Rationalizing Time Inconsistent Behavior: The Case of Late Payments*

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August 29, 2016

Abstract

Consumers often sign contracts in which they consume a good over a period of time, paying for it a fee due at a later period; such as in TV cable, internet, and cell phone. In such contracts, buyers do not always read or understand terms and conditions, thus underestimating the penalty involved in not honoring the contract. As a result, consumers may initially agree to pay the contract fee on time, yet sub-optimally decide to pay such fee late once it is due (plus penalties). In this paper, we find that such preference reversal can be explained by present bias, but only under restrictive parameter conditions. However, allowing for bounded rationality and memory loss helps rationalize such behavior for less restrictive parameter values. We further show how a seller can increase profits by setting fees and penalties that lead consumers to fall prey to preference reversals over time.

Keywords: Dynamic Inconsistency, Contracts, Rational Behavior, Memory Loss.
JEL classification: D01, D22, D84, D82, D86.

*We thank Ana Espinola-Arredondo for her insightful comments and suggestions.
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1 Introduction

In the U.S., 28.4 million households pay at least one bill late per month (Wall Street Journal 3/15/2007), and penalties for late payment increased from $7 billion in 2000 to $22 billion in 2004 (Wall Street Journal April 2004). More importantly, 91% of adults paying their bills late recognize that, at the time of making a big financial decision (such as buying a car, or refinancing their mortgage), they were confident of making a correct financial decision. While such behavior could be explained by unexpected events occurring after the signature of the contract, it could also be rationalized by a consumer's inability to fully understand the details of the contract, especially those referred to penalties. In fact, 75% of adults paying a bill late either agree or strongly agree that they could have used financial advice before signing the contract.

Similar paying patterns emerge in the "Consumer Financial Literacy Survey" for the 2009-2015 period, and in other studies.[1] Interestingly, the most common reason that consumers use to justify their late payments is forgetfulness (61%), and also being busy with work and family obligations (39%). Lack of available funds (42%) is, thus, not the main reason. Therefore, people who have the ability to pay on time, and sign up expecting so do so, often end up paying their bills late. In this paper, we present different competing reasons to explain such surprising behavior. We also identify the conditions under which the seller takes advantage of such behavior from consumers by strategically designing fees and penalties.

Our paper seeks to answer two questions: Why do consumers sign contracts anticipating to pay them on time, but when contract fees are due consumers change their minds and choose to pay fees late incurring additional penalties?; and, Under which conditions do sellers maximize profits by strategically designing fees and penalties that induce consumers to pay their bills late?

While looking into the first question, an immediate answer is that consumers exhibit present bias and hyperbolic discounting which, to some extent, explains why a person would pay a bill late despite suffering a penalty. As we show, this explanation is, however, insufficient since it can only sustain late payments under restrictive parameter conditions. We then offer two more reasons for such consumer behavior. The first is misinterpretation, understood as that, even when a consumer is presented with all the information needed to make an optimal decision, he lacks the skills or motivation to fully understand the contract and penalties. According to Smithers (2011), for instance, 93% of British citizens recognize that they did not carefully read terms and conditions before signing up online for a product or a service. Tugend (2013) finds that software contracts are, on average, 74,000 words long (similar in length to the first Harry Potter book), leading most customers to click in the "I Agree" button to accept the contract without reading it. Hence, misinterpretation of contract conditions seems relatively prevalent, especialy in certain markets such as online and software contracts. One could argue that individuals, after misinterpreting the

[1] In the 2009-2015 period, the "Consumer Financial Literacy Survey" reports that an average of 26.83% of U.S. adults recognize not paying all bills on time. In addition, an average of 77.33% adults who paid a bill late agree or strongly agree that they could have used financial advice. Similarly, according to the Citi Simplicity Survey (2013), "59% of Americans have paid a bill late in their lifetime (including credit card, utility, cable, etc.), and 88% of those have done so in the past 12 months."

conditions in a contract and facing penalties, would carefully read future contracts before signing them thus not being subject to misinterpretation again. However, the volume of individuals paying their bills late suggests that consumers do not significantly change the amount of time they spend evaluating contracts.

The second reason we examine is memory loss: as time proceeds, the consumer is likely to forget the details of the contract. Decreasing retention abilities have been well documented in the psychological literature since Ebbinghaus (1885) and Craik and Lockhart (1972) both in short and long periods of time; for a survey of this literature see Schacter (1999). More recently, Ericson (2011) experimentally shows that individuals suffer from overconfidence in their own memory, that is, they overestimate their future ability to remember events, which leads them to make suboptimal financial decisions in the present.[2] Consumers' overconfidence in their own ability to remember future tasks can, hence, affect their ability to paying bills on time and avoid penalties in a contract. Our model explains how a consumer is more likely to sign a contract expecting to pay on time but then end up paying late when he misinterprets the conditions of the contract and/or suffers from memory loss, i.e. there is a higher chance of "preference reversal."

A misinterpreting consumer can be an opportunity for a seller to obtain higher profits by setting steep penalties and higher fees. However, a seller may not always find it optimal to do so since high penalties and fees can deter well informed consumers from the contract. While answering the second question, we present the conditions under which the seller chooses to sell to misinterpreters alone. We find that correctly interpreting the contract can help prevent this type of consumer from entering into the contract. In other words, the presence of misinterpreters induces the seller to focus only on this type of buyers, which can be more significantly exploited, at the expense of losing all sales to informed buyers. This result can then be understood as a market failure similar to the "lemons" problem, whereby the presence of a large proportion of bad quality cars prevents sales of good quality cars. Indeed, since the seller cannot observe the buyers' types (informed vs. misinterpreter) we find that, if there is a sufficiently high proportion of misinterpreters, the seller's expected profits become larger by focusing on misinterpreters alone. Further, we find that remembering the details of the contract can be especially useful to the consumer if a large proportion of consumers forget those details. In such a situation, the seller sets very high penalties to exploit the misinterpreters and the informed buyers pay their fee on time, thus avoiding any exploitation.

In our model, the seller selects the fee (price for the good) and the penalties that buyers suffer if they were to pay the fee late. Regarding fees, we find that choosing a very high fee deters well informed individuals from signing the contract, while misinterpreters sign it. A lower fee, however, induces all types of individuals to accept the contract, but generating a smaller profit per consumer. The seller, therefore, faces a trade-off when setting optimal fees, since lower fees reduce her margins

[2] In the experiment, subjects had to choose between receiving a large payment ($20), conditional on them remembering to claim their payment with a six-month delay, or a small payment ($5 to $20 in increments of $0.75) that would automatically be sent to them after six months. Ericson (2011) finds that, while three quarters of the subjects choose the large payment and said they would remember to claim their payment after six months, only half of them claimed it, thus reflecting their overconfidence in their own memory.

but expand the number of buyers signing the contract. We find that the seller chooses a fee that induces full participation if the proportion of misinterpreters is sufficiently low relative to the degree of misinterpretation. This is because, if the degree of misinterpretation is too high or there are many misinterpreters, the seller is attracted to heavily exploit this segment of consumers even if that entails giving up informed types who would not be willing to accept the contract.

If the seller sets a fee such that everyone participates, her choice of penalties for late payments determine whether all type of customers pay late or only misinterpreters do. Specifically, if the seller sets high (low) penalties, only misinterpreting consumers (all type of consumers, respectively) pay their fees late. When setting penalties, the seller faces a similar trade-off as when designing optimal fees: she can either collect low penalties from all customers, or high penalties from only misinterpreters. Our findings show that the seller sets a relatively low penalty if the proportion of informed consumers at the due date is high and the degree of misinterpretation is low. Intuitively, few customers can be exploited and the extent to which they can be exploited is small, thus making it unattractive for her to focus on misinterpreters alone.

Our results identify conditions under which the seller uses fees and penalties to serve both types of consumers (those fully understanding the contract and those who misinterpret it), or only serve those consumers who do not understand the details of the contract. In order to guarantee that all types of consumers are served, governments may regulate the severity of the penalties that the seller can impose on consumers who pay their bills late, or make contract information more easily available to consumers. Furthermore, our findings suggest that stringent regulations on late penalties (essentially banning firms from charging these penalties) would lead the seller to use fees alone to maximize profits. The seller, however, can still set fees to either attract misinterpreters alone (if their proportion is large) or all types of customers otherwise. Hence, setting limits on the penalties that firms can charge for late payments do not necessarily solve the market failure described above in which only a segment of customers are served, and heavily exploited, by the seller.

Related literature. Present bias is a common explanation to rationalize consumers' dynamically inconsistent behavior. Laibson (1997) considers a quasi-hyperbolic discounting to account for present bias, while O'Donoghue and Rabin (1999) extend his model to allow for the present bias parameter to change over time. Both papers explain how caring more about the present than the future can lead consumers to pay their bill late. However, this does not provide a complete answer to the question. In particular, as the due date for a bill approaches, there is a small time difference between the current and the future (when consumers are penalized). For this small time period, present bias can only explain late bill payments under restrictive parameter conditions.

Hoch and Loewenstein (1991) analyze consumers who do not have enough willpower and self-control to make optimal choices, and the factors affecting self-control. Caillaud and Jullien (2000) use revealed preferences to understand time inconsistent behavior. In their model, the consumer reveals a set of preferences at each time period, and his preferences change over time.[3] Similarly,

[3] Gul and Pesendorfer (2001) use a similar idea for a finite time horizon.

our paper examines how a consumer's preferences change over time, and how such a preference reversal can be used by a seller to its advantage.

Grenadiera and Wang (2007) analyze time inconsistent preferences in investment decisions. After an entrepreneur makes an initial investment, her utility function changes in future periods, which leads her to suboptimal investment decisions in the first period. While an investment is analogous to signing a contract, the consumer in our model, however, faces the probability of not understanding the details of the contract, and can suffer from memory loss over time. (For a detailed literature review of models explaining time inconsistent behavior, see Koszegi (2014).)

Our model also builds on the empirical literature analyzing consumers' memory loss affects their economic decisions, such as the results in Ericson's (2011) experiment described above. Karlan et al. (2016) examine data from three banks in Peru, Bolivia and the Philippines, and show that, among people who have recently opened a savings account, reminders increase the probability of meeting their commitments. Similarly, Calzolari and Nardotto (2012) conducted a field experiment on a sample of individuals joining a gymnasium, and found that a weekly email reminder increases attendance by up to 25%.

Our paper is structured as follows. Section 2 describes the model, and section 3 presents the consumer's problem for each of our behavioral settings. In section 4 we examine the seller's problem for each type of consumer he may face. Section 5 discusses our main conclusions and policy implications.

2 Model

Consider a setting with discrete time $t \in \{1, \ldots n, ...T\}$, where $T < \infty$. A monopolist offers a contract at time $t = 0$ to the consumer, who can choose to sign it or not. If he signs it, the contract provides x units of a good to the consumer at $t = 0$, and the due date for payment is $t = n$. Hence, the time periods between n and 0 can be understood as the paying cycle, e.g., a month for cable bills. The consumer can pay late (i.e., at period $t > n$) but must pay by the final period T where $T \geq n$.[4] This setting embodies as special cases contracts providing $x = 1$ unit of the good, contracts due immediately after signing (i.e., $n = 1$), and those allowing the consumer to enjoy the good for $n - 1$ periods and pay at the "due date" period n (e.g., $n = 30$ days in monthly billing cycles).

The consumer faces a fee $F > 0$ for the good, implying that his discounted utility at the time period t when the consumer chooses to pay the fee F is $\beta\delta^t[u(x) - F]$, where $u(x)$ denotes her utility from x units, where $u'(x) > 0$ and $u''(x) < 0$; $\delta \in (0, 1)$ represents the consumer's discount factor; and $\beta \leq 1$ denotes the consumer's present bias, as in Laibson (1997). (Note that if $\beta = 1$ present bias is absent, and the consumer exhibits standard exponential discounting.) If, in contrast, the consumer does not pay fee F, her discounted utility is $\beta\delta^t u(x)$.

[4] This assumption can be rationalized by considering that, if the consumer does not pay by T she faces strict legal action, which causes a high disutility.

If she pays one period late, the fee F increases to K_1F, where penalty K_1 satisfies $K_1 > 1$.[5] Similarly, if she is $i = t - n$ periods late, she pays a total fee of K_iF, where $1 < K_1 < K_2 < ... < K_{T-n}$, thus indicating that penalties increase in time. We indicate periods after n as i, e.g., if $t = n + 1$ then $i = 1$ since $i = t - n$ by definition. Applying backward induction, we start by solving the consumer's problem (whether he signs the contract and, if so, when does he pay) and then move on to the producer (designing the optimal contract).

3 Consumer's Problem

We now analyze our model, starting from the contexts most common in literature (full rationality, with or without present bias), and subsequently move to bounded rationality settings. Proofs of all results are provided in the appendix.

3.1 Case 1: Full rationality

Let us first consider a setting in which the consumer perfectly understands fees and penalties, and $\beta \leq 1$, which allows for him to exhibit present bias (if $\beta < 1$) or not (if $\beta = 1$). As we next show, exponential discounting is incompatible with preference reversal (i.e., consumers do not change their mind about paying the contract on time), but present bias allows for such preference reversal to exist.

Lemma 1. *If the consumer does not exhibit present bias, $\beta = 1$, he pays at the due date every contract he agreed to sign at $t = n$. If the consumer exhibits present bias, $\beta < 1$, she signs the contract at $t = 0$ expecting to pay at the due date $t = n$ if $K_i \geq \frac{1}{\delta^i}$ for all i periods, but does not pay when the bill is due if there exists a period i such that $K_i < \frac{1}{\beta\delta^i}$.*

Hence, the consumer signs the contract at $t = 0$ and expects to pay at $t = n$ if penalties are sufficiently high, i.e., $K_i \geq \frac{1}{\delta^i}$ for every late period i; but once the bill is due at $t = n$, the consumer chooses not to pay if the penalty for some period i is sufficiently low, i.e. $K_i < \frac{1}{\beta\delta^i}$. Intuitively, at $t = 0$ the decision about whether to pay on time or at a later period i is in the future, and thus unaffected by present bias. Once the due date arrives, however, present bias affects the consumer's decisions about whether to pay at $t = n$ or at a later period $t = n + i$. Both conditions on the penalty K_i, however, can only hold for a restricted set of (K_i, δ^i)-pairs, that is, $K_i \in \left(\frac{1}{\delta^i}, \frac{1}{\beta \cdot \delta^i}\right]$. In addition, the distance $\frac{1}{\delta^i} - \frac{1}{\beta \cdot \delta^i} = \frac{1-\beta}{\beta \cdot \delta^i}$ is decreasing in the present bias parameter β; as illustrated in Example 1 below. In our setting, β is likely to be close to one, since the type of contracts we analyze are due after a few days/weeks of being signed.[6] Such a high value of β entails a small

[5] In the U.S., for instance, Time Warner Cable (TV services) and Verizon (phone) charge a penalty of up to 1.5% of the monthly fee in case of late payment, i.e., $K_1 = 0.015$. These penalties are larger in other countries. Reliance India Mobile, for example, charges a late penalty equivalent to 2.5% of the monthly fee in case of late payment.

[6] Takeuchi (2011) discusses that if the time gap is very small, there is even a chance of $\beta > 1$ (future bias). While we do not consider the case for future bias, note that $\beta > 1$ implies $\frac{1}{\beta\delta^i} < \frac{1}{\delta^i}$, entailing that the condition on K_i in Lemma 1 cannot hold, and thus the consumer would not exhibit preference reversal under any parameter values. Balakrishnan et al. (2015) estimated β's from empirical studies ranging from 0.901 to 0.937, close to one.

region of K_i for which preference reversal can occur.

Example 1. Consider a consumer with discount factor $\delta = 0.95$, a contract that sets a penalty $K_1 F$ the period immediately after the bill is due, i.e., $i = 1$. (For simplicity assume that the final period is $T = n + 1$.) In this setting, the consumer expects to pay on time if the fee is sufficiently high, $K_1 \geq \frac{1}{0.95} = 1.052$; but does not pay once the bill is due if $K_1 < \frac{1}{\beta\delta^1} = \frac{1.052}{\beta}$. Hence, preference reversal requires K_1 to satisfy $1.052 \leq K_1 < \frac{1.052}{\beta}$. For a present bias parameter of $\beta = 0.92$, such as those reported in the empirical studies in Balakrishnan et al (2015), this condition on K_1 becomes extremely narrow, i.e., $1.052 \leq K_1 < 1.143$.

3.2 Case 2: Bounded rationality

The introduction of present bias in the previous section allowed for preference reversals to arise, but under a restrictive set of parameter values. We next explore a model in which the consumer's cognitive ability is bounded, whereby we assume that she does not fully understand the severity of the fees involved in the contract. In particular, consider that, with probability $p \in [0, 1]$ she correctly interprets fee F, but with probability $1 - p$ she incorrectly infers that a lower fee $f < F$ is due at $t = n$. A similar argument applies to the increasing rate of penalty, K_i, which the consumer correctly assesses with probability p, or interprets at a lower rate $k_i < K_i$ with probability $1 - p$. (Similarly as for K_i, the misinterpreted k_i satisfies $1 < k_1 \leq k_2 \cdots \leq k_{T-n}$.)[7]

A common example would be customers signing contracts with several pages of details and clauses without carefully reading them (e.g., quickly clicking on the "I Agree" button in the case of online agreements). While we allow for the consumer to misunderstand fees with probability $1 - p$, we assume that such probability does not change across time.[8] The following proposition identifies conditions under which such a preference reversal can arise.

Proposition 1. *Under a setting of bounded rationality, the consumer signs the contract at $t = 0$ expecting to pay at $t = n$ if $K_i \geq \frac{pF+(1-p)\left(1-\delta^i k_i\right)f}{\delta^i pF}$ for all i periods, but does not pay when the bill is due if there exists a period i such that $K_i < \frac{pF+(1-p)\left(1-\beta\delta^i k_i\right)f}{\beta\delta^i pF}$.*

Similarly as under full rationality (Lemma 1), the consumer expects to pay at $t = n$ if the penalty is sufficiently high, but does not pay at $t = n$ if the penalty of at least one period i is sufficiently low (second condition on K_i). Unlike Lemma 1, however, the range of K_i's where a preference reversal occurs is wider under bounded than under full rationality, as the next corollary shows.

[7] For generality, we allow for the misterpreted fee to satisfy $f < F$ in the consumer problem. For the seller problem, we will assume a specific functional form $f = F \cdot (1 - \alpha)$, where $\alpha \in [0, 1]$ denotes the degree of the consumer's misinterpretation. Similarly, we allow a general value of the misterpreted penalty $k_i < K_i$ in the consumer's problem. In the seller's problem, we will later assume $k_i = K_i \cdot (1 - \gamma)$, where $\gamma \in [0, 1]$ captures the degree of the consumer's misinterpretation.

[8] The probability of misunderstanding fees could, however, change across time if, for instance, the consumer faces a lower probability p of remembering the fees he originally understood. For completeness, we explore such a possibility in Appendix A whereby we allow for probability p to decrease over time.

Corollary 1. *The range of parameters supporting preference reversal under bounded rationality is greater than under full rationality, that is,*

$$\left[\frac{pF + (1-p)\left(1-\beta\delta^{i}k_{i}\right)f}{\beta\delta^{i}pF} - \frac{pF + (1-p)\left(1-\delta^{i}k_{i}\right)f}{\delta^{i}pF}\right] \geq \left[\frac{1}{\beta\delta^{i}} - \frac{1}{\delta^{i}}\right].$$

Example 2. Consider an extension of Example 1 where $\delta = 0.95$ and $i = 1$. Let us assume a fee of $F = 100$, a misinterpreted fee of $f = 90$ and misinterpreted penalty of $k_i = 1.05$ where the probability of correctly interpreting the fee is $p = 0.8$. In this setting, the consumer expects to pay on time if the fee is sufficiently high, $K_1 \geq 1.042$; but does not pay once the bill is due if $K_1 < \left(\frac{1.29}{\beta} - 0.23625\right)$. Hence, preference reversal requires K_1 to satisfy $1.053 \leq K_1 < \left(\frac{1.29}{\beta} - 0.23625\right)$. For a present bias parameter of $\beta = 0.92$, such as those reported by Balakrishnan et al (2015), this condition on K_1 becomes $1.053 \leq K_1 < 1.165$. The magnitude of the range here is 0.112; thus expanding the range of K_1 values than under full rationality (with or without present bias).

4 Seller's Problem

The seller maximizes profits using two variables: the fee F and the stream of penalties $\{K_i\}_{i=1}^{T-n}$. For tractability, this section assumes that the misterpreted fee f satisfies $f = F \cdot (1-\alpha)$, where $\alpha \in [0, 1]$ denotes the degree of the consumer's misinterpretation; and, similarly, that the misterpreted penalty satisfies $k_i = K_i \cdot (1-\gamma)$, where $\gamma \in [0, 1]$ represents the degree of misinterpretation.

4.1 Determining optimal stream of penalties

We first find the optimal stream of penalties $\{K_i\}_{i=1}^{T-n}$ as a function of F. Section 4.2 identifies the optimal F.

4.1.1 Case 1: Full rationality

As shown in Lemma 1, when the consumer does not exhibit present bias, he pays the fee at the due date, and thus late penalties do not apply. With present bias, however, there is a possibility for preference reversal. The producer can anticipate consumer's behavior, setting the stream of penalties K_i described in the following proposition.

Proposition 2. *Under present bias, the seller sets a penalty* $K_i = \frac{1}{\beta\delta^i} - \epsilon$ *for every late period* i *where* $\epsilon \to 0$, *which induces the consumer to pay the bill at the last period* $t = T$.

This result follows directly from Lemma 1. Since there are no probabilities involved, the producer can accurately predict the consumer's decision: he induces the consumer to not pay until the last period, which gives the seller the maximum possible revenue $K_{T-n}F$.

Example 3. Consider similar parameter values as in the previous example, i.e., $\delta = 0.95$, $\beta = 0.92$, and $t = n + 1$. Hence, penalty K_1 becomes 1.144.

4.1.2 Case 2: Bounded rationality

Introducing present bias gave us a solution for the stream of penalties $\{K_i\}_{i=1}^{T-n}$. However, the payment behavior that it predicted (paying in the last period) is not generally observed. We next examine the optimal stream of penalties in Case 2. For illustration purposes, Figure 1a depicts the probability of the consumer interpreting the fees correctly in the case for bounded rationality where the probability of misterpreting fees, p_t, is assumed to be constant at $p_t = p_0$ from $t = 0$ until the due date $t = n$. After the due date, the seller would send a notice to the buyer, thus increasing the probability of correctly interpreting the fee, reaching a probability $p_T = 1$ in the last period T.

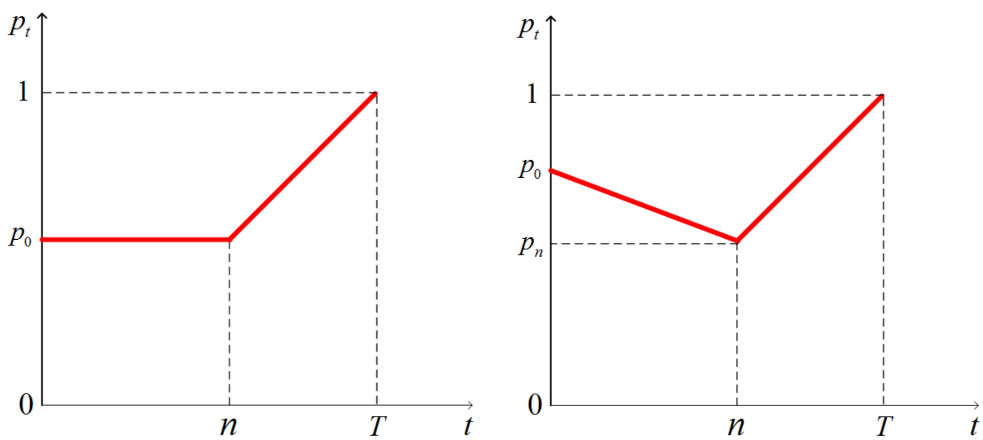

Fig. 1a. p_0 constant until $t = n$. Fig. 1b. p_0 decreasing until $t = n$.

Figure 1b illustrates the probability of the consumer interpreting the fees correctly decreases over time. At $t = 0$, the consumer interprets the fee correctly with probability p_0. As time progresses, however, the probability of the consumer interpreting the fee correctly decreases, entailing that $p_0 > p_n$, which could be due to memory loss or other behavioral biases. However, as in Figure 1a, the probability of correctly interpreting the fee reaches $p_T = 1$ in the last period. We next identify the optimal stream of penalties when probability p_t is constant until the due date $t = n$. (Appendix A shows that our results are not qualitatively affected in when such probability decreases over time.)

Proposition 3. *Under a setting of bounded rationality, the seller sets the optimal stream of penalties as follows:*

i) If $\gamma \geq \overline{\gamma}$, the seller's optimal penalty is $K_i = \left(\frac{1}{\beta \cdot \delta^i} \frac{1}{(1-\gamma)} - \epsilon\right)$ for every late period i and $\epsilon \to 0$.

ii) *Otherwise, the seller's optimal penalty is* $K_i = \frac{1}{\beta\delta^i} - \epsilon$ *, where*

$$\overline{\gamma} \equiv 1 - \frac{\beta\delta^{T-n}}{1 - p_n\beta\delta^{T-n}} \left(\sum_{i=1}^{T-n} \frac{1}{\beta\delta^i} p_{i+n} \prod_{j=n}^{i+n-1} (1 - p_j) \right)$$

In this setting, the seller has two penalty options. First, she can keep the value of the penalty relatively low, which induces everyone (even the correctly informed buyers who did not misinterpret fees) to pay in the final period. Second, the seller can set a high penalty that induces informed (uninformed) buyers to not pay late (pay late, respectively). The seller will prefer to induce everyone to pay late if few buyers are uninformed (i.e., probability p_n is low), or if the extent to which they are uninformed is not high (i.e., γ is not close to zero). The seller decides the exact value of the penalties by maximizing the profit over the entire period. We can also see that as p_n increases, the seller's total profit from setting a penalty that makes only misinterpreters pay late decreases. This is because more people pay on time and less of them pay a penalty.[9]

Example 4. Assume, as in previous examples, that $\delta = 0.95$, $\beta = 0.92$ and $\gamma = 0.2$. In addition, $T = n + 2$, $p_n = 0.4$, $p_{n+1} = 0.8$, and $p_T = 1$. Then, if the seller seeks to induce both types of buyers to pay in the last period, she sets a penalty of $K_1 = 1.144$ and $K_{T-n} = 1.204$. If the seller seeks only uninformed buyers pay late, the seller sets a penalty of $K_1 = 1.43$ and $K_{T-n} = 1.505$. Given our parameter values, the seller chooses to include everyone since the condition in Proposition 3 holds, i.e., $1.26 > 1.204$.

4.2 Determining the optimal fee

The value of the optimal fee F depends on the utility of the consumer.

4.2.1 Case 1: Full rationality

Lemma 2. *In the case of full rationality with present bias, the seller sets a fee*

$$F = \frac{u(x)\left(1 + \beta \cdot \sum_{t=1}^{T} \delta^t\right)}{\beta\delta^n}.$$

Hence, this fee collapses to $F = \frac{u(x)\sum_{t=0}^{T}\delta^t}{\delta^n}$ in the case that the consumer does not exhibit present bias.

Example 5. Consider a case where $\delta = 0.95$, $T = 5$, $n = 3$ and $u(x) = 1$. In this setting, the optimal fee is $F = 3.396 + \frac{1}{\beta}$ which decreases in present bias β. Intuitively, the consumer is willing

[9] Note that $p_T = 1$ in the last period, implying that if a consumer forgets to pay on time, he will pay at least in the last period.

to pay less at the payment period n as his present bias parameter approaches 1 (which entails he does not exhibit present bias). That is, when $\beta < 1$, he assigns a smaller utility weight to future payoffs than when he does not experience present bias ($\beta = 1$), ultimately helping the seller set a higher fee when the consumer is not subject to present bias than otherwise.

4.2.2 Case 2: Bounded rationality

Since in this case the seller is uncertain about whether the consumer understood the details of the contract, fee F depends on f and p. Proposition 4 and the subsequent discussion analyses the optimal fee when the seller chooses a stream of penalties, K_i's, that induce late payment only among misinterpreters; whereas Proposition 5 explains the optimal fee when the seller induces late payment from all types of consumers.

Proposition 4. *In the case of bounded rationality, if the seller chooses a stream of K_t's to induce late payment only amongst misinterpreters (Proposition 3 i), the seller selects a fee F_A that induces every type of buyer to participate in the contract if and only if p satisfies $p \geq \tilde{p}$. Otherwise, the seller chooses a fee F_B, which only induces misterpreters to accept the contract, where $F_A \equiv \frac{u(x)\left(1+\beta\sum_{t=1}^{T}\delta^t\right)}{\beta\delta^n}$, $F_B \equiv \frac{u(x)\cdot\left(1+\beta\sum_{t=1}^{T}\delta^t\right)}{(1-\alpha)\beta\delta^n}$, and*

$$\tilde{p} \equiv \alpha - \frac{p_n(1-\alpha)}{\left(\sum_{i=1}^{T-n}\left(\prod_{j=n}^{i+n-1}(1-p_j)\right)p_{i+n}K_i\right)}.$$

In this case, the seller faces a tradeoff. She can either set fee F so high that only uninformed consumers participate (denoted as F_B), or set a relatively low fee that induces all types of consumers to participate (denoted as F_A). The benefit of everyone participating lies in larger sales, which increase profit. However, selling to only uninformed types allows for a higher fee and guarantees a penalty from late payments. In particular, if p is high enough (there are enough people who correctly understand the contract and, thus, will not participate for a high fee), the seller accommodates every type of buyer to maximize profits. However, if p is low, the seller can benefit from leaving informed consumers out of the market and obtaining a larger margin from misterpreters alone. The following example illustrates the conditions that determine the seller's choice for F.

Example 6. Similarly as in previous examples, assume $\delta = 0.95$ and $\beta = 0.92$, which yields fees $F_A = 4.905$ and $F_B = 6.13$. In addition, consider $\alpha = 0.2$, $T = n + 2$, $p = 0.4$, $p_{n+1} = 0.8$, $p_T = 1$ which yields $K_1 = 1.43$ and $K_{T-n} = 1.5$. The fee that induces all buyers to participate is $F_A = 4.9$, while that inducing uninformed buyers alone is $F_B = 6.13$. In this context, the seller chooses to attract all types of buyers since the expected profit is greater, i.e., $5.62 > 5.31$.

Proposition 5. *If the optimal stream of penalties K_t is set to induce everyone to pay in the final period (Proposition 3 ii), then the seller chooses the fee F_A that induces full participation if* *and only if p satisfies $p \geq \widehat{p} \equiv \alpha$. Otherwise, the seller chooses a fee F_B, which only induces misterpreters to accept the contract.*

The above condition shows that, even if every type of buyer is induced to pay in the last period, the seller has a decision to make regarding whether she should choose fee F_A, which attracts all buyers to the contract, or F_B, which only attracts misinterpreters to sign the contract. Figures 2a-2b combine the conditions identified on Propositions 3-5. In particular, they depicts cutoff $\overline{\gamma}$ on the vertical axis (from Proposition 3), and probability cutoffs $\tilde{p}$ and $\widehat{p}$ on the horizontal axis (from Proposition 3 and 4, respectively), where $\tilde{p} < \widehat{p}$. According to Proposition 3, when penalties are largely underestimated by misterpreters, $\gamma \geq \overline{\gamma}$ as depicted in Figure 2a, the seller sets penalties to induce only this group of customers to pay late. In addition, if their proportion is relatively high (i.e., low p, in the shaded area of Figure 2a), the seller sets fees so only misterpreters sign the contract, giving up well-informed individuals; whereas when their proportion is low (high values of p in the unshaded region), the seller sets fees to induce full participation. Similar results apply when penalties are not significantly underestimated, i.e., $\gamma < \overline{\gamma}$ as illustrated in Figure 2b, where the seller does not find it profitable to focus on misterpreters alone since they cannot be heavily exploited. In this case, she chooses high (low) fees if the proportion of well-informed individuals is low (high), thus inducing the participation of misinterpreters alone (both types of buyers, respectively). Our findings help examine the effect of regulations that ban penalties. In particular, Proposition 3 (case ii) can be sustained under all parameter values, implying that the seller uses fee F as his only instrument to attract one of both types of customers. In particular, he sets fee F_A when misinterpreters are relatively frequent, $p \geq \widehat{p}$, thus attracting this type of customers alone; or fee F_B otherwise, which attracts all types of customers. Therefore, if $p \geq \widehat{p}$ well-informed customers are not served, and a stringent regulation on penalties does not necessarily solve the market imperfection described above.

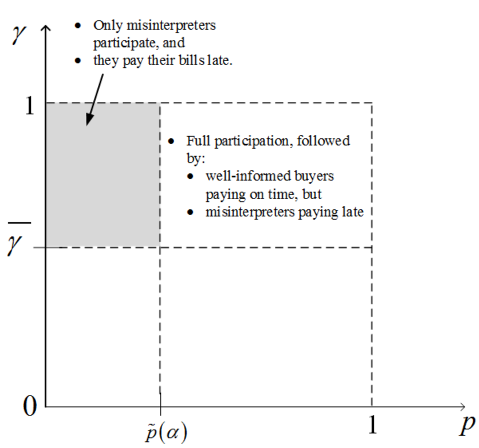

Fig. 2a. Only misinterpreters pay late.

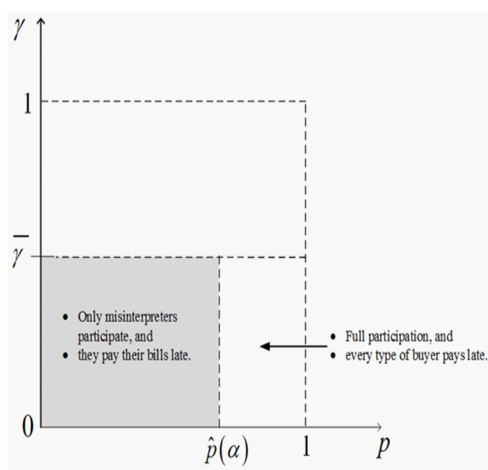

Fig. 2b. All type of buyers pay late.

Example 7. Assume the same parameter values as in Example 6. The fee that induces all buyers to participate is $F_A = 4.905$, while the one attracting uninformed buyers alone is $F_B = 6.13$. In this context, the seller chooses to attract every type of buyer since the expected payoff is greater, i.e., $5.9 > 4.4$.

Corollary 2. *As the degree of misinterpretation, α, increases, the initial probability p needed to accommodate all consumers increases, i.e., $\frac{\partial \tilde{p}}{\partial \alpha} > 0$ and $\frac{\partial \check{p}}{\partial \alpha} > 0$ where $\alpha \in [0, 1]$.*

The above corollary implies that, regardless of the decision on K_i (whether this stream induces late payment from misinterpreters alone or from every type of buyer), as the degree of misinterpretation increases, the seller can exploit this type of customers more heavily (charging a higher fee F_B), and thus she will be more tempted to sell to misinterpreters alone.

Example 8 (Summary). Assume the same parameter values as in Examples 6 and 7, except for $p_0 = 0.8 > p$. It is straightforward to see that we obtain the same result: a higher value of p_0 makes it more likely that the seller will choose to set a fee F that attracts every type of buyer. In case all buyers are included, the stream of $\{K_i\}_{i=1}^{T-n}$ depends on p_n, and in our example $p_n = p$, the stream of penalties too remains the same. The table below summarizes the previous examples, also including the net utilities for buyer and profits for the seller.

Case	*Optimal F*	*Optimal K_1*	*Optimal $K_2 = K_{T-n}$*	*Payment period*	*Expected profits*
1	4.9	1.144	1.2	$t = T = 5$	5.11
2	4.9	1.43	1.5	0.4 at $t = 3$; 0.48 at $t = 4$; 0.12 at $t = 5$	5.62

Case	*Type of consumer*	*Utility at each period*	*Weighted sum of utilities*	*Expected profits + utility*
1	Informed	-0.88 at $t = T = 5$	-0.88	4.23
2	Both	0.1 at $t = n = 3$; -2.007 at $t = 4$; -2.35 at $t = T = 5$	-1.205	4.415

Summary of Examples

The "Utility at each period" column gives a particularly interesting result. First, notice that in Case 1, the number is negative, but close to zero, since the seller decides to make the buyers pay as late as possible to maximize profits. The buyers in this case are all perfectly informed and the seller is only able to exploit the buyers' present bias and discount factor. In Case 2, those buyers paying on time obtain a small positive utility. However, those paying even slightly late get a very large negative utility. This is because the seller decides to choose a stream of $\{K_i\}_{i=1}^{T-n}$ that are so high that the informed buyers pay on time. However, the uninformed ones make an inefficient decision and end up paying large penalties, entailing highly negative utilities.

In Case 2 the seller decides to include everyone while offering a contract. Higher values of p_0 mean that the seller is more likely to include everyone since she cannot profit enough by including only misinterpreters. When it comes to setting penalties, a lower value of p_n makes it more likely that the seller will induce only misinterpreters to pay late (as explained after Proposition 3). If p_n is high, all buyers (including the informed type) will be induced to pay late. Intuitively, correctly interpreting the contract at $t = n$ is beneficial if many other buyers misinterpret it at the same period. In other words, information is more valuable to people if only a few of them have it.

5 Conclusions

The paper offers a set of results. In the first part of the paper, we find that the range of penalties for which the consumer is likely to pay late (and regret it ex post) increases as we introduce present bias, bounded rationality and memory loss. This explains why people often pay bills late, or sign contracts that are not optimal for them. While the consumer is provided with all the information he needs, the seller anticipates that a proportion of buyers are incapable of processing all that information, thus taking advantage of this situation; as examined in the second part of the paper. Specifically, if the seller finds that there is a large proportion of misterpreters (buyers who do not fully understand contracts), she can set prices in such to significantly exploit this segment of customers. Seller can extract further rents if rationaly bounded buyers suffer from memory loss, i.e., they forget the details of the contract over time.

When the seller offers a unique contract, since he does not have observe a buyer's type, some well informed buyers may end up rejecting the contract, giving rise to a lemons problem. In other words, the seller may find profitable that the market is incomplete. This equilibrium result, may not be a problem is some industries. However, in markets like health and education, governments may want to intervene by either minimizing the extent to which firms can impose penalties (or choose prices to leave well informed consumers out). The government may also want to focus on ensuring the public is well informed so that they do not succumb to such sub-optimal behavior. Finally, our paper can be extended to consider sellers that offer a menu of contracts inducing self selection from each type of buyer, and how information rents depend on the degree of misterpretated fees and penalties. In addition, this paper identify equilibrium conditions under which behavior leads to sub-optimal equilibria. Empirical research in this field would be interesting in markets such as electricity and cable.

6 Appendix

6.1 Appendix A. Bounded rationality with decreasing p

In this appendix, we modify Case 2 allowing for the probability of misinterpretation $1-p$ to increase over time, that is, the probability of correctly interpreting fees, p, decreases over time, which can be rationalized due to memory loss or other behavioral biases.

Consumer. We next analyze under which conditions preference reversal can emerge in this setting.

Proposition A1. *Under bounded rationality with decreasing p, the consumer signs the contract at $t=0$ expecting to pay at $t=n$ if $K_i \geq \frac{p_0 \cdot F+(1-p_0)\left(1-\delta^i \cdot k_i\right) \cdot f}{\delta^i \cdot p_0 \cdot F}$ for all i periods, but does not pay when the bill is due if there exists a period i for which $K_i < \frac{p_n \cdot F+(1-p_n)\left(1-\beta \cdot \delta^i \cdot k_i\right) \cdot f}{\beta \cdot \delta^i \cdot p_n \cdot F}$.*

Proof. This proof is symmetric to that of Proposition 1. The only difference is that in the proof, p has a time subscript (i.e., $p_0 \neq p_n$). (Q.E.D.)

As the next corollary demonstrates, the introduction of a decreasing probability p further expands the set of penalties K_i for which preference reversal arises.

Corollary A1. *The range of parameters supporting preference reversal under bounded rationality with decreasing probability p is greater than under a constant probability p.*

Proof. The difference

$$\frac{p_n \cdot F+(1-p_n)\left(1-\beta \cdot \delta^i \cdot k_i\right) \cdot f}{\beta \cdot \delta^i \cdot p_n \cdot F}-\frac{p_0 \cdot F+(1-p_0)\left(1-\delta^i \cdot k_i\right) \cdot f}{\delta^i \cdot p_0 \cdot F}>0$$

If we consider $p_0 = p$ and $p_0 > p_n$, we can see that the first term is greater than the second and, hence, the expression is greater than zero. (Q.E.D.)

Seller. From the producer's point of view, the decision making about $\{K_i\}_{i=1}^{T-n}$ is unaffected by a decreasing probability p, since the seller is only concerned with the probability of the consumer remembering at time $t=n$ and beyond. However, the difference with Case 2 lies in the fact that now p_n satisfies $p_n < p_0$. Regarding fees, the conditions for optimal fees are identical to those identified in Case 2. However, Case 2 allows for a higher value at p_0, making it more likely that the seller sets a fee that is low enough for every type of buyer to sign the contract.

6.2 *Proof of Lemma 1*

The consumer signs contract expecting to pay on time if, for all period i,

$$u(x)+\sum_{t=1, t \neq n}^{T} \beta \cdot \delta^t \cdot u(x)+\delta^n \cdot[u(x)-F] \geq u(x)+\sum_{t=1, t \neq n+i}^{T} \beta \cdot \delta^t \cdot u(x)+\delta^{n+i} \cdot[u(x)-K_i \cdot F]$$

which after solving yields $K_i \geq \frac{1}{\delta^i}$. The consumer does not pay at $t = n$ after having agreed to do so at $t = 0$ if there is a period $i > n$ such that

$$[u(x) - F] + \sum_{t=1}^{T-n} \beta \cdot \delta^t \cdot u(x) < u(x) + \sum_{t=1, t \neq i}^{T-n} \beta \cdot \delta^n \cdot u(x) + \beta \cdot \delta^i \cdot [u(x) - K_i \cdot F]$$

after solving for K_i, we obtain $K_i < \frac{1}{\beta \cdot \delta^i}$. When the consumer does not exhibit present bias, $\beta = 1$, the above conditions on K_i become $K_i \geq \frac{1}{\delta^i}$ and $K_i < \frac{1}{\delta^i}$, thus being incompatible with each other.

6.3 *Proof of Proposition 1*

The consumer signs the contract at $t = 0$ expecting to pay on time if

$$\begin{aligned} & u(x) + \sum_{t=1,\, t \neq n}^{T} \beta \cdot \delta^t \cdot u(x) + \beta \cdot \delta^n \cdot [u(x) - p_0 \cdot F - (1 - p_0) \cdot f] \\ \geq \quad & u(x) + \sum_{t=1,\, t \neq n+i}^{T} \beta \cdot \delta^t \cdot u(x) + \beta \cdot \delta^{n+i} [u(x) - p_0 \cdot K_i \cdot F - (1 - p_0) \cdot k_i \cdot f] \end{aligned}$$

which solving for K_i yields

$$K_i \geq \frac{p_0 \cdot F + (1 - p_0) \left(1 - \delta^i \cdot k_i\right) \cdot f}{\delta^i \cdot p_0 \cdot F}$$

The consumer does not pay at $t = n$ after having agreed to do so at $t = 0$ if there is a late period i such that

$$\begin{aligned} & u(x) + \sum_{t=n+1}^{T} \beta \cdot \delta^{t-n} \cdot u(x) + \beta \cdot \delta^{n-n} \cdot [u(x) - p_n \cdot F - (1 - p_n) \cdot f] \\ < \quad & u(x) + \sum_{t=n+1,\, t \neq n+1}^{T} \beta \cdot \delta^{t-n} \cdot u(x) + \beta \cdot \delta^{n+i-n} [u(x) - p_n \cdot K_i \cdot F - (1 - p_n) \cdot k_i \cdot f] \end{aligned}$$

or, after solving for K_i,

$$K_i < \frac{p_n \cdot F + (1 - p_n) \left(1 - \beta \cdot \delta^i \cdot k_i\right) \cdot f}{\beta \cdot \delta^i \cdot p_n \cdot F}$$

6.4 *Proof of Corollary 1*

The difference

$$\begin{aligned} & \frac{p \cdot F + (1 - p) \left(1 - \beta \cdot \delta^i \cdot k_i\right) \cdot f}{\beta \cdot \delta^i \cdot p \cdot F} - \frac{p \cdot F + (1 - p) \left(1 - \delta^i \cdot k_i\right) \cdot f}{\delta^i \cdot p \cdot F} \\ = \quad & \frac{p \cdot F \cdot (1 - \beta) + f \cdot (1 - \beta) \cdot (1 - p)}{\beta \cdot \delta^i \cdot p \cdot F} > 0 \end{aligned}$$

since all the terms in the final expression are greater than 0. Hence, we obtain the result of the corollary.

6.5 *Proof of Proposition 2*

From Proposition 1, we know that under present bias with full rationality no consumer will pay on time if $K_i < \frac{1}{\beta \cdot \delta^i}$. Since the sellers would like them to pay as late as possible to collect maximum penalty, he sets the fee at a fee marginally less than $\frac{1}{\beta \cdot \delta^i}$. The consumers then pay in the final period. Hence, $K_i = \frac{1}{\beta \cdot \delta^i} - \epsilon$ where $t > n$ and $\epsilon \to 0$.

6.6 *Proof of Proposition 3*

In case of bounded rationality with memory loss, the seller has two options for the stream of K_i. One stream would induce every type of consumer to pay late, while the other stream would induce only uninformed types to pay late. If she chooses to induce late payment from only uninformed types, she can charge a higher penalty. Here, $t>n$ for every period t.

The penalty she can charge an uninformed buyer is $\frac{1}{(1-\gamma)} \cdot \frac{1}{\beta \cdot \delta^{t-n}}$. Thus, if she chooses to induce late payment only in the uninformed type, her expected profit will be

$$F \cdot p_n + \frac{F}{(1-\gamma)} \cdot \left(\sum_{i=1}^{T-n} \frac{1}{\beta \cdot \delta^i} \cdot p_{i+n} \cdot \prod_{j=n}^{i+n-1} (1-p_j) \right)$$

This is because, under the condition of a higher penalty, the ones who have the correct interpretation (represented by the probability in that period) will pay the fee and not delay any further. If, in contrast, the seller induces every type of consumer to pay in the final period, her expected profit will be $\frac{F}{\beta \cdot \delta^{T-n}}$. Thus, the seller chooses the optimal penalty by comparing these two profits, that is, the seller sets a high fee that attracts only correctly informed buyers if

$$F \cdot p_n + \frac{F}{(1-\gamma)} \cdot \left(\sum_{i=1}^{T-n} \frac{1}{\beta \cdot \delta^i} \cdot p_{i+n} \cdot \prod_{j=n}^{i+n-1} (1-p_j) \right) \geq \frac{F}{\beta \cdot \delta^{T-n}}$$

which, rearranging, yields

$$p_n + \frac{1}{(1-\gamma)} \left(\sum_{i=1}^{T-n} \frac{1}{\beta \cdot \delta^i} \cdot p_{i+n} \cdot \prod_{j=n}^{i+n-1} (1-p_j) \right) \geq \frac{1}{\beta \cdot \delta^{T-n}},$$

after solving for γ, we obtain

$$\gamma \geq 1 - \frac{\beta \cdot \delta^{T-n}}{1 - p_n \beta \cdot \delta^{T-n}} \left(\sum_{i=1}^{T-n} \frac{1}{\beta \cdot \delta^i} \cdot p_{i+n} \cdot \prod_{j=n}^{i+n-1} (1-p_j) \right) \equiv \overline{\gamma}$$

Hence, the seller sets penalty $K_i = \left(\frac{1}{\beta \cdot \delta^i} \cdot \frac{1}{(1-\gamma)} - \epsilon\right)$ for every late period i. Otherwise she sets it as $K_i = \frac{1}{\beta \cdot \delta^i} - \epsilon$ where $\epsilon \to 0$.

6.7 *Proof of Lemma 2*

The fee a consumer is willing to pay in the case of full rationality with present bias is

$$u(x) + \beta \cdot \delta \cdot u(x) + \cdots + \beta \cdot \delta^T \cdot u(x) \geq \beta \cdot \delta^n \cdot F$$

which can be compactly expressed as

$$u(x) + u(x) \cdot \beta \cdot \sum_{t=1}^{T} \delta^t \geq \beta \cdot \delta^n \cdot F$$

and, solving for F yields $F \geq \frac{u(x) \cdot (1 + \beta \cdot \sum_{t=1}^{T} \delta^t)}{\beta \cdot \delta^n}$. Therefore, the seller charges $F = \frac{u(x) \cdot (1 + \beta \cdot \sum_{t=1}^{T} \delta^t)}{\beta \cdot \delta^n}$.

6.8 *Proof of Proposition 4*

If the seller chooses a stream of penalties to induce late payment among only misinterpreters, he must then optimize the following function with respect to F to get the optimal fee.

$$\max \left\{ \sum_{i=0}^{T-n} \left(\prod_{j=n-1}^{i+n-1} (1-p_j) \right) \cdot p_{i+n} \cdot K_i \cdot F_A, (1-p) \cdot \left(\sum_{i=1}^{T-n} \left(\prod_{j=n}^{i+n-1} (1-p_j) \right) \cdot p_{i+n} \cdot K_i \cdot F_B \right) \right\}$$

Here, he chooses F_A if it is optimal for him to include every type of buyer and F_B if he wants only misinterpreters to sign the contract. Mathematically, $F_A = \frac{1}{1-\alpha} \cdot F_B$.

If the seller has chosen to induce late payment only among misinterpreters, she chooses a contract that will induce everyone to participate if

$$\sum_{t=0}^{T-n} \left(\prod_{j=n-1}^{i+n-1} (1-p_j) \right) \cdot p_{i+n} \cdot K_i \cdot F_A \geq (1-p) \cdot \left(\sum_{i=1}^{T-n} \left(\prod_{j=n}^{t-1} (1-p_j) \right) \cdot p_{i+n} \cdot K_i \cdot F_B \right)$$

$$\Rightarrow \sum_{i=0}^{T-n} \left(\prod_{j=n-1}^{i+n-1} (1-p_j) \right) \cdot p_{i+n} \cdot K_i \cdot F_A \geq \frac{1-p}{1-\alpha} \cdot \left(\sum_{i=1}^{T-n} \left(\prod_{j=n}^{t-1} (1-p_j) \right) \cdot p_{i+n} \cdot K_i \cdot F_A \right)$$

which, solving for p, yields

$$p \geq \alpha - \frac{p_n \cdot (1-\alpha)}{\left(\sum_{i=1}^{T-n} \left(\prod_{j=n}^{i+n-1} (1-p_j) \right) \cdot p_{i+n} \cdot K_i \right)} \equiv \tilde{p}$$

6.9 *Proof of Proposition 5*

If the seller chooses an optimal stream of penalties that induces everyone to pay in the last period, her choice to fee F solves

$$\max\left\{\frac{1}{\beta\cdot\delta^T}\cdot F_A,(1-p)\,\frac{1}{\beta\cdot\delta^T}\cdot F_B\right\}$$

Since $F_B=\frac{F_A}{1-\alpha}$, we have

$$\max\left\{\frac{1}{\beta\cdot\delta^T}\cdot F_A,\frac{(1-p)}{(1-\alpha)}\frac{1}{\beta\cdot\delta^T}\cdot F_A\right\}$$

Cancelling out common terms, we obtain, $\max\left\{1,\frac{(1-p)}{(1-\alpha)}\right\}$. Therefore, the seller will choose a fee that induces every type of buyer to participate if $1>\frac{(1-p)}{(1-\alpha)}$, i.e., if $p>\gamma$.

6.10 *Proof of Corollary 2*

Differentiating $\tilde{p}$ with respect to α yields

$$1+\frac{p_n}{\left(\sum_{i=1}^{T-n}\left(\prod_{j=n}^{t-1}(1-p_j)\right)\cdot p_{i+n}\cdot K_i\right)}>0$$

For $\frac{\partial\widehat{p}}{\partial\alpha}$, we can see that, since $\widehat{p}=\alpha$, as α increases, $\widehat{p}$ also increases.

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