi(x)dx - \int_{s-\frac{a-bp_m^*}{bt}}^{s} \phi(x)dx \right]$$

Given the normal density function $\phi(x)$ in equation (1), we can find $\frac{\partial \Pi_m}{\partial s} = 0$ for $s = 0$ and $\frac{\partial \Pi_m}{\partial s} < 0$ for $s > 0$. Thus, the unique optimal location under mill pricing and constant location cost is the city center, $s_m^* = 0$.

Under discriminatory pricing, the profit function is shown in expression (18), taking derivative with respect to s, we obtain

$$\frac{\partial \Pi_d}{\partial s} = \int_s^{s+\frac{a}{bt}} tn\phi(x)\frac{a-bt(x-s)}{2}dx - \int_{s-\frac{a}{bt}}^{s} tn\phi(x)\frac{a-bt(s-x)}{2}dx$$

Now let $r = |x-s|$, where r is the distance from the firm's location and $r \in [0, \frac{a}{bt}]$. Each consumer at distance r has a demand $q_r = \frac{a-btr}{2} \geq 0$ under price policy (14). Then $\frac{\partial \Pi_d}{\partial s}$ becomes

$$\frac{\partial \Pi_d}{\partial s} = t\int_0^{\frac{a}{bt}} n[\phi(s+r) - \phi(s-r)]q_r dr$$

Given the normal density function $\phi(x)$, it follows $\frac{\partial \Pi_d}{\partial s} = 0$ for $s = 0$ and $\frac{\partial \Pi_d}{\partial s} < 0$ for $s > 0$. Thus, similarly to mill pricing, the unique optimal location under discriminatory pricing and constant location cost is the city center, $s_d^* = 0$.

D Comparison of equilibrium outcomes in Case 3

Based on Proposition 1, we know $s_m^* = s_d^* = 0$. We next compare market radius, profits, output, and welfare in mill and discriminatory pricing.

Market radius. Using (5) and (16), the market radius under mill pricing and spatial price discrimination are $radius_m = |R_m - s_m^*| = \frac{a-bp_m^*}{bt}$ and $radius_d = |R_d - s_d^*| = \frac{a}{bt}$. Because $p_m^* \in (0, \frac{a}{2b})$, it follows that $radius_m < \frac{a}{bt}$, so the market area is larger under discriminatory pricing than under mill pricing when the firm's location is Given.

Profits. Under discriminatory pricing, the market area can be divided into three regions $x \in [-\frac{a}{bt}, -\frac{a-bp_m^*}{bt})$, $x \in [-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]$, and $x \in (\frac{a-bp_m^*}{bt}, \frac{a}{bt}]$. For any market x in the market interval $[-\frac{a}{bt}, -\frac{a-bp_m^*}{bt})$ and $(\frac{a-bp_m^*}{bt}, \frac{a}{bt}]$, the firm can make positive net revenue above location cost under discriminatory pricing, while zero net revenue above location cost under mill pricing since the demand in this market interval is zero. Under mill pricing, for any market x in $[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]$, the net revenue above location cost (4) is maximized at $p_m = \frac{a-bt|x|}{2b}$ with a value of $\frac{n\phi(x)(a-bt|x|)^2}{4b}$, which is the optimal net revenue above location cost (12) under price discrimination. Since p_m^* is a constant mill price, p_m^* cannot be equal to $\frac{a-bt|x|}{2b}$ and maximize the net revenue for every market $x \in [-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]$. Thus, discriminatory pricing yields higher aggregate revenue than under mill pricing. Given the same location cost of the given location, discriminatory pricing is more profitable than mill pricing.

Output. Since p_m^* solves the first order condition (7), this means $\int_{-\frac{a-bp_m^*}{bt}}^{\frac{a-bp_m^*}{bt}} n\phi(x)(a - 2bp_m^* - bt|x|)dx = 0$. Using (8) and (17), we can calculate the output difference between the two pricing systems:

$$\begin{aligned}
Q_d - Q_m &= \int_{-\frac{a}{bt}}^{-\frac{a-bp_m^*}{bt}} \frac{n\phi(x)(a-bt|x|)}{2}dx + \int_{\frac{a-bp_m^*}{bt}}^{\frac{a}{bt}} \frac{n\phi(x)(a-bt|x|)}{2}dx \\
&\quad - \frac{1}{2}\int_{-\frac{a-bp_m^*}{bt}}^{\frac{a-bp_m^*}{bt}} n\phi(x)[a-2bp_m^* - bt|x|]dx \\
&= \int_{-\frac{a}{bt}}^{-\frac{a-bp_m^*}{bt}} \frac{n\phi(x)(a-bt|x|)}{2}dx + \int_{\frac{a-bp_m^*}{bt}}^{\frac{a}{bt}} \frac{n\phi(x)(a-bt|x|)}{2}dx \\
&> 0
\end{aligned}$$

Thus, the output of the monopolist is higher under spatial price discrimination than mill pricing. From above equation, we can clearly see that the output difference between discriminatory and mill pricing is equal to the output gain from the extra market area under spatial price discrimination.

Social welfare. Using (11) and (20), we can calculate the social welfare difference between the two pricing regimes:

$$\begin{aligned} W_d - W_m = & \underbrace{\int_{-\frac{a}{bt}}^{-\frac{a-bp_m^*}{bt}} \frac{3n\phi(x)(a-bt|x|)^2}{8b} dx + \int_{\frac{a-bp_m^*}{bt}}^{\frac{a}{bt}} \frac{3n\phi(x)(a-bt|x|)^2}{8b} dx}_{>0,\text{Welfare gain from extra market regions } [-\frac{a}{bt}, -\frac{a-bp_m^*}{bt}) \text{ and } (\frac{a-bp_m^*}{bt}, \frac{a}{bt}]} \\ & + \underbrace{\frac{1}{2}(\Pi_{m|x\in[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]} - \Pi_{d|x\in[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]})}_{<0,\text{Welfare loss in the nearby market interval } [-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]} \\ & \gtreqless 0 \end{aligned}$$

where $\Pi_{m|x\in[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]}$ and $\Pi_{d|x\in[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]}$ are the profits under mill pricing and discriminatory pricing in market area $[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]$, respectively. As we argued previously, $\Pi_{m|x\in[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]} < \Pi_{d|x\in[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]}$. This implies that in market area $[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]$, spatial price discrimination reduces welfare. Relative to mill pricing, discriminatory pricing regime serves extra market regions $[-\frac{a}{bt}, -\frac{a-bp_m^*}{bt})$ and $(\frac{a-bp_m^*}{bt}, \frac{a}{bt}]$, where the welfare increases. The sign of $W_d - W_m$ depends on the welfare gain from $[-\frac{a}{bt}, -\frac{a-bp_m^*}{bt})$ and $(\frac{a-bp_m^*}{bt}, \frac{a}{bt}]$ and the welfare loss from $[-\frac{a-bp_m^*}{bt}, \frac{a-bp_m^*}{bt}]$. Thus, the welfare therefore may be higher or lower under discrimination than under mill pricing.

E Simulation description

Under mill pricing, the two-stage game can be formulated as constrained optimization problem

$$\begin{aligned} \max_{p_m, s} \quad & \pi_m(p_m, s) = \int_{s-\frac{a-bp_m}{bt}}^{s+\frac{a-bp_m}{bt}} n\phi(x) p_m[a - b(p_m + t|x-s|)] dx - F(s, n) \\ \text{s.t.} \quad & \int_{s-\frac{a-bp_m}{bt}}^{s+\frac{a-bp_m}{bt}} n\phi(x)[a - 2bp_m - bt|x-s|] dx = 0 \end{aligned} \tag{25}$$

The two-stage game under discriminatory pricing can be formulated as constrained optimization problem

$$\begin{aligned} \max_{p_d, s} \quad & \pi_d(p_d, s) = \int_{s-\frac{a}{bt}}^{s+\frac{a}{bt}} n\phi(x) p_d[a - b(p_d + t|x-s|)]dx - F(s,n) \\ \text{s.t.} \quad & p_d = \frac{a - bt|x-s|}{2b} \end{aligned} \tag{26}$$

The integrals can be approximated with Monte Carlo Simulation. Generally, suppose $q(x)$ is density function of x and that we want to compute $\int g(x)q(x)dx$. We can simulate N draws $(x_1, ..., x_N)$ from $q(x)$, and let $N^{-1}\sum_{i=1}^{N} g(x_i)$ be the approximation of $\int g(x)q(x)dx$. In practice, many researchers adopt this technique to approximate integral in their studies (Berry et al., 1995; Dubé et al., 2012; Lee and Seo, 2015).

We set $a = b = \sigma = \sigma_F = 1$, and $A = 0.15$. Now we simulate $n = 100,000$ artificial consumers drawn from $\phi(x)$. We only analyze the case where $s \geq 0$. Analogous results apply when $s \leq 0$. In footnote section 3.1, we also show that $p_m \in (0, \frac{a}{2b})$. Thus, the constrained optimization problem under mill pricing becomes

$$\begin{aligned} \max_{p_m, s} \quad & \pi_m(p_m, s) = \frac{1}{n}\sum_{i=1}^{n}\left(1(s - \frac{a-bp_m}{bt} \leq x_i \leq s + \frac{a-bp_m}{bt}) n p_m[a - b(p_m + t|x_i - s|)]\right) - F(s,n) \\ \text{s.t.} \quad & \frac{1}{n}\sum_{i=1}^{n}\left(1(s - \frac{a-bp_m}{bt} \leq x_i \leq s + \frac{a-bp_m}{bt}) n[a - 2bp_m - bt|x_i - s|]\right) = 0 \\ & 0 < p_m < \frac{a}{2b}, s \geq 0 \end{aligned} \tag{27}$$

where indicator function $1(s - \frac{a-bp_m}{bt} \leq x_i \leq s + \frac{a-bp_m}{bt})$ takes 1 if x_i is in the interval $(s - \frac{a-bp_m}{bt}, s + \frac{a-bp_m}{bt})$ and 0 otherwise.

Under discriminatory pricing, the constrained optimization problem becomes

$$\begin{aligned} \max_{s} \quad & \pi_d(s) = \frac{1}{n}\sum_{i=1} n\left(1(s - \frac{a}{bt} \leq x_i \leq s + \frac{a}{bt})\frac{n(a - bt|x_i - s|)^2}{4b}\right) - F(s,n) \\ \text{s.t.} \quad & s \geq 0 \end{aligned} \tag{28}$$

where $1(s - \frac{a}{bt} \leq x_i \leq s + \frac{a}{bt})$ takes 1 if x_i is in the interval $(s - \frac{a}{bt}, s + \frac{a}{bt})$ and 0 otherwise.

In this paper, we solve the Mathematical Program with Equilibrium Constraints (MPEC) with KNITRO optimization solver (Su and Judd, 2012; Dubé et al., 2012). After we find the equilibrium p_m^*, s_m^* and s_d^*, we can use Monte Carlo approximation to get the equilibrium profits, outputs, consumer surplus, and welfare under both pricing regimes. For example, equilibrium output under mill pricing can be approximated by $Q_m^* = \frac{1}{n}\sum_{i=1}^{n}(1(s_m^* - \frac{a-bp_m^*}{bt} \leq$

$x_i \leq s_m^* + \frac{a-bp_m^*}{bt})[a-b(p_m^*+t|x_i-s_m^*|)]])$. We replicate the Monte Carlo simulation 1000 times and find the mean of each variable.

To get Figure 2, we first generate a sequence of location $(s_1, ..., s_{ns})$ and calculate MCL, MRL_m, and MRL_d at each location. For a given location s_k, the marginal cost of location can be obtained by $MCL(s_k) = \frac{Ans_k}{\sqrt{2\pi}\sigma_F^3} e^{-\frac{s_k^2}{2\sigma_F^2}}$.

Under discriminatory pricing, the marginal revenue of location at s_k can be approximated by

$$\begin{aligned} MRL_d(s_k) = & \frac{1}{n}\sum_{i=1}^{n}\left(1(s_k - \frac{a}{bt} \leq x_i \leq s_k)\frac{nt(a-bt|x_i-s_k|)}{2}\right) \\ & - \frac{1}{n}\sum_{i=1}^{n}\left(1(s_k \leq x_i \leq s_k + \frac{a}{bt})\frac{nt(a-bt|x_i-s_k|)}{2}\right) \end{aligned} \tag{29}$$

Under mill pricing, we need to find the optimal price at location s_k. To achieve this, we solve

$$\begin{aligned} \max_{p_m} \pi_m(p_m, s_k) = & \frac{1}{n}\sum_{i=1}^{n}\left(1(s_k - \frac{a-bp_m}{bt} \leq x_i \leq s_k + \frac{a-bp_m}{bt})np_m[a-b(p_m+t|x_i-s_k|)]\right) \\ & - F(s_k, n) \\ \text{s.t.} \quad & 0 < p_m < \frac{a}{2b} \end{aligned}$$

By solving above problem with KNITRO, we get the optimal price p_m^*. Then we can approximate marginal revenue of location at s_k under mill pricing by Monte Carlo simulation.

$$\begin{aligned} MRL_m(s_k) = & \frac{1}{n}\sum_{i=1}^{n}\left(1(s_k - \frac{a-bp_m^*}{bt} \leq x_i \leq s_k)nbtp_m^*\right) \\ & - \frac{1}{n}\sum_{i=1}^{n}\left(1(s_k \leq x_i \leq s_k + \frac{a-bp_m^*}{bt})nbtp_m^*\right) \end{aligned} \tag{30}$$

Finally, we can plot MCL, MRL_m, and MRL_d and obtain figure 2.

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