Explaining Hypothetical Bias Variations Using Income Elasticity of Demand*

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Abstract

Experimental methods have been widely used to elicit consumer preferences. However, the estimates are challenged because the existing literature indicates that individuals overstate their economic valuation in hypothetical settings, thus giving rise to the so-called "hypothetical bias." Although many studies seek to experimentally test which factors emphasize or ameliorate the hypothetical bias, no studies analyze its theoretical foundations. In this paper, we provide a theoretical model to incorporate the underlying causes of hypothetical bias. Our results also help to explain experimental regularities, such as that hypothetical bias increases in a commodity's income elasticity of demand.

Keywords: Experiments; Hypothetical Bias; Income Elasticity; Willingness to Pay.
JEL classification: C9, D01, D10

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1 Introduction

Field and laboratory experiments in the last decades have sought to elicit consumers' willingness to pay for various goods, both private and public in nature, including environmental services and other nonmarket goods. However, many studies have shown the presence of what is often referred to as *hypothetical bias* (HB): subjects reporting a higher willingness-to-pay in the "hypothetical" treatment of the experiment (when they do not have to pay for the good) than in the "real" treatment (in which subjects may actually pay for the good). For instance, in a recent survey of the literature, Harrison and Rutström (2008) find a positive bias in 34 of 39 studies, implying that the median willingness-to-pay under the hypothetical setting is 67% higher than under the real setting. Similarly, List and Gallet (2001) examine a large set of experimental articles, concluding that, while the experimental methodology and the type of goods considered largely vary across studies, a positive HB still arises in 27 of 29 articles. Given the broad range of studies finding a positive bias, several scholars, such as Unnevehr et al. (2010), emphasize that there are still open questions about the HB and how to explain this gap. Similarly, Carson (2012) points out that, while contingent valuation is preferable than other methods in consumer choice studies, it can only remain a valuable tool if we discern why HB emerges, and why it varies so significantly across different types of goods. Finally, many researchers note that, due to the lack of a theory about the causes of HB, our ability to identify which factors are responsible for the bias is rather limited; see Carson et al. (1996), Harrison (1996), and Murphy et al. (2005). A better understanding of the HB is, hence, necessary and timely; Loomis (2011).

In this study, we present an individual decision-making model which helps explain different experimental regularities: First, our theoretical results show that the HB is positive under relatively general conditions (and it only becomes zero under very special conditions); and, second, the HB is larger for those goods with a high income elasticity (luxury goods). Hence, our study helps rationalize the experimental observation of HB increasing when individuals are asked to reveal their willingness to pay for goods that can be regarded as luxuries, while it decreases when the good is a necessity. We demonstrate that the HB can be mathematically expressed as a function of the income elasticity of demand, along with other parameters.

The following subsection elaborates on the experimental studies that, mostly during the last two decades, identified the presence of HB in many goods, ranking the reported HBs and the good used in each experiment. We then present a tractable model that explains the connection between HB and income elasticity of demand. Section 3 applies our model to settings in which subjects' utility function is quasilinear, Stone-Geary (thus considering the Cobb-Douglas type as a special case), and the generalized Constant Elasticity of Substitution function (which embodies the standard CES as a special case). In addition, we analyze an example of contributions to public goods. These examples offer testable predictions that future researchers can use to investigate to which extent our model predicts subjects' behavior in controlled experiments, and in which cases it differs. Finally, section 4 concludes.

1.1 Related literature

Real versus Hypothetical payments. Harrison (1996) argues that based solely on what can be inferred from the data, the willingness-to-pay in hypothetical treatments could accurately describe subjects' valuation of the good they face in the experiment. In contrast, the willingness-to-pay reported in the real treatment, as in Davis and Holt (1993) and Shogren et al. (2001), could misstate the participants' actual preferences for the object. To overcome this problem, some experiments include *both* hypothetical and real payment scenarios. For example, Lusk and Schroeder (2004) compare hypothetical and actual payments in a study of consumer demand for beef rib-eye steaks. They conclude that average willingness-to-pay for steaks in the hypothetical setting was about 1.2 times that in the non-hypothetical setting. As we describe below, the presence of a positive HB in experiments with several types of goods, and its significant variation from one good to another, has remained a puzzle among experimentalists in the last decades.[1]

In an attempt to remove HB, some studies include a previous stage in which subjects are allowed to talk about the characteristics of the object before the beginning of the experiment; see Cummings and Taylor (1999). While they demonstrate that HB can be partially reduced, it is still present for most subjects. Similarly, List (2001) examined the presence of HB in a well-functioning marketplace auctioning sports cards. In this context, he found that, while subject communication mitigates HB for certain consumers, HB is essentially unaffected for bidders with experience in the market. Blumenschein et al. (2008) found a similar result, showing that communication does not reduce HB in an experiment on a diabetes management program.

Luxury goods. Each of the above papers measured HB for a single good. However, List and Gallet (2001) and Murphy et al. (2005) estimate the factors that influence HB in a set of 29 previous studies. While existing studies do not include income elasticity as an explanatory variable of HB variations, Table 1 reports the HB in a sample of studies, indicating that HB is larger for luxury goods (e.g., fine chocolates sold for $15 a piece) than for necessities (bread, sold at $1.1). Our model helps explain such a variation in the observed HB between different types of goods.[2]

[1] Another issue in these experiments is that subjects' willingness-to-pay could be influenced by outside options; see Harrison (1996). In order to address this issue, experimenters reminded subjects that the products they face in the lab are the same as those found in grocery stores, and designed experiments that mimic the environment that consumers face in real life.

[2] Note that Table 1 only reports anecdotal evidence, rather than a statistical analysis, between HB and income elasticity. Other studies reporting a positive HB are, for instance, Bohm (1972) analyzing TV programs (which finds an HB of 16%), and Frykblom (1997) which presented subjects a copy of the book "The Environment," a volume of the Swedish National Atlas, and found an average HB of 60%.

Study	*Commodity*	*Hypothetical Bias*
Ginon et al., 2011	Bread	10%
Johannesson et al, 1998	Chocolate	19%
Lusk and Schroeder, 2004	Beef	20%
Kealy et al., 1990	Candy bar	30%
Bishop et al., 1983	Goose hunting permits	60%
Cummings, Harrison and Rutström, 1995	Electric juice-maker	163%
Cummings, Harrison and Rutström, 1995	Calculator	163%
Neill et al., 1994	Paintings by Navajo artist	290%
Johannesson et al, 1998	Fine chocolates	701%
Cummings, Harrison and Rutström, 1995	Fine chocolates	873%
Neill et al., 1994	16th century map	2400%

Table 1. A sample of studies on Hypothetical Bias for private goods.

An alternative source of HB could be consumers' lack of experience with the good, which might explain the high HB coefficients in the certain experiments, such as those using rare paintings in Neill et al. (1994). However, this explanation does not justify the HB subjects exhibit when facing relatively well-known goods, such as bread; and more importantly, why the HB experiences a six-fold increase when subjects face a standard calculator or fine chocolates. Since most of the goods in this literature are well-known (see goods in Table 1), the underlying reason supporting an increase in HB cannot solely originate from uncertainty, but instead connect with income elasticity as we describe in the next section.[3]

2 Model

A common elicitation method widely used in the literature is Becker, Degroot, and Maschak method (BDM). The sequential structure of this method is illustrated in Figure 1, and follows the next steps:

1. Each participant $i \in N$ submits a bid $p \in [0,1]$, where the maximum bid allowed in the experiment (e.g., $\overline{p}$) is normalized to one, and $N \geq 2$ denotes the number of participants;
2. One of the participants is randomly chosen with probability $\frac{1}{N}$ (often with a spinning wheel or other randomization mechanism that assigns the same probability to all participants);
3. A price $p' \in [0,1]$ is randomly selected. Similarly to the selection of one participant in step (2), price p' is uniformly distributed (e.g., the realization of p' is drawn using a spinning wheel in many experiments);

[3] Our model would, hence, predict that if subjects are familiarized with both a necessity and luxury good (such as bread and chocolates, as those used in the experiments reported in Table 1), HB would still exist for luxury goods, and be larger than for necessity goods.

4. After price p' has been determined, the subject selected in step (2) pays p' if and only if the bid he submitted in step (1) higher than p', i.e., if $p > p'$.

Since p' is uniformly distributed, the probability described in step (4) is $\Pr(p > p') = F(p) = p$, where $F(\cdot)$ is the cumulative distribution function of p'. (If, instead, the participant's bid is lower than p' the object is unassigned, which happens with probability $1 - F(p) = 1 - p$.) Therefore, the joint probability that a participant submitting bid p ends up with the commodity is $\frac{1}{N} \times p$. Recall that in this case, he pays a price p' rather his initial bid p, where $p > p'$. Intuitively, every subject i anticipates that by submitting a higher bid p he increases the probability of exceeding the randomly selected price p' and thus receiving the object. Unlike in auction settings, however, the bids submitted by other subjects do not affect the probability of participant i being selected as the winner of the object (step 2) nor the price that he pays in step 4 (which is the randomly selected price p' as long as his bid satisfies $p > p'$).

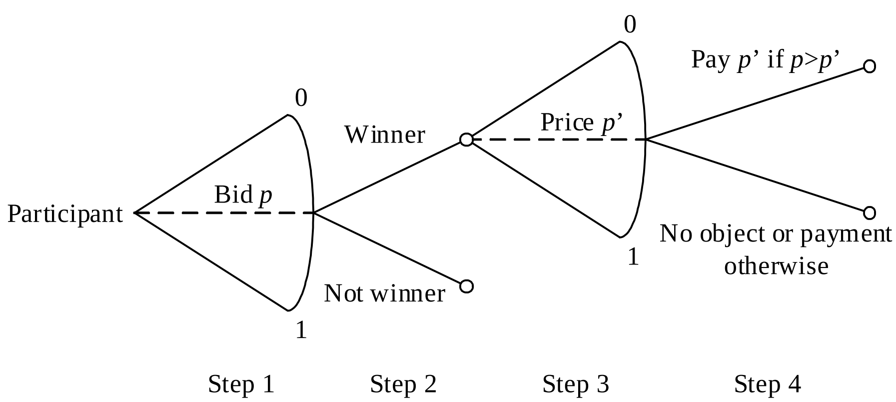

Figure 1. BDM mechanism.

Consider a consumer with utility function $u(x, y)$, which is continuous and non-decreasing, and is strictly quasiconcave in both x and y. For simplicity, x represents the good that the experiment presents to the subject, whereas y denotes all other goods (composite commodity y) whose price is normalized to \$1. Let $w > 0$ represent the consumer's income. We allow individuals to be uncertain as to whether they will have to pay for the good in the real treatment, R. In particular, following BDM, we consider that, upon revealing a willingness-to-pay b for the good, subjects assign a probability $q_R \equiv \frac{1}{N} \times p$ to receiving the good, while $\alpha_R \equiv \frac{p'}{p}$ represents the share of p that the individual is required to pay at the end of the experiment.[4] Hence, q_R and α_R are a objective probability and share, respectively. Finally, since p' follows a uniform distribution, the expected share that the individual pays becomes $E[\alpha_R] = \int_0^1 \frac{p'}{p} dp' = \frac{1}{2p}$. Similarly, the participant's expected value of the joint probability $q_R \alpha_R$ is $E[q_R \alpha_R] = \frac{p}{N} \frac{1}{2p} = \frac{1}{2N}$.

[4]Note both q_R and α_R are well-defined probabilities since $p \in [0, 1]$ and thus $p < N$, and given that the object is only assigned to the individual if $p > p'$.

In the hypothetical treatment, the subject submits a bid b, and he assigns a subjective probability q_H of receiving the object, and pays a share α_H of his bid. In this setting, we assume that probability q_H satisfies $0 \leq q_H \leq q_R$ and that the share α_H satisfies $0 \leq \alpha_H \leq \alpha_R$. For completeness, our model not only allows for the case in which $q_H = 0$, when participants believe that they will not receive the object with certainty, as in stated preference contingent valuation studies; but also the case in which participants believe that their choices during the hypothetical treatment can lead them to receive the good, i.e., $q_H > 0$ (but smaller than q_R), as suggested by the literature on consequentiality (see Poe and Vossler, 2011, and Carson et al. 2014).[5,6]

In this context, the individual's utility maximization problem in each treatment becomes

Real treatment	**Hypothetical treatment**
$\max_{x,y\geq 0} \; u(x,y)$	$\max_{x,y\geq 0} \; u(x,y)$
subject to $\frac{1}{2N}px + y \leq w$	subject to $q_H\alpha_H px + y \leq w$

Rearranging the budget constraints in each problem, yields $x + \frac{2N}{p}y \leq \frac{2N}{p}w \equiv w_R$ for the real and $x + \frac{1}{q_H\alpha_H p}y \leq \frac{1}{q_H\alpha_H p}w \equiv w_H$ for the hypothetical treatment, where $w_R \leq w_H$ holds as long as $2N \leq \frac{1}{q_H\alpha_H}$, i.e., when the probability q_H of receiving the object and/or the share α_H that the individual pays are sufficiently small relative to the number of participants. For instance, if either $q_H = 0$ or $\alpha_H = 0$ (or both), this inequality holds for all number of bidders N, and thus $w_R \leq w_H$ is satisfied. Alternatively, if $\alpha_H = 1/2$ and $N = 10$, the inequality holds as long as the probability of receiving the object, q_H, is smaller than $1/10$.[7] Hence, w_R is determined by observables (N, p and w), while w_H is determined by both observables p and w and unobservables (belief q_H and share α_H). At the end of the hypothetical treatment, participants could be asked to report the probability they sustained on receiving the good, q_H, and the share of their submitted price to be paid, α_H, helping the experimenter recover w_H comparing it against w_R.

Generally, when $w_R \neq w_H$, the demand for the good when the subject plays in the real treatment differs from that in the hypothetical treatment, i.e., his Walrasian demand satisfies $x(p, w_R) \neq x(p, w_H)$. Since the hypothetical bias (HB) measures the difference in his willingness to pay for the good, we first obtain the inverse of the above Walrasian demands, i.e., $x^{-1}(p, w_H) \equiv p(x, w_H)$ and $x^{-1}(p, w_R) \equiv p(x, w_R)$, and then define the HB as

$$HB \equiv p(x, w_H) - p(x, w_R)$$

[5] For generality, our model also allows for the possibility that, upon receiving the good (i.e., if $q_H > 0$), the participant pays nothing for the good, $\alpha_H = 0$, or a positive share of his initial bid, $\alpha_H > 0$.

[6] Besides experiments in the context of willingness to pay, other studies also find that individuals do not make random decisions in the absence of economic incentives. For example, higher incentives only have a marginal effect on improving performance in labor markets; for a review see Camerer and Hogarth (1999).

[7] Note that, even if the subject fully pays for the good when he is chosen, i.e., $\alpha_H = 1$, the above condition on wealth levels holds as long as $2N \leq \frac{1}{q_H}$, or $q_H \leq \frac{1}{2N}$, which becomes less demanding as the number of participants grows.

or in continuous form $HB \equiv \frac{\partial p(x,w)}{\partial w}$. While the hypothetical bias can arise from different reasons, it could emerge from consumers' uncertainty about the utility level that the good will provide. However, most of the products used in experiments are relatively homogeneous and well-known by participants, e.g., bread and a standard calculator, thus reducing the potential of uncertainty to explain the observed HB. More importantly, the experimental evidence summarized in subsection 1.1 suggests that the HB is larger for luxury goods, which indicates that the primitive reason giving rise to the HB must be connected with income elasticity, as we next describe.[8]

Let us first define income elasticity in terms of the inverse demand function $p(x, w)$, as follows $\varepsilon_{w,p} = \frac{\partial p(x,w)}{\partial w} \frac{w}{p(x,w)}$. Using this expression we can now represent the HB in the more compact coefficient described in Proposition 1. (All proofs are relegated to the appendix).

Proposition 1. *The hypothetical bias (HB) can be expressed as* $HB(\varepsilon) = \varepsilon_{w,p} \frac{\theta_x}{x(p,w)}$*, where* $\theta_x \equiv \frac{p(x,w) \cdot x}{w}$ *represents the budget share that the consumer spends on good* x*.*

Thus, the experimentally observed HB should: (1) increase in the income elasticity of the good presented to the subject, $\varepsilon_{w,p}$, i.e., becoming particularly large for luxury goods where $\varepsilon_{w,p} > 1$, as suggested in several experimental settings; and (2) in the consumer's budget share on this good, θ_x. Intuitively, as the income effect of a good increases (because $\varepsilon_{w,p}$ increases, θ_x increases, or both), then its HB also raises.

In addition, our analysis allows for an unrestricted value of $x > 0$. However, for the case in which $x(p, w) = 1$ (as it is the case in many experimental treatments where subjects are asked to reveal their willingness-to-pay for a single unit of the good), the expression of the HB becomes $\varepsilon_{w,p} \cdot \theta_x$. Hence, for a given HB, e.g., 0.2 as reported for beef by Lusk and Schroeder (2004), we can describe the $(\theta_x, \varepsilon_{w,p})-$pairs that yield such an HB with the function $\theta_x = \frac{0.2}{\varepsilon_{w,p}}$; as depicted in the level set of figure 1. Intuitively, for the HB coefficient to remain constant as the good becomes more luxurious (larger income elasticity), the subject's budget share θ_x must decrease. Otherwise, the HB unambiguously increases, graphically represented by $(\theta_x, \varepsilon_{w,p})-$pairs to the northeast of level set $HB(\varepsilon) = 0.2$.

[8] Other articles consider another type of uncertainty, namely the payment and provision uncertainty that experimental subjects face; see Mitani and Flores (2013), Blumenschein et al. (2008), and Carson and Groves (2007).

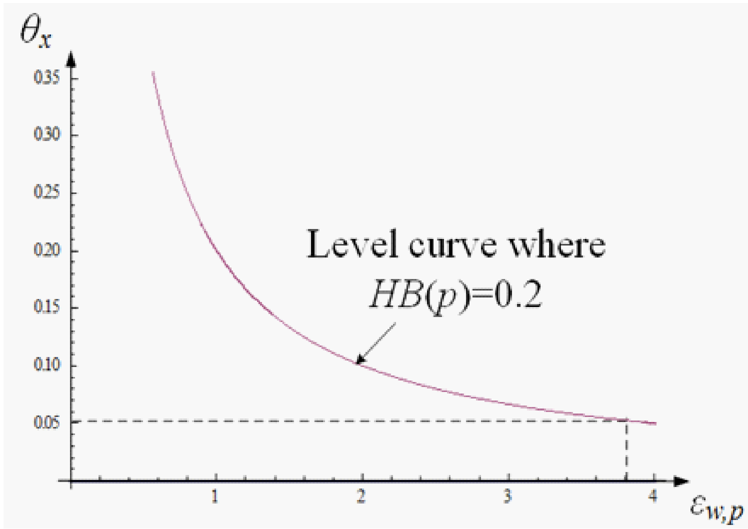

Figure 1. Level curve of $(\theta_x, \varepsilon_{w,p})-$pairs yielding $HB(\varepsilon) = 0.2$.

Our results also extend to public goods, thus suggesting that the HB would be larger for luxurious public goods (e.g., fancy public parks and gardens with a high $\varepsilon_{w,p}$ value) and for goods with a large budget share. While these comparative statics hold for HB in private goods, as described in the articles reported in Table 1, the few experimental studies testing for the presence of HB in public goods do not yet allow us to observe a similar pattern for this type of goods.

2.1 Linearities and HB

Since the HB coefficient depends on the income elasticity of demand, quasilinearity in the utility function $u(x, y)$ can play a critical role in the emergence of HB. In particular, consider $u(x, y) = v(f(x), g(y))$, where $v' > 0$, $f' > 0$ and $g' > 0$ (positive marginal utilities), but f'' can be zero if good x enters linearly or $f'' < 0$ if it enters non-linearly. A similar argument applies to good y and g''. The next proposition evaluates the HB coefficient in each of these cases.

Proposition 2. *If the utility function is quasilinear with respect to good x (good y), i.e., $f'' < 0$ and $g'' = 0$ ($f'' = 0$ and $g'' < 0$, respectively), then its HB coefficient is zero. However, if both goods enter non-linearly, i.e., $f'', g'' < 0$, income effects are present for both goods, i.e., $\varepsilon_{w,p} > 0$, and their HB is positive. Finally, if both goods enter linearly, $f'', g'' = 0$, income effects (and HBs) are only positive for the good/s consumed in positive amounts.*

Hence, a utility function such as $u(x, y) = ax^{\alpha} + by^{\beta}$, where $\alpha, \beta \leq 1$ will exhibit a positive HB in both goods as long as $\alpha, \beta \neq 1$. If, in contrast, the utility function is quasilinear in good x, i.e., $\alpha \neq 1$ but $\beta = 1$, then income effects for good x are zero, i.e., $\varepsilon_{w,p} = 0$, implying that the HB is zero for good x but positive for good y.[9] Finally, if the utility function is linear in both goods, $\alpha, \beta = 1$ (as in the case of perfect substitutes), then income effects arise for the good being consumed in positive amounts (the good with the highest marginal utility per dollar, e.g., good x if $\frac{a}{p} > b$), thus yielding a positive HB; whereas the income effect (and HB) for the good not consumed is zero.[10]

Our results therefore suggest that experimental subjects, which recurrently exhibit positive HBs, must have a utility function that is either: (1) non-linear in both goods; or (2) quasilinear in good y (i.e., good y enters non-linearly while good x enters linearly); or (3) a linear utility function (thus regarding goods as perfect substitutes) but only consume good x, i.e., $x > 0$ and $y = 0$. Since good y embodies all goods different from x, the third option (where $y > 0$) seems unrealistic. More experimental studies are, however, needed in order to disentangle whether HB arises because consumer preferences fit the non-linearity in (1) or the quasilinearity in (2).

Application to Public Goods. As described in the Introduction, List and Gallet (2001) and Murphy et al. (2005) demonstrate that HB exists for both public and private goods, with the bias being higher for public goods. In order to show that our model can also account for this experimental observation, let us next evaluate the HB for an economy with 2 consumers $i = \{A, B\}$, one private good x, and one public good Y. Denote the price of private good by $p_x > 0$, while that of the public good is $p_y > 0$. The income of both individuals is w, and their utility function is

$$u(x^i, Y) = v\left(f(x^i), g(Y)\right).$$

where $Y \equiv y^i + y^j$ denotes aggregate contributions to the public good for every individual $i \neq j$. Similarly as in section 2, let us consider that $v' > 0$, and that the marginal utility of the private and public good are positive, i.e., $f' > 0$ and $g' > 0$ respectively. In addition, we allow for these goods to enter nonlinearly, i.e., $f'', g'' \leq 0$.[11] In this setting, we can apply the results from Proposition 2 to the context of private and public goods, as the next Corollary describes.

Corollary 1. *If the utility function is quasilinear with respect to the private (public) good then its HB coefficient is zero, while that of the public good (private good, respectively) is positive. If the utility function is non-linear in both the private and public good, then the HB of both goods is positive. If both goods enter linearly, only the good consumed in positive amounts has a positive HB coefficient.*

Our results hence suggest that, for quasilinear utility functions such as $u(x^i, Y) = ax^i + bg(Y)$,

[9] An alternative quasilinear functional form often used in applications is $u(x, y) = a \ln x + by$. We explore this utility function in the numerical simulations in the next section.

[10] If the marginal utility per dollar coincides across goods, $\frac{a}{p} = b$, a continuum of utility maximizing bundles arises. In this context, if the consumer chooses a bundle with strictly positive amounts of both goods, a marginal increase in income would yield positive income effects for both goods and, as a consequence, a positive HB. For this reason, Proposition 1 states that income effects are positive for the good consumed in positive amounts (in the case of corner solutions), or the goods consumed in positive amounts (in the case of an strictly interior solution).

[11] Hence, if the utility function is quasilinear with respect to the private (public) good, $f'' < 0$ and $g'' = 0$ ($f'' = 0$ and $g'' < 0$, respectively). If, instead, both goods enter non-linearly (linearly), $f'' < 0$ and $g'' < 0$ ($f'' = g'' = 0$, respectively).

where $a, b > 0$ and $g(Y)$ is nonlinear in the total donations to the public good, Y, the HB coefficient of the public (private) good should be null (positive, respectively). In contrast, utility functions such as $u(x^i, Y) = f(x^i) + bY$, where $f(x^i)$ is nonlinear in the private good, yield a positive HB for the public good but a null HB for the private good. When both goods enter nonlinearly, such as in $u(x^i, Y) = \log x^i + \log Y$, Corollary 1 demonstrates that the HB is positive for both types of goods. This result goes in line with recurrent experimental observations whereby HB arises for both public and private goods, thus suggesting that individual preferences rarely fit the quasilinear description and, instead, exhibit non-linearities in both goods.

3 Application to different utility functions

In this section we apply the above results to utility functions recurrently used in the experimental literature, thus providing a set of testable predictions. In particular, we examine the Stone-Geary utility function, used by Cronin (1982, 1983); the Cobb-Douglas utility function, considered in Burtless and Hausman (1978) and Carter and Castillo (2002); the quasilinear utility function, as in Banks et al. (1989), Fehr et al. (1996) and Mckelvey and Page (2000); and the CES utility function, used in Schechter (2007). While these experimental studies do not test for the presence of HB, experimentalists measuring HB in different types of goods can use our theoretical predictions to better identify subjects' underlying preferences. In addition, our results provide some unsuspected comparative statics that also allow for experimental tests.

Example 1: *Stone-Geary utility function.* Let us first consider that the consumer's utility function is given by

$$u(x, y) = (x - a)^{\alpha}(y - b)^{\beta}$$

where $\alpha, \beta > 0$, and $a, b > 0$ denote the minimal amounts of goods x and y that the consumer needs to survive. For simplicity, we assume that these minimal amounts are not extremely high, i.e., $a, b < w$. In this setting, the Walrasian demand becomes $x(p, w) = \frac{\alpha\beta p + \alpha(w-d)}{(\alpha+\beta)p}$, thus implying that the inverse demand function is $p(x, w) = \frac{\alpha(w-b)}{(\alpha+\beta)x - \beta\alpha}$. As a consequence, income elasticity is $\varepsilon_{w,p} = \frac{w}{w-b}$, which becomes 1 when the above parameters satisfy $a = b = 0$, as in the case of Cobb-Douglas utility function. Therefore, the HB coefficient in this context is

$$HB(\varepsilon) = \frac{w}{w - b} \frac{\theta_x}{x(p, w)} = \frac{p}{w - b}$$

where $\theta_x = \frac{p\frac{\alpha\beta p + \alpha(w-d)}{(\alpha+\beta)p}}{w} = \frac{\alpha\beta p + \alpha(w-d)}{(\alpha+\beta)w}$ represents the budget share of good x. Hence, the HB increases in the price of good x, and in the minimal amount that the consumer needs of all other goods, b. Intuitively, when the consumer needs more units of good y to survive, his consumption of good x is low in relative terms. Hence, a marginal change in income yields a large increase in the demand of good x, which ultimately increases the HB coefficient. Figure 2 depicts $HB(\varepsilon)$ for a value of $b = 1$.

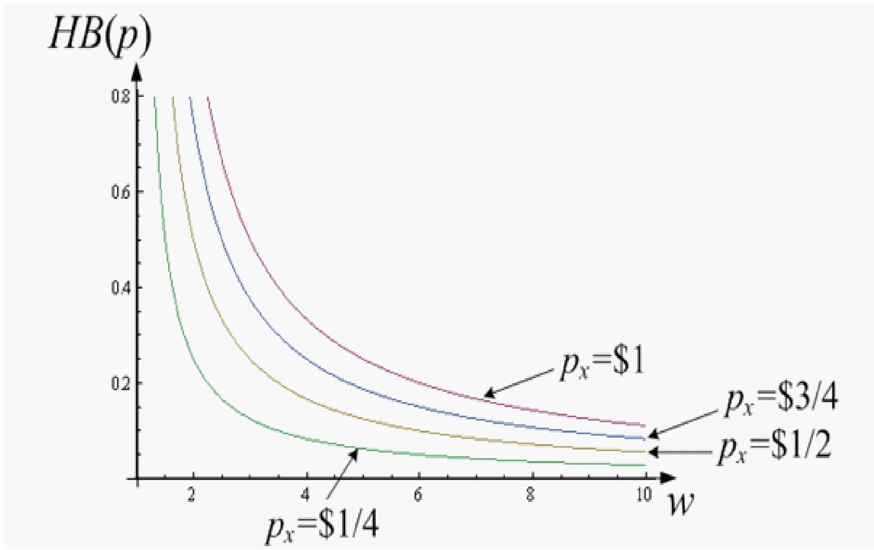

Figure 2. $HB(\varepsilon)$ for a Stone-Geary utility function, where $b = 1$.

Importantly, our above discussion embodies the Cobb-Douglas utility function as a special case, namely, when $a = b = 0$. In this case, the HB decreases to $\frac{p}{w}$. Graphically, all the HB curves depicted in figure 2 (where $b = 1$) would experience a downward shift.

Example 2: *Quasilinear utility function.* Let us now consider that the consumer's utility function is given by

$$u(x, y) = a \cdot x + b \cdot \ln y$$

In this setting, the Walrasian demand becomes $x(p, w) = \frac{a \cdot w - b \cdot p}{a \cdot p}$, thus implying that the inverse demand function is $p(x, w) = \frac{a \cdot w}{b + ax}$. As a consequence, income elasticity is $\varepsilon_{w,p} = 1$, ultimately yielding an HB of

$$HB(\varepsilon) = 1 \frac{\theta_x}{x(p, w)} = \frac{p}{w}$$

where $\theta_x = \frac{p \cdot \frac{aw - bp}{ap}}{w}$. Hence, similarly as for the Stone-Geary and Cobb-Douglas utility functions considered above, the HB increases in the price of good x, but decreases in the individual's wealth level; as depicted in figure 3.

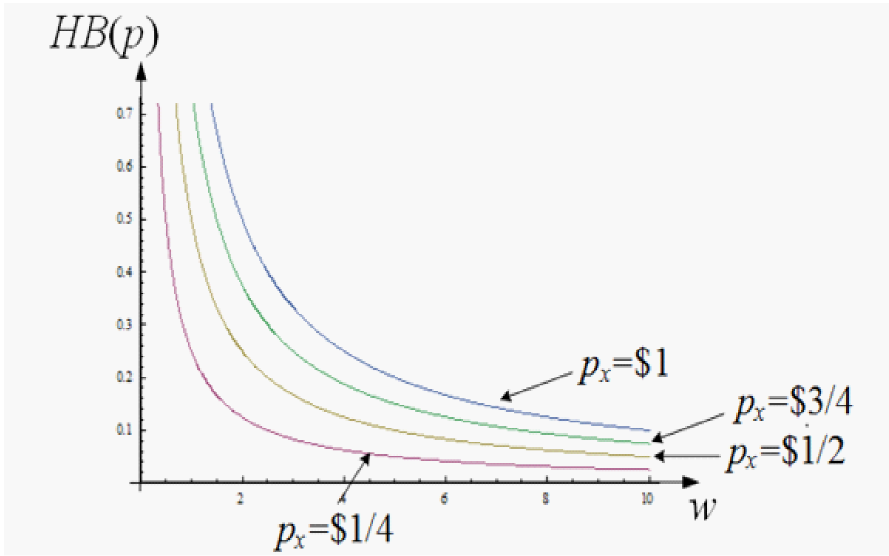

Figure 3. $HB(\varepsilon)$ for quasilinear utility.

Example 3: *Generalized CES utility function.* Let us now consider a generalized CES utility function

$$u(x, y) = \left[(x + b)^{\delta} + y^{\delta}\right]^{\frac{1}{\delta}}$$

where parameter b captures whether good x is a luxury relative to the composite good y, which occurs when $b < 0$, or if instead good y is a luxury, i.e., when $b > 0$. Note that in the special case in which $b = 0$ the above utility function coincides with a standard CES utility function. In addition, parameter δ satisfies $0 \neq \delta \leq 1$. As it is well known if, besides $b = 0$, parameter $\delta \to 0$ $u(x, y)$ represents a Cobb-Douglas utility function; if $\delta = 1$, it represents preferences for substitutes (linear utility function); while if $\delta \to -\infty$, it represents preferences for complements (Leontieff utility function). In this context, the HB becomes

$$HB(\varepsilon) = \frac{p}{w} \cdot \frac{\left[(b + x)^2 + 4wx\right]^{1/2} - (b + x)}{\left[(b + x)^2 + 4wx\right]^{1/2}}$$

(See appendix 1 for more details about the Walrasian demand under the generalized CES utility function, and its associated income elasticity.) The left-hand panel of figure 4 illustrates that HB is increasing in prices and decreasing in wealth, similarly as for previous utility functions.[12] The right-hand panel of the figure, however, examines a new dimension that the previous functional forms could not capture: the degree to which good x is considered a luxury or a necessity (embodied in parameter b). In particular, when the good is a luxury ($b < 0$), the curve representing the

[12] For simplicity, the figure assumes $b = 2$, but similar results arise for other values of parameter b.

HB coefficient shifts upwards, indicating that experimental subjects fall more prey of the HB. In contrast, when the good is a necessity ($b > 0$), the HB coefficient decreases, ultimately reflecting that HB is likely small in experimental settings.[13]

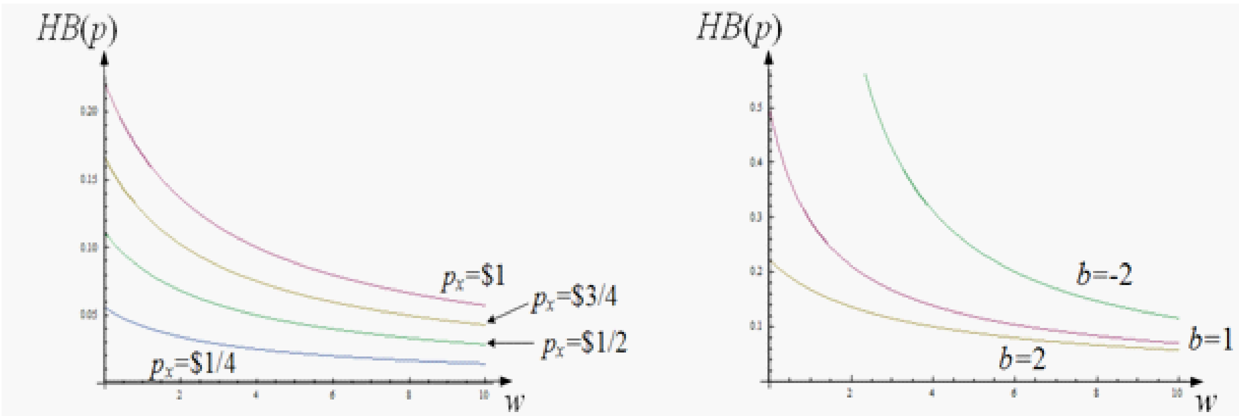

Figure 4. $HB(\varepsilon)$ for the generalized CES utility function.

Finally, note that in the special case in which $b = 0$, the above utility function becomes the standard CES utility function, and the HB coefficient simplifies to

$$HB(\varepsilon) = \frac{p}{w} \cdot \left(1 - \frac{x^{1/2}}{(4w + x)^{1/2}}\right).$$

In the case in which the consumer only purchases one unit of the item, $x = 1$, the HB coefficient reduces to $\frac{p}{w} \cdot \left(1 - \frac{1}{(4w+1)^{1/2}}\right)$. As for previous utility functions, the HB increases in the price of good x, but decreases in income, w.

Example 4. *Public goods.* Consider an individual decision maker with utility function $u(x^i, Y) = a \log x^i + b \log Y$. In this setting, it is straightforward to find the demand function for the public good, $y^i(p, w) = \frac{w}{3p_y}$, and for the private good, $x^i(p_x, w) = \frac{w - p_y y^i}{p_x} = \frac{2w}{3p_x}$. (See appendix 2 for more details.) Solving for p_y and p_x in order to obtain the inverse demand functions, and using the definition $HB(\varepsilon) = \varepsilon_{w,p} \frac{\theta_x}{x(p,w)}$, we obtain an HB coefficient for the public and private good of

$$HB_y(\varepsilon) = \frac{p_y}{w} \quad \text{and} \quad HB_x(\varepsilon) = \frac{p_x}{w}$$

where $HB_y(\varepsilon) > HB_x(\varepsilon)$ if and only if prices satisfy $p_y > p_x$. While there are no studies measuring the HB of private and public goods for the same group of experimental subjects, articles using metadata of experiments involving both private and public goods, such as List and Gallet (2001) and Murphy et al. (2005), conclude that experimentalists generally assume a more expensive price

[13] For simplicity, the right-hand panel of figure 4 considers a price $p = 1$, but similar results emerge for other price levels.

for the public than the private good, i.e., $p_y > p_x$, and that these experiments find a higher HB for public than private goods, a result and assumption that go in line with our findings.

4 Conclusions and further research

Our paper, hence, provides a theoretical foundation for two repeatedly observed results in experiments on market and nonmarket valuation: (1) positive HBs, which arise under most experimental methodologies and types of goods; and (2) a larger HB for luxury goods than for goods regarded as necessities. Interestingly, our theoretical results offer several unexplored experimental tests. First, while studies abound on the presence of HB, to our knowledge all experimental articles measure subjects' valuation focusing on a single good; thus not providing a direct comparison of how HB varies as different goods are presented to the same individuals. Second, our results provide at least two testable implications that should hold regardless of the underlying preference relation of the subjects participating in the experiment: (1) HB increases in the income elasticity of the good; and (2) it also increases in the budget share of that good. Nonetheless, when we examine quasilinearity and nonlinearity in subjects' utility function our results are relatively flexible, which ultimately allow for a battery of tests in controlled experiments.

5 Appendix

5.1 Appendix 1 - *Generalized CES utility function*

In this context, the Walrasian demand of good x is

$$x(p,w) = \frac{w - b\sqrt{p}}{\sqrt{p} + p}$$

In this setting, we can only find the expression of the inverse demand function $p(x,w)$ for specific values of r. In particular, if $r = \frac{1}{2}$, then $p(x,w) = \frac{\left[(b+x)^2+4wx\right]^{1/2}+(b+x)}{\left[(b+x)^2+4wx\right]^{1/2}}$, which yields an income elasticity of $\varepsilon_{w,p} = \frac{\left[(b+x)^2+4wx\right]^{1/2}-(b+x)}{\left[(b+x)^2+4wx\right]^{1/2}}$. Thus, the budget share is $\theta_x = \frac{px(p,w)}{w} = \frac{p\left(w-b\sqrt{p}\right)}{\left(\sqrt{p}+p\right)w}$. As a consequence, the HB is

$$HB(\varepsilon) = \varepsilon_{w,p}\frac{\theta_x}{x(p,w)} = \frac{p}{w} \cdot \frac{\left[(b+x)^2 + 4wx\right]^{1/2} - (b+x)}{\left[(b+x)^2 + 4wx\right]^{1/2}}. \quad \blacksquare$$

5.2 Appendix 2 - *Public good contributions*

In order to find the Walrasian demand for the public and private good, let us first identify each individual's best response function. The utility maximization problem of individual i is that of selecting his consumption of private good, x, and his contribution to the public good, y^i, to solve:

$$\max_{x,y^i} \; a \log x^i + b \log G$$

$$\text{subject to} \;\; p_x x^i + p_y y^i = w, \quad \text{and } y^i + y^j = G$$

Since $x^i = \frac{w - p_y y^i}{p_x}$, the above problem can be more compactly expressed as

$$\max_{y^i} \; a \log \left(\frac{w - p_y y^i}{p_x} \right) + b \log(y^i + y^j)$$

taking first order condition with respect to y^i yields

$$\frac{b}{y^i + y^j} + \frac{ap}{p_y y^i - w} = 0$$

and solving for y^i we obtain a best response function of

$$y^i(y^j) = \frac{bw}{(a+b)p_y} - \frac{a}{a+b} y^j$$

By symmetry, the best response function of individual j is

$$y^j(y^i) = \frac{bw}{(a+b)p_y} - \frac{a}{a+b} y^i$$

Simultaneously solving for y^i and y^j, we find the demand function for the public good, $y^i(p,w) = \frac{bw}{(2a+b)p_y}$, and for the private good, $x^i(p_x, w) = \frac{w - p_y y^i}{p_x} = \frac{2aw}{(2a+b)p_x}$. Solving for p^y in $y^i(p,w)$, yields the inverse demand function for the public good $p_y(y^i, w) = \frac{bw}{(2a+b)y^i}$. This inverse demand entails an income elasticity of $\varepsilon_{w,p} = 1$. Hence, using the definition $HB(\varepsilon) = \varepsilon_{w,p} \frac{\theta_x}{x(p,w)}$, we obtain an HB coefficient for the public good of

$$HB_y(\varepsilon) = \frac{p_y}{w}$$

Solving for p_x in $x^i(p_x, w) = \frac{2aw}{(2a+b)p_x}$ entails an inverse demand for the private good of $p_x(x,w) = \frac{2aw}{(2a+b)x}$. Hence, the income elasticity of this good also becomes $\varepsilon_{w,p} = 1$ for the private good. As a consequence, the HB for the private good is

$$HB_x(\varepsilon) = \frac{p_x}{w}$$

Finally, comparing both biases, we obtain that $HB_y(\varepsilon) > HB_x(\varepsilon)$ if and only if prices satisfy $p_y > p_x$. ■

5.3 *Proof of Proposition 1*

Using $HB(\varepsilon) = \frac{\partial p(x,w)}{\partial w}$, we can rewrite $\varepsilon_{w,p} = \frac{\partial p(x,w)}{\partial w} \frac{w}{p(x,w)}$ as

$$\varepsilon_{w,p} = HB(\varepsilon)\frac{w}{p(x,w)}$$

Solving for $HB(\varepsilon)$ we obtain,

$$HB(\varepsilon) = \varepsilon_{w,p}\frac{p(x,w)}{w}$$

Finally, we can multiply and divide by $x(p,w)$ on the right-hand side of the equality, to obtain

$$HB(\varepsilon) = \varepsilon_{w,p}\frac{\theta_x}{x(p,w)}$$

where $\theta_x \equiv \frac{p(x,w)\cdot x}{w}$ represents the budget share that the consumer spends on good x. ■

5.4 *Proof of Proposition 2*

Case 1. The utility maximization problem of individual i is that of selecting his consumption of good x and y to solve:

$$\max_{x,y} v(f(x), g(y))$$

$$\text{subject to } px + y = w.$$

Since $y = w - px$, the above problem can be more compactly expressed as

$$\max_{x} v(f(x), g(w - px))$$

Taking first order condition with respect to x yields

$$v'f' + v'g'(-p) \leq 0$$

which in the case of interior solutions reduces to $f' = pg'$. When the utility function is quasilinear with respect to good x, i.e., $f'' < 0$ and $g'' = 0$, f' is decreasing in x while pg' is constant. On one hand, the left-hand side of the first-order condition $f' = pg'$ does not depend on income, w, since it originates from $\frac{\partial v}{\partial f}\frac{\partial f(x)}{\partial x}$ where w is absent. Similarly, the right-hand side, pg', does not depend on w either since $g''(w - px) = 0$. As a consequence, the solution of first-order condition $f' = pg'$, i.e., the Walrasian demand for good x, is independent of income. Therefore, $\frac{\partial x(p,w)}{\partial w} = 0$, ultimately implying that income-elasticity of demand is null, and that the HB is also null.

Case 2. A similar argument applies to the case in which the utility function is quasilinear with respect to good y, i.e., $f'' = 0$ and $g'' < 0$, whereby we can solve for x in the budget constraint, $x = \frac{w-y}{p}$, in order to express the utility function in terms of good y alone.

Case 3. In the case in which both goods enter non-linearly, i.e., $f'' < 0$ and $g'' < 0$, the above first-order condition $f' = pg'$ depends on income, w. In particular, while f' is independent of income, pg' is a function of on w since $g'' < 0$, e.g., if $g(w - px) = (w - px)^{1/2}$, then $g' = \frac{1}{2}(w - px)^{-1/2}$. In particular, since w enters positively into $g(\cdot)$, an increase in w produces a shift in the g' function, ultimately increasing the Walrasian demand for good x In this setting, income-elasticity of demand is thus positive, and HB is also positive.

Case 4. Finally, when both goods enter linearly, i.e., $f'' = g'' = 0$, both the left- and right-hand side of first-order condition $f' = pg'$ is constant in x. In this context, the consumer only demands positive amounts of good x, which occurs when $f' > pg'$; or of good y otherwise. Since an increase in income does not alter the sign of this inequality (given that both f' and pg' are independent on income), a marginal increase in w is entirely spent in the good whose demand was originally positive. Only for this good are wealth effects positive, and its associated HB positive. ■

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