

Imperfect Price Deflation in Production Systems

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Nominal prices that are measured with error create challenges for econometric estimation. A straightforward approach for a model with linear input demands, such as generated from a quadratic normalized profit function, is proposed where the nominal price is measured with error. Nominal measurement error is likely because expected output price is measured imperfectly by actual output price. An approach using generalized method of moments is developed to estimate such errors-in-variables systems that avoids use of extra-sample data or additional structural specifications. Monte-Carlo examination of small sample properties shows promise. Measurement error is statistically significant using aggregate U.S. agricultural data.

Keywords: errors-in-variables, expected prices, GMM estimation, input demands

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The connection between economic theory and measurement is often tenuous and filled with compromises. There has long been a concern that substantially wrong conclusions can be reached if there is serious measurement error (e.g., Greene 2000). For example, standard errors-in-variables (EIV) models can lead to biased and inconsistent estimates and underidentification. Many of the proposals for dealing with EIV require extra sample data and/or very sophisticated methods based on simulation or characteristic functions (e.g., Hausman, et al. 1991; Cragg 1997; Amemiya 1990; Schennach 2004; Li and Vuong 1998; and Newey 2001).

A wide variety of articles in agricultural economics have considered EIV. An incomplete but representative list of articles includes: (i) EIV biases in production function estimation (Griliches 1957), (ii) an EIV AIDS demand system (Buse 1994) and reported income not being the income that belongs in demand equations or Engel curves (Newey 2001; Lewbel 1996), and (iii) EIV analyses of the conventional capital asset pricing model (Bjornson and Innes 1992; Johnson, Mittelhammer and Blayney 1994). Furthermore, as noted by McCallum (1976), linear rational expectations models can also be viewed as EIV models, so a host of other models implicitly include EIV (e.g., Aradhya and Holt 1989; Sheffrin 1983).¹

The particular problem addressed in this paper deals with price deflation. It is common in agricultural economic studies to deflate by a numeraire, which may be a price index or an expected price. For example, under zero-degree homogeneity in typical normal-

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ized profit function approaches such as popularized by Lau (1978), output price generally becomes a numeraire that normalizes or deflates all input prices. Lau (1978), and Lau and Yotopolous (1972) called this the “Unit-Output-Price” or UOP profit function. A common form explored in this paper is the normalized quadratic profit or value function, which has had widespread application in agricultural economics (e.g., Vasavada and Chambers 1986; Shumway 1983; Shumway and Lim 1993; Arnade 1992; Lopez, Adelaja, and Andrews 1988; Fabiosa, Jensen, and Yan 2004).

This article examines relatively simple methods to deal with the mis-measured numeraire problem in the class of input demand functions based on the normalized quadratic profit function. The model can be viewed as a nonlinear EIV model or it can be transformed into one that is linear in the numeraire but has some non-standard properties.² We show that homogeneous (deflated) economic systems have advantages over most EIV models. In order to simplify the issues involved, we begin with a bivariate regression model with no intercept for illustrative purposes and consider various lessons learned about estimation techniques. More general multivariate models with an intercept and other explanatory variables are then considered. We show that they can be identified and estimated without extra-sample data or a structural model for the market price, as is commonly necessary in structural rational expectations approaches to instrumental variables problems. Second, no additional assumptions are needed about the correlation of functions of the variable that is measured with error and the instruments, as are commonly required in the instrumental variables approach to linear EIV models. These two points are under-appreciated in the EIV literature. In the case of this article, conventional generalized method of moments (GMM) can be used to estimate the parameters of the mean part of the model. We also show how to interpret and estimate the GMM framework with a traditional IV approach that can be implemented with most commercial computer software. This approach is examined with single equation Monte-Carlo methods, which consider bias, mean-square error, and inference relative to sample size. Finally, the approach is applied to a sample of U.S. agricultural production data based on a system specification of factor demand equations derived from a normalized profit function.

The Basic Problem

Most empirical researchers deflate prices to represent zero-degree homogeneity, even though homogeneity is sometimes rejected by empirical tests (Shumway 1995). Nevertheless, economic theory compels most production economists to estimate equations of the form (where the data are normalized to yield a zero constant term):

$$(1) \quad \textbf{Model 1:} \quad x_t = \beta(w_t / p_t) + \varepsilon_t, \quad E(\varepsilon_t) = 0, \quad E(\varepsilon\varepsilon^\top) = \sigma^2 \mathbf{I}_T, \quad t = 1, \dots, T,$$

where x_t is a factor demand (e.g., fertilizer or acreage of a crop), w_t is the factor demand price, p_t is output price, ε_t is a random shock, t is time or some index of the observations, and $\beta < 0$ is an unknown scalar parameter to be estimated.³ Because the model is linear in parameters and data, an additive random error with classical properties is assumed (A

single input demand model of agricultural input demand is found in Griliches 1959).

The least squares estimator associated with (1) is (where all summations are over $t = 1, \dots, T$ unless otherwise indicated)

$$(2) \quad \hat{\beta} = \left\{ \sum_t (w_t / p_t)^2 \right\}^{-1} \sum_t x_t w_t / p_t = \beta + \left\{ \frac{1}{T} \sum_t (w_t / p_t)^2 \right\}^{-1} \frac{1}{T} \sum_t (w_t / p_t) \varepsilon_t .$$

Assume w_t/p_t is statistically independent of ε_t , so that $\text{plim} \left\{ \frac{1}{T} \sum_t (w_t / p_t) \varepsilon_t \right\} = 0$. Then the least squares estimator is consistent so long as

$$(3) \quad \text{plim} \left\{ \frac{1}{T} \sum_t (w_t / p_t)^2 \right\} = \Omega_{w/p}$$

is positive and finite (White 1984).

While measurement error is often viewed as endemic to economic analysis, problematic measurement is likely more serious for p_t than w_t in agriculture because p_t is typically unknown to the decision maker at the time most input decisions are made. Thus, assuming risk neutrality, decisions must be based on some type of output price expectation. We do not seek to develop a formal econometric model to choose among the many possible ways in which producers might form expectations for output prices, or estimate the predicted price with extra sample information to use as an instrumental variable in (1). Instead, we identify and develop simple econometric methods that can be used to obtain consistent parameter estimates for (1) when some unobservable p_t^* is used by the decision maker in (1), while the econometrician only observes p_t . This produces the EIV problem. Alternatively, p_t might represent an aggregate price index such as the producer price index often used to deflate price data.⁴ In each case, some p_t^* may apply in the true data generating process for x_t while p_t is used by the econometrician. Thus, the true model 1 is represented as

$$\textbf{Model 1:} \quad x_t = \beta(w_t / p_t^*) + \varepsilon_t, \quad E(\varepsilon_t) = 0, \quad E(\varepsilon \varepsilon^T) = \sigma^2 \mathbf{I}_T, \quad t = 1, \dots, T .$$

The Main EIV Assumptions

The main assumptions applied throughout this article are that w is exogenous and that

$$(4) \quad \begin{aligned} p_t &= p_t^* \delta_t, \quad \delta_t > 0, \quad E(\delta_t) = 1, \quad E(1/\delta_t) = \lambda, \quad E(1/\delta_t^2) = \theta, \quad t = 1, \dots, T, \\ E(\delta \delta^T) &= \nu \mathbf{I}_T, \quad E(\delta \varepsilon^T) = \mathbf{0}_{T \times T}, \\ \text{plim} \left\{ \frac{1}{T} \sum_t (w_t / p_t^*) \varepsilon_t \right\} &= 0, \quad \text{plim} \left\{ \frac{1}{T} \sum_t (w_t / p_t^*)^2 \right\} = \Omega_{w/p^*}, \\ \text{plim} \left\{ \frac{1}{T} \sum_t p_t^* w_t \right\} &= \Omega_{p^*w}, \quad \text{plim} \left\{ \frac{1}{T} \sum_t w_t^2 \right\} = \Omega_{ww}, \end{aligned}$$

with all expectations and probability limits assumed to exist.⁵ We also assume that w_t is independent of both δ_t and ε_t , and that δ_t and ε_t are independent random variables. Throughout, a multiplicative EIV structure is maintained in keeping with the common presumption that the conditional variance of p_t increases when the mean increases. The approach with an additive error is transparent with the transformation $\delta_t = 1 + \zeta_t / p_t^*$, $E(\zeta_t) = 0$, if ζ_t is the random additive error with support within $(-p_t^*, \infty)$.

As implied by virtually any econometrics textbook, under these assumptions the ordinary least squares (OLS) estimator of β with x_t regressed against w_t/p_t (labeled $\hat{\beta}_1$) is inconsistent. However, when the numeraire is measured with error, (4) yields via the weak law of large numbers:

$$\begin{aligned}
 \text{plim } \hat{\beta}_1 &= \text{plim} \left\{ \left[\sum_t (w_t / p_t)^2 \right]^{-1} \sum_t x_t (w_t / p_t) \right\} \\
 (5) \quad &= \frac{\beta \text{plim} \left\{ \frac{1}{T} \sum_t (w_t / p_t^*)^2 (1 / \delta_t) \right\} + \text{plim} \left\{ \frac{1}{T} \sum_t (w_t / p_t^*) (1 / \delta_t) \varepsilon_t \right\}}{\text{plim} \left\{ \frac{1}{T} \sum_t (w_t / p_t^*)^2 (1 / \delta_t)^2 \right\}} \\
 &= \frac{\beta \Omega_{w/p^*} E(1 / \delta_t) + \mu_{w/p^*} E(1 / \delta_t) E(\varepsilon_t)}{\Omega_{w/p^*} E(1 / \delta_t^2)} = \frac{\beta \lambda}{\theta}.
 \end{aligned}$$

Because $1/\delta_t$ is a convex function, it follows by Jensen's inequality that $\lambda > 1$. In addition, by the Cauchy-Schwarz inequality (Roussas 1973),

$$(6) \quad \theta \equiv E(1 / \delta_t^2) > E(1 / \delta_t) E(1 / \delta_t) = \lambda^2 > E(1 / \delta_t) = \lambda.$$

Thus, OLS estimates have attenuation bias (bias towards zero) and the interpretation of the estimated parameter as a marginal response of the conditional mean of x_t given w_t/p_t is incorrect. The size of the bias depends on the ratio of the mean and second moment about zero of $1/\delta_t$. Moreover, the tails of the distribution of $1/\delta_t$ could be thick enough that its moments do not exist. If so, the bias of the OLS estimator would be undefined.

Consistent Estimation in the Presence of EIV

By far, instrumental variables (IV) estimation is the most common approach for EIV models (Hausman, et al. 1991). Indeed, IV appears to be the estimator of choice for linear models when proper instruments are present and replicate or validation measurements are not possible (Carroll, Ruppert, and Stefanski 1998, Chapter 5). Much of the recent literature focuses on nonlinear EIV where the standard IV estimator is inconsistent. For example, discrete choice models are nonlinear and often presumed to be subject to substantive measurement errors. A laundry list of methods is found in Carroll, Ruppert, and Stefanski (1998) ranging from total least squares to Bayesian methods to nonstandard likelihood

methods.⁶ If p_t^* is viewed as fixed, then likelihood methods are inconsistent because the number of parameters grows at the same rate as the sample size. However, if p_t^* is a random draw from a distribution with density $h(p_t^*, \phi)$, where ϕ represents the vector of parameters for h , then a consistent estimator may be found if one knows h .

EIV and GMM Approaches to Consistent Estimation

For the model in this article, a much simpler least squares approach produces a consistent estimate of β . Multiplying both sides of (1) by the observed price, $p_t = p_t^* \delta_t$, obtains an equivalent regression model that is now linear with simple heteroskedasticity,

$$(7) \quad \textbf{Model 2:} \quad p_t x_t = \beta w_t \delta_t + p_t \varepsilon_t, \quad E(\varepsilon_t) = 0, \quad E(\varepsilon \varepsilon^\top) = \sigma^2 \mathbf{I}_T, \quad t = 1, \dots, T.$$

However, equivalent treatment requires consideration of heteroskedasticity in the error term and the role of δ_t on the right hand side, i.e., where the regression equation is $p_t x_t = \beta w_t + \xi_t$, $\xi_t = (\delta_t - 1)\beta w_t + p_t^* \delta_t \varepsilon_t$, $E(\xi_t) = 0$. A consistent estimate of β is obtained by least squares regression of $p_t x_t$ on w_t ,

$$(8) \quad \hat{\beta}_2 = \left\{ \sum_t w_t^2 \right\}^{-1} \sum_t p_t x_t w_t,$$

which yields

$$(9) \quad \text{plim } \hat{\beta}_2 = \left[\text{plim} \left\{ \frac{1}{T} \sum_t w_t^2 \right\} \right]^{-1} \beta \text{plim} \left\{ \frac{1}{T} \sum_t w_t^2 \delta_t \right\} = \beta \Omega_{ww} / \Omega_{ww} = \beta,$$

by the assumptions that $E(\delta_t) = 1$ and that δ_t is statistically independent of w_t and ε_t . Thus, the problem involves only EIV and not simultaneous equations bias. This suggests that if the measurement error can be represented additively, then the inconsistency caused by EIV can be eliminated. Of course, valid inference requires consideration of heteroskedasticity, for which common robust methods are available (White 1980; Newey and West 1987).

Alternatively, consider a class of IV or GMM estimators assuming availability of an instrument z_t that is statistically independent of ε_t and δ_t .⁷ In this case, the IV estimators for models 1 and 2 and their probability limits using actual data are:

$$(10) \quad \begin{aligned} \tilde{\beta}_1 &= \left\{ \sum_t z_t (w_t / p_t) \right\}^{-1} \sum_t z_t x_t, \quad \text{plim } \tilde{\beta}_1 = \beta / \lambda, \\ \tilde{\beta}_2 &= \left\{ \sum_t z_t w_t \right\}^{-1} \sum_t z_t p_t x_t, \quad \text{plim } \tilde{\beta}_2 = \beta, \end{aligned}$$

respectively, where the probability limits are established as above assuming appropriate asymptotic moments exist. For non-degenerate distributions, $\theta > \lambda > 1$, attenuation bias occurs for IV as well as OLS estimation of model 1, although OLS attenuation bias is larger because $\theta/\lambda > \lambda > 1$. For example, suppose δ has a uniform distribution on support $(.5, 1.5)$. Then $\lambda = 1.098$, $\theta = 1.333$, and $\theta/\lambda = 1.214$. Hence, in large samples, OLS parameter estimates will be 82.4 percent of the true parameter while IV estimates will be 91.0 percent of the true parameter.

The central message in this development is that if a variable is suspected to have measurement error, then the variable is best included linearly rather than nonlinearly on the right-hand-side if simple estimation methods are to be used. More specifically, estimates can be misleading when deflation is accomplished by dividing by a variable that is measured with error. If measurement error is suspected in the deflator, IV or GMM estimation can reduce bias but does not eliminate it. Better yet, the bias in large samples can be eliminated using GMM on the transformed model 2.

The impact of nonlinearity in EIV models has received considerable visibility (Griliches and Ringstad 1977; Amemiya 1990), but little discussion applies in the special context of prices in microeconomic systems. This article demonstrates important special cases as are customary in “real” models of behavior involving expectations or relative prices.⁸ If deflation is by a price or representation of prices that is not measured well, then the model may be better transformed as in model 2. Although heteroskedasticity and serial correlation can be introduced thereby, robust standard error calculation can attain consistent inference (White 1980; Newey and West 1987).

Extending Models 1 and 2 to Several Regressors

Although the above lessons are instructive, they can be misleading in applications. For example, the typical inclusion of an intercept and/or other regressors necessitates generalizations for models of the form:

$$(11) \quad x_t = \alpha + (\mathbf{w}_t / p_t^*)^\top \boldsymbol{\beta} + \mathbf{z}_t^\top \boldsymbol{\gamma} + \varepsilon_t, \quad E(\varepsilon_t) = 0, \quad t = 1, \dots, T, \quad E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^\top) = \sigma^2 \mathbf{I}_T,$$

where $\mathbf{w}_t \in \mathbb{R}_+^{K_1}$ contains K_1 input prices and $\mathbf{z}_t \in \mathbb{R}_+^{K_2}$ represents K_2 non-price variables such as technical change, fixed factors, or other shifters of the factor demand. Without restating, the main assumptions (see (4)) and properties of instruments are extended to (11).

Suppose the econometrician assumes incorrectly that the model is

$$(12) \quad x_t = \alpha + (\mathbf{w}_t / p_t)^\top \boldsymbol{\beta} + \mathbf{z}_t^\top \boldsymbol{\gamma} + \varepsilon_t, \quad E(\varepsilon_t) = 0, \quad t = 1, \dots, T, \quad E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^\top) = \sigma^2 \mathbf{I}_T.$$

Let $\mathbf{w}_{1t} = [1, (\mathbf{w}_t / p_t)^\top, \mathbf{z}_t^\top]^\top$ denote the $K \times 1$ vector of regressors for observation t , with $K = 1 + K_1 + K_2$, and define the $T \times K$ regressor matrix by $\mathbf{W}_1 = [\mathbf{w}_{11}, \dots, \mathbf{w}_{1T}]^\top$. Let $\boldsymbol{\theta} = [\alpha, \boldsymbol{\beta}^\top, \boldsymbol{\gamma}^\top]^\top$ be the $K \times 1$ vector of parameters. The least squares estimator of $\boldsymbol{\theta}$ is

$$(13) \quad \hat{\theta}_1 = (\mathbf{W}_1^\top \mathbf{W}_1)^{-1} \mathbf{W}_1^\top \mathbf{x}.$$

By inserting x_t from (11) into (13), straightforward manipulation reveals that, in general, all coefficients are estimated inconsistently by $\hat{\theta}_1$. As suggested by (13), attenuation bias cannot be established without an assumption about the correlation among regressors (Nelson 1995). However, if z is uncorrelated with w/p and no intercept term is included, then attenuation bias holds for the OLS estimate of β .

Unfortunately, a consistent estimator of θ is more complicated than that suggested by $\hat{\beta}_2$ in (8). Analogous to model 2, consider transforming the model by multiplying (11) by p_t . Defining matrices $\mathbf{W}_2 = [\mathbf{p}, \mathbf{W}, \mathbf{D}(p_t)\mathbf{Z}]$ and $\mathbf{W}_2^* = [\mathbf{p}, \mathbf{D}(\delta_t)\mathbf{W}, \mathbf{D}(p_t)\mathbf{Z}]$, where $\mathbf{D}(\cdot)$ denotes a diagonal $T \times T$ matrix with t^{th} diagonal element indicated in parentheses, \mathbf{W} is the $T \times K_1$ matrix of input prices, and \mathbf{Z} is the $T \times K_2$ matrix of non-price demand shifters, the assumptions in (4) imply

$$(14) \quad \begin{aligned} E(\mathbf{W}_2) &= E[\mathbf{D}(\delta_t)\mathbf{p}^*, \mathbf{W}, \mathbf{D}(p_t^*)\mathbf{D}(\delta_t)\mathbf{Z}] \\ &= [\mathbf{p}^*, \mathbf{W}, \mathbf{D}(p_t^*)\mathbf{Z}] \\ &= E[\mathbf{D}(\delta_t)\mathbf{p}^*, \mathbf{D}(\delta_t)\mathbf{W}, \mathbf{D}(p_t^*)\mathbf{D}(\delta_t)\mathbf{Z}] = E(\mathbf{W}_2^*). \end{aligned}$$

Thus, multiplying element-by-element by \mathbf{p} , the true model in (11) becomes

$$(15) \quad \begin{aligned} \mathbf{D}(p_t)\mathbf{x} &= \mathbf{D}(p_t)(\mathbf{W}_1\theta + \varepsilon) \\ &= [\mathbf{p}, \mathbf{D}(\delta_t)\mathbf{W}, \mathbf{D}(p_t)\mathbf{Z}]\theta + \mathbf{D}(p_t)\varepsilon \\ &= \mathbf{W}_2^*\theta + \mathbf{D}(p_t)\varepsilon, \end{aligned}$$

where the second line follows from $p_t \mathbf{w}_t^\top / p_t^* = p_t^* \delta_t \mathbf{w}_t^\top / p_t^* = \delta_t \mathbf{w}_t^\top$. The model in (15) also can be written in terms of the observable data matrix \mathbf{W}_2 as

$$(16) \quad \begin{aligned} \mathbf{D}(p_t)\mathbf{x} &= [\mathbf{p}, \mathbf{W} + \mathbf{D}(\delta_t)\mathbf{W} - \mathbf{W}, \mathbf{D}(p_t)\mathbf{Z}]\theta + \mathbf{D}(p_t)\varepsilon \\ &= [\mathbf{p}, \mathbf{W}, \mathbf{D}(p_t)\mathbf{Z}]\theta + (\mathbf{D}(\delta_t) - \mathbf{I}_T)\mathbf{W}\beta + \mathbf{D}(p_t)\varepsilon \\ &\equiv \mathbf{W}_2\theta + \xi, \end{aligned}$$

where $\xi = (\mathbf{D}(\delta_t) - \mathbf{I}_T)\mathbf{W}\beta + \mathbf{D}(p_t^*)\mathbf{D}(\delta_t)\varepsilon$, which satisfies $E(\xi) = \mathbf{0}$ and

$$(17) \quad E(\xi\xi^\top) = (v-1)\mathbf{D}((\mathbf{w}_t^\top\beta)^2) + v\sigma^2\mathbf{D}(p_t^{*2}).$$

The composite error vector ξ is thus heteroskedastic and the individual variances are both data and parameter dependent.

The least squares estimator of this model using observed (EIV) data is

$$\begin{aligned}
 \hat{\theta}_2 &= (\mathbf{W}_2^\top \mathbf{W}_2)^{-1} \mathbf{W}_2^\top (\mathbf{D}(p_t) \mathbf{x}) \\
 (18) \quad &= (\mathbf{W}_2^\top \mathbf{W}_2)^{-1} \mathbf{W}_2^\top (\mathbf{W}_2 \theta + \xi) \\
 &= \theta + (\mathbf{W}_2^\top \mathbf{W}_2)^{-1} \mathbf{W}_2^\top \xi.
 \end{aligned}$$

Unlike the simple model with no intercept or non-price regressors, the least squares estimator is now inconsistent because some right hand side variables are correlated with the error term:

$$(19) \quad \text{plim } \hat{\theta}_2 = \theta + \left[\text{plim} \left\{ \frac{1}{T} \mathbf{W}_2^\top \mathbf{W}_2 \right\} \right]^{-1} \text{plim} \left\{ \frac{1}{T} \mathbf{W}_2^\top \xi \right\} \neq \theta.$$

In particular, the covariance between \mathbf{W}_2 and ξ includes the terms

$$(20) \quad \text{plim} \left\{ \frac{1}{T} \sum_t p_t \xi_t \right\} = (v-1) \text{plim} \left\{ \frac{1}{T} \sum_t p_t^* \mathbf{w}_t^\top \beta \right\} \equiv (v-1) \beta^\top \boldsymbol{\Omega}_{p^* \mathbf{w}},$$

and

$$(21) \quad \text{plim} \left\{ \frac{1}{T} \sum_t p_t \mathbf{Z}_t \xi_t \right\} = (v-1) \text{plim} \left\{ \frac{1}{T} \sum_t p_t^* \mathbf{Z}_t (\mathbf{w}_t^\top \beta) \right\} \equiv (v-1) \boldsymbol{\Omega}_{p^* \mathbf{z} (\mathbf{w}^\top \beta)}.$$

Neither vanishes, in general. When measurement error is present, $v-1 > 0$ because this is the variance of the measurement error, so that least squares will be inconsistent.

However, IV/GMM can be applied as a simple alternative to least squares to obtain consistent estimates.⁹ Using the $T \times K$ data matrix $\mathbf{Z}^* = [\mathbf{1}, \mathbf{W}, \mathbf{Z}]$ as instruments, where $\mathbf{1}$ is a $T \times 1$ vector of ones and \mathbf{Z}^* is presumed independent of ε and δ , yields

$$(22) \quad E \left[\mathbf{Z}^{*\top} \mathbf{D}(p_t) \mathbf{x} \right] = E \left[\mathbf{Z}^{*\top} (\mathbf{W}_2 \theta + \xi) \right] = \mathbf{Z}^{*\top} E(\mathbf{W}_2) \theta.$$

The expectation on the far right hand side follows from equation (14). The IV estimator of θ we propose for this exactly identified case is

$$(23) \quad \hat{\theta}_{\text{IV}} = \left[\mathbf{Z}^{*\top} \mathbf{W}_2 \right]^{-1} \mathbf{Z}^{*\top} \mathbf{D}(p_t) \mathbf{x} = \left[\mathbf{Z}^{*\top} \mathbf{W}_2 \right]^{-1} \mathbf{Z}^{*\top} (\mathbf{W}_2 \theta + \xi) = \theta + \left[\mathbf{Z}^{*\top} \mathbf{W}_2 \right]^{-1} \mathbf{Z}^{*\top} \xi.$$

Note that $\text{plim } \hat{\theta}_{\text{IV}} = \theta$ because

$$(24) \quad \text{plim} \left\{ \frac{1}{T} \mathbf{Z}^{*\top} \mathbf{W}_2 \right\} = \text{plim} \frac{1}{T} \begin{bmatrix} \mathbf{t}^\top \mathbf{p}^* & \mathbf{t}^\top \mathbf{W} & \mathbf{t}^\top \mathbf{D}(p_t^*) \mathbf{Z} \\ \mathbf{W}^\top \mathbf{p}^* & \mathbf{W}^\top \mathbf{W} & \mathbf{W}^\top \mathbf{D}(p_t^*) \mathbf{Z} \\ \mathbf{Z}^\top \mathbf{p}^* & \mathbf{Z}^\top \mathbf{W} & \mathbf{Z}^\top \mathbf{D}(p_t^*) \mathbf{Z} \end{bmatrix} = \text{plim} \left\{ \frac{1}{T} \mathbf{Z}^{*\top} \mathbf{W}_2^* \right\},$$

while

$$(25) \quad \text{plim} \left\{ \frac{1}{T} \mathbf{Z}^{*\top} \mathbf{D}(p_t) \boldsymbol{\varepsilon} \right\} = \text{plim} \left\{ \frac{1}{T} \mathbf{Z}^{*\top} \mathbf{D}(p_t^*) \mathbf{D}(\delta_t) \boldsymbol{\varepsilon} \right\} = \mathbf{0},$$

because $E(\delta_t \varepsilon_t) = 0 \quad \forall t = 1, \dots, T$ and \mathbf{Z}^* is independent of $\boldsymbol{\delta}$ and $\boldsymbol{\varepsilon}$.

Importantly, this approach does not require finding an instrument and assuming it is correlated with right hand side variables. We use as instruments only data at hand in the regression model. Thus, instrumental variables (GMM) using only the available data can be used in a straightforward way to consistently estimate the parameters of the conditional mean, $\boldsymbol{\theta}$.¹⁰

Estimating the Magnitude of Measurement Error

The results thus far imply that linearizing the homogeneous system in (11) permits consistent estimation with IV/GMM without requiring additional data. Moreover, while not the focus of the EIV literature in economics, this approach also permits consistent estimation of ν by including an additional moment for p_t . This moment is far from having the conventional IV interpretation using cross moments of errors and exogenous variables. In particular, multiplying both sides of model 2 by p_t^2 yields

$$(26) \quad p_t^2 x_t = \alpha p_t^2 + p_t \delta_t \mathbf{w}_t^\top \boldsymbol{\beta} + p_t^2 \mathbf{z}_t^\top \boldsymbol{\gamma} + p_t^2 \varepsilon_t, t = 1, \dots, T$$

Applying the diagonal matrix notation above gives

$$(27) \quad \mathbf{D}(p_t^2) \mathbf{x} = \mathbf{D}(p_t) \mathbf{W}_2^* \boldsymbol{\theta} + \tilde{\boldsymbol{\varepsilon}},$$

with $\tilde{\boldsymbol{\varepsilon}} = \mathbf{D}(p_t^2) \boldsymbol{\varepsilon}$, $E(\tilde{\boldsymbol{\varepsilon}}) = \mathbf{0}$, and $E(\tilde{\boldsymbol{\varepsilon}} \tilde{\boldsymbol{\varepsilon}}^\top) = E(\delta_t^4) E(\varepsilon_t^2) \mathbf{D}(p_t^{*4}) \equiv \kappa \sigma^2 \mathbf{D}(p_t^{*4})$. In turn, (27) can be written in terms of the observable data, \mathbf{W}_2 , as

$$(28) \quad \mathbf{D}(p_t^2) \mathbf{x} = \mathbf{D}(p_t) \mathbf{W}_2 \tilde{\boldsymbol{\theta}} + \tilde{\boldsymbol{\xi}},$$

where $\tilde{\boldsymbol{\theta}} = [\alpha, \nu \boldsymbol{\beta}^\top, \boldsymbol{\gamma}^\top]^\top$ and $\tilde{\boldsymbol{\xi}} = \mathbf{D}(p_t) (\mathbf{D}(\delta_t) - \nu \mathbf{I}_T) \mathbf{W} \boldsymbol{\beta} + \mathbf{D}(p_t^2) \boldsymbol{\varepsilon}$ satisfies $E(\tilde{\boldsymbol{\xi}}) = \mathbf{0}$ and

$$(29) \quad E(\tilde{\xi}\tilde{\xi}^\top) = (\kappa - v^2)\mathbf{D}\left((p_t^* \mathbf{w}_t^\top \boldsymbol{\beta})^2\right) + \kappa\sigma^2 \mathbf{D}(p_t^{*4}).$$

The composite error vector $\tilde{\xi}$ is again heteroskedastic and the individual variances are both data and parameter dependent. One simple IV estimator for v from (28) would be to use ι as the only instrument, which gives the moment condition,

$$(30) \quad \iota^\top \mathbf{D}(p_t^2) \mathbf{x} = \iota^\top \mathbf{D}(p_t) \mathbf{W}_2 \hat{\boldsymbol{\theta}}_{\text{IV}}.$$

Simple IV estimators for $(\boldsymbol{\theta}, v)$ can then be found as the joint solutions to (23) and (30).

Even if one ignores the covariance structure, simple IV using (23) and (30) produces a consistent estimator of $\tilde{\boldsymbol{\theta}}$. Moreover, because a consistent estimator of $\boldsymbol{\theta}$ is available from (23), a consistent estimator of v is obtained by using ι as the only instrument and solving (30) for \hat{v}_{IV} in terms of $\hat{\boldsymbol{\theta}}_{\text{IV}}$. Including the additional instruments in \mathbf{Z}^* leads to an over-identified system. If ι is used as the only instrument, then from (30) an appropriate estimator of v is

$$(31) \quad \hat{v}_{\text{IV}} = [\iota^\top \mathbf{D}(p_t) \mathbf{W} \hat{\boldsymbol{\beta}}_{\text{IV}}]^{-1} \iota^\top \mathbf{D}(p_t^2) [\mathbf{x} - (\iota \hat{\alpha}_{\text{IV}} + \mathbf{Z} \hat{\boldsymbol{\gamma}}_{\text{IV}})] = 1 + (\mathbf{p}^\top \mathbf{W} \hat{\boldsymbol{\beta}}_{\text{IV}})^{-1} \mathbf{p}^\top \hat{\xi}_{\text{IV}}.$$

Since $\text{plim}(\frac{1}{T} \mathbf{p}^\top \hat{\xi}_{\text{IV}}) = (v-1) \boldsymbol{\beta}^\top \boldsymbol{\Omega}_{p^*w}$ and $\text{plim}(\frac{1}{T} \mathbf{p}^\top \mathbf{W} \hat{\boldsymbol{\beta}}_{\text{IV}}) = \boldsymbol{\beta}^\top \boldsymbol{\Omega}_{p^*w}$, \hat{v}_{IV} consistently estimates v .

The structure of the covariance matrix is informative. For the two-equation system composed of (16) and (28), it includes, in addition to (17) and (29), the covariance between ξ and $\tilde{\xi}$,

$$(32) \quad E(\xi\tilde{\xi}^\top) = (\eta - v^2) \mathbf{D}(p_t^*) \mathbf{D}\left((\mathbf{w}_t^\top \boldsymbol{\beta})^2\right) + \eta\sigma^2 \mathbf{D}(p_t^{*3}),$$

where $\eta = E(\delta_t^3)$. Thus, the 2×2 covariance matrix for the t^{th} observation is

$$(33) \quad \boldsymbol{\Omega}_t = E \begin{bmatrix} \xi_t^2 & \xi_t \tilde{\xi}_t \\ \xi_t \tilde{\xi}_t & \tilde{\xi}_t^2 \end{bmatrix} = \begin{bmatrix} (v-1)(\mathbf{w}_t^\top \boldsymbol{\beta})^2 + v\sigma^2 p_t^{*2} & (\eta - v^2) p_t^* (\mathbf{w}_t^\top \boldsymbol{\beta})^2 + \eta\sigma^2 p_t^{*3} \\ (\eta - v^2) p_t^* (\mathbf{w}_t^\top \boldsymbol{\beta})^2 + \eta\sigma^2 p_t^{*3} & (\kappa - v^2) (p_t^* \mathbf{w}_t^\top \boldsymbol{\beta})^2 + \kappa\sigma^2 p_t^{*4} \end{bmatrix},$$

which depends on the data, so the errors are clearly heteroskedastic. Though (33) is somewhat messy, it is easy to see that a two-stage technique implies that all of the parameters can be identified and estimated consistently.¹¹

Most conventional GMM software packages cannot be used to estimate this system because they assume the same instruments are used in each equation.¹² However, conventional GMM software can be applied to the system composed of (16) and (28) with \mathbf{Z}^* as

instruments in both equations to obtain consistent parameter estimates. This system of $2K$ equations is over-identified with $K+1$ parameters and $K-1$ over-identifying restrictions.

Efficient Estimation of Over-Identified Systems

One can proceed with the typical IV representation as in the previous section or by using a GMM framework. We use the latter approach here because valid inference requires accounting for the covariance matrix in (33). The vector of sample moments implied by (16) and (28) can be written as

$$(34) \quad \begin{aligned} \mathbf{G}_1(\boldsymbol{\theta}) &\equiv \frac{1}{T} \mathbf{Z}^{*\top} (\mathbf{D}(p_t)\mathbf{x} - \mathbf{W}_2\boldsymbol{\theta}) \equiv \frac{1}{T} \mathbf{Z}^{*\top} \boldsymbol{\xi}, \\ \mathbf{G}_2(\tilde{\boldsymbol{\theta}}) &\equiv \frac{1}{T} \mathbf{Z}^{*\top} (\mathbf{D}(p_t^2)\mathbf{x} - \mathbf{D}(p_t)\mathbf{W}\tilde{\boldsymbol{\theta}}) \equiv \frac{1}{T} \mathbf{Z}^{*\top} \tilde{\boldsymbol{\xi}}. \end{aligned}$$

Let $\mathbf{G}(\tilde{\boldsymbol{\theta}}) = [\mathbf{G}_1(\boldsymbol{\theta})^\top \mathbf{G}_2(\tilde{\boldsymbol{\theta}})^\top]^\top$. Then, under fairly general regularity conditions, the GMM estimator (e.g., Newey and McFadden 1994),

$$(35) \quad \hat{\boldsymbol{\theta}} = \arg \min \mathbf{G}(\tilde{\boldsymbol{\theta}})^\top \mathbf{A} \mathbf{G}(\tilde{\boldsymbol{\theta}}),$$

will be consistent where \mathbf{A} is a $2K \times 2K$ weighting matrix and K is the number of instruments. Efficiency requires that \mathbf{A} is proportional to the inverse of the covariance matrix of moments. Thus, with \mathbf{Z}^* as instruments and no serial correlation in either ξ_t or $\tilde{\xi}_t$,

$$(36) \quad \mathbf{A}^{-1} = \frac{1}{T^2} (\mathbf{Z}^{*\top} \otimes \mathbf{I}_2) \boldsymbol{\Omega} (\mathbf{Z}^* \otimes \mathbf{I}_2).$$

where $\boldsymbol{\Omega} = \text{cov}(\boldsymbol{\xi}, \tilde{\boldsymbol{\xi}})$ is a $2T \times 2T$ covariance matrix with (33) as a typical 2×2 block for the t^{th} observation, \mathbf{I}_2 is the 2×2 identity matrix, and \otimes is the Kronecker product for matrix multiplication. The point of (33) and (36) is that the error covariance matrix is non-scalar, even in the absence of serial correlation. Statistical tests will be inconsistent unless this attribute of the model is properly considered. Serial correlation further complicates valid inferences. However, the heteroskedasticity-autocorrelation consistent (HAC) approach of Newey and West (1987) is a useful way to account for general non-scalar error covariance structures.

A Simple Monte-Carlo Study Comparing Models 1 and 2 Given EIV

Although OLS and conventional instrumental variables (CIV) produce inconsistent estimates for $\boldsymbol{\theta}$ while simple GMM estimates are consistent, small sample comparisons of bias and mean-square error are also of interest. Tables 1 and 2 present some Monte-Carlo comparisons among the three estimators: OLS, CIV and GMM (IV on the transformed model using White's heteroskedasticity consistent covariance matrix estimator). As al-

ways, some inductive lessons can be learned, but care must be taken in extrapolating the results to other data generating processes. In all cases, the parameters are $\alpha = 5$; $\beta_1 = 3$; $\beta_2 = 2$; $\gamma_1 = 1.5$; and $\gamma_2 = 2.5$. The data for the matrix of regressors were generated from independent uniform distributions and are fixed in repeated samples. The relevant sample means are 2.15 for w_1/p^* , 3.70 for w_2/p^* , 15.03 for z_1 , and 11.78 for z_2 . With this orthogonal design, the estimates of γ should include no systematic bias, which provides a way to check the effects of sampling and computer related (e.g., rounding or random number generator) issues.

This data structure generates attenuation bias for OLS estimators of coefficients of the deflated input prices because of orthogonality in the generated right hand side variables. Regression errors are generated from a normal distribution with mean 0 and standard deviation 15. The measurement errors, δ , are generated from a uniform distribution with mean 1 and varying levels of ν as noted in the tables that follow. The number of observations, T , varies from 45, to 200, to 4500 with 2500 replications each. To compare the estimators on a relatively equal footing with a parsimonious number of moments, the GMM moment conditions used, analogous to (34), are

$$(37) \quad \begin{aligned} G_1(\theta) &= \frac{1}{T} \mathbf{Z}^{*\top} \xi = \frac{1}{T} \mathbf{Z}^{*\top} (\mathbf{D}(p_t) \mathbf{x} - \mathbf{D}(p_t) \iota \alpha - \mathbf{W} \beta - \mathbf{D}(p_t) \mathbf{Z} \gamma), \\ G_2(\tilde{\theta}) &= \frac{1}{T} \mathbf{t}^\top \tilde{\xi} = \frac{1}{T} \mathbf{t}^\top (\mathbf{D}(p_t^2) \mathbf{x} - \mathbf{D}(p_t^2) \iota \alpha - \mathbf{D}(p_t) \mathbf{W} \beta \nu - \mathbf{D}(p_t^2) \mathbf{Z} \gamma). \end{aligned}$$

The first column of results in table 1 is from an OLS regression using the observed data (see (5)). The second column is the conventional linear IV estimator with the mis-measured price as the numeraire. This estimator is inconsistent as shown in (10). The third column of estimates contains the GMM/IV results using a vector of ones, \mathbf{w} , and \mathbf{z} as instruments (\mathbf{Z}^*) analogous to (23) in order to estimate θ . Also, a vector of ones analogous to (30) is used to additionally estimate ν . (Note that different instruments are being used for the two equations as in (37)). The first set of results in the upper portion of the table contains relatively little measurement error variance with $V(\delta) = \nu - 1 = .01$. For a small variance of measurement error, OLS might be expected to dominate because in the limit with no measurement error OLS is unbiased and has minimum mean-squared error (MSE). As is apparent from table 1, the intercept has the largest bias across all methods. Focusing on the estimates of β , moving from left to right, all OLS slope estimates have substantial attenuation bias. For a sample size of 45 and measurement error variance .01, the biases of OLS estimates of β are approximately 4 to 7 times higher than GMM estimates. This increases to as much as 7 times higher for $T = 200$ and approximately 20 times higher when the sample size is 4500. Thus, increasing sample size for a given value of ν leads to larger relative biases for OLS. With small measurement error variance ($\nu - 1 = .01$), the CIV estimator generally has smaller bias than OLS and also smaller bias than GMM for sample sizes 45 and 200. But for the large sample size, CIV has lower bias than OLS but GMM dominates both OLS and CIV.

For measurement error variance $\nu - 1 = .10$, the same basic pattern emerges. Attenua-

tion bias is large for OLS and GMM dominates OLS. However, in contrast to the case with small measurement error, OLS bias can be 140 times larger than GMM for large T . Further, the biases of OLS and CIV are substantially larger compared to the case with small error variance. A similar pattern emerges qualitatively for measurement error variance $\nu - 1 = .20$. As T gets large, the bias is as high as 250 times larger for OLS than GMM and near 50 times larger for CIV relative to GMM.

Evidently from table 1, GMM performs well in terms of bias at $T = 200$. While GMM performs reasonably well for small sample sizes and small measurement errors, it does not do very well for small sample sizes and large measurement errors. However, perusing the lower left hand portion of the table, GMM continues to outperform OLS and CIV in these cases as well.

Estimates of ν by GMM also seem to have reasonable bias properties. However, the true value of ν is only slightly larger than 1.0 so a $-.07$ bias is approximately a 6% bias. When sample sizes are 200 or more, ν appears to be estimated reasonably well.

Table 2 contains the corresponding MSE calculations. Indeed for low and moderate measurement error and low sample sizes, OLS (and sometimes CIV) dominates GMM in MSE for β and α even though the opposite is true for bias. When sample size increases to $T = 200$, the MSE of OLS is also smaller than GMM and CIV for small measurement error ($\nu - 1 = .01$). However, for moderate and high measurement error, the MSE of GMM dominates that of OLS. For large samples ($T = 4500$), GMM does not dominate CIV or OLS for small measurement error (when ν is small OLS does as well as GMM and CIV does better). When measurement error is moderate to large, GMM clearly dominates both CIV and OLS. Finally, moving left to right, the GMM estimator behaves as though it is mean-square error consistent in contrast to OLS and CIV.

Given that OLS and CIV may be MSE dominant for some levels of measurement error and sample sizes, but can have considerable bias, an analysis of the effects on inference conventionally associated with these three approaches is useful. Table 3 contains these simulations. For OLS, the variance of the estimator is estimated as

$$(38) \quad \hat{V}(\hat{\theta}_{OLS}) = [(x - W_1 \hat{\theta}_{OLS})^\top (x - W_1 \hat{\theta}_{OLS}) / (N - 5)] (W_1^\top W_1)^{-1},$$

while the estimate of the CIV variance is

$$(39) \quad \hat{V}(\hat{\theta}_{CIV}) = [(x - W_1 \hat{\theta}_{CIV})^\top (x - W_1 \hat{\theta}_{CIV}) / (N - 5)] (Z^{*\top} W_1)^{-1} (Z^{*\top} Z^*) (W_1^\top Z^*)^{-1}.$$

Because all data are drawn independently from identical distributions, (17) implies that ξ and $\tilde{\xi}$ are heteroskedastic but are not serially correlated.¹³ The conventional GMM heteroskedasticity consistent estimator is used in the simulation of GMM tests. The asymptotic White robust covariance matrix of moments is estimated by

$$(40) \quad \hat{\Omega} = 1/T^2 \begin{bmatrix} Z^{*\top} D(e_i^2) Z^* & Z^{*\top} D(e_i \tilde{e}_i) \mathbf{1} \\ \mathbf{1}^\top D(e_i \tilde{e}_i) Z^* & \mathbf{1}^\top D(\tilde{e}_i^2) \mathbf{1} \end{bmatrix}$$

where \mathbf{e} and $\tilde{\mathbf{e}}$ are vectors of estimated residuals corresponding to ξ and $\tilde{\xi}$, respectively. The remaining component is the matrix of gradients of the moment conditions,

$$(41) \quad \Phi \equiv \frac{\partial \mathbf{G}}{\partial \tilde{\boldsymbol{\theta}}^\top} = \begin{bmatrix} \frac{\partial \mathbf{G}_1}{\partial \boldsymbol{\theta}^\top} & \frac{\partial \mathbf{G}_1}{\partial v} \\ \frac{\partial \mathbf{G}_2}{\partial \boldsymbol{\theta}^\top} & \frac{\partial \mathbf{G}_2}{\partial v} \end{bmatrix} = \begin{bmatrix} 1/T \mathbf{Z}^{*\top} \mathbf{W}_2 & \mathbf{0}_K \\ 1/T \mathbf{t}^\top \mathbf{W}_3 & 1/T \mathbf{t}^\top \mathbf{D}(p_t) \mathbf{W} \boldsymbol{\beta} \end{bmatrix},$$

where $\mathbf{W}_3 = [\mathbf{D}(p_t) \mathbf{p}, v \mathbf{D}(p_t) \mathbf{W}, \mathbf{D}(p_t^2) \mathbf{Z}]$, implying the robust covariance estimate is

$$(42) \quad \hat{V}(\hat{\boldsymbol{\theta}}_{GMM}) = \hat{\Phi} \hat{\Omega}^{-1} \hat{\Phi}^\top.$$

Using $z = |\hat{\theta}_{iGMM} - \tilde{\theta}_i| / \sqrt{\hat{V}_{ii}(\hat{\boldsymbol{\theta}}_{GMM})}$, $i = 1, \dots, 5$, (where \hat{V}_{ii} is the i^{th} diagonal element of \hat{V}) and rejecting when $z > 1.96$, implying a size of .05 for type I errors under the null hypothesis that the individual parameter is the true value, table 3 reports the size of the three tests corresponding to (38)–(42). Traditional OLS inference produces relatively reliable sizes when measurement error and sample size are low and for tests on parameters of non-price coefficients. At $T = 200$ and $v - 1 = .01$, type I errors are too large by almost .04 for one estimate of $\boldsymbol{\beta}$. Increasing the sample size beyond 200 provides very unreliable tests with rejection occurring much more frequently than is warranted. Increasing measurement error to moderate levels further increases unreliability of tests for both α and $\boldsymbol{\beta}$ yielding virtually certain rejection in some cases (e.g., the rejection probability of 1.0 for β_1 at $T=45$ and $v - 1 = .20$). Moderate to large measurement error with large sample sizes exacerbates the problem, as evidenced by the test sizes of 1.0 for α and $\boldsymbol{\beta}$. Thus, ignoring EIV and estimating by OLS with the conventional covariance formula leads to a useless rate of over-rejection of tests for zero price parameters.

Using the CIV approach in (39) yields empirical sizes that are worse than OLS for the small sample size and small measurement error, but biased toward non-rejection. However, as the sample size grows to 200, CIV outperforms OLS and does so demonstrably for $T = 4500$ (compare the empirical size of 0.620 for OLS to 0.052 for CIV for β_1). Across all sample sizes, for small measurement error, GMM performs about as well as or better than conventional approaches. For moderate and high levels of measurement error ($v - 1 = .10$ or $.20$), the GMM approach impressively dominates both OLS and CIV for tests on both α and $\boldsymbol{\beta}$.

Overall, these results from Tables 1-3 give some confidence upon which to proceed with inference using robust t -tests accounting for heteroskedasticity, at least for sample sizes of 200 observations or more. These simulations show that small sample sizes and high levels of measurement error lead to imprecise and highly variable estimates by each of the methods. However, for small sample sizes and low but positive levels of measure-

ment error, GMM out-performs OLS in terms of bias (location) but is worse in terms of mean-squared error. In general, hypothesis tests have poor size unless a method is used that centers the estimates on the true parameter values, at least asymptotically. Applying GMM/IV on the transformed regression equation yields such an approach. For sample sizes of approximately 200, the GMM estimates have reasonably good bias properties and yield proper sizes for conventional tests when using a robust heteroskedasticity consistent estimator of the parameter covariance matrix. However, it is well-known that inference based upon GMM estimates will often lead to an over-rejection of the null hypothesis in small samples (e.g., Burnside and Eichenbaum 1996). This is consistent with our simulation results.

The general conclusion in the literature seems to be that the usual first-order asymptotic theory can give poorly behaved estimates of the variance of the efficient GMM estimator in small samples. This has led researchers to consider more refinements of asymptotic theory (more exacting expansions) or simply to bootstrap standard errors. Bootstraps of t-ratios for over-identified systems require that moment conditions hold exactly whereas GMM, which is a minimum distance estimator, only requires them to hold approximately. This requires a re-centering procedure (Hall and Horowitz 1996) or a re-weighting procedure (Brown and Newey 2002). In these as well as a wide variety of cases, bootstrapping has been shown to be of substantial value in small samples. Though we do not provide bootstrap estimates of our empirical application below due to the large and complex system and relatively small number of temporal observations, we expect that a properly executed block bootstrap would likely improve inference in most cases (Horowitz 2001).

A final caution must be added to the conclusions of our simulations. Many of the applications of the type considered here (price deflation) are in systems. Indeed, the application that follows is a 5-equation system.¹⁴ Here multiple equations provide information on shared parameters and can be estimated more precisely if the standard system assumptions are valid. Thus, there may be reasons to be less sanguine about the conclusions of the Monte-Carlo analysis regarding sample size. In the system context, there are not 200 observations on the systems data generating process. Rather the 200 observations are divided among equations. Thus, while the system degrees of freedom may be high as measured conventionally by $5T-K$ where K is the total number of parameters in the system and T is the number of annual time periods, there is reason to be cautious about extrapolation of the single equation results to a 5 equation system where many issues come into play (e.g., Binkley and Nelson 1988; Fiebig and Kim 2000).

An Empirical Illustration

Finally, we illustrate the approach discussed here using the data set in Ball, et al. (1997). This data set may not be the best showcase of these methods, but it is perhaps the most widely used data set for estimation of technical change, productivity, and factor demands and output supplies in U.S. agriculture. The data are annual, 1948-1994, for U.S. aggregate agriculture. The single output data has seven inputs: chemicals, fuels, feeds, hired labor, other inputs, self-employed labor, and capital. A time trend is included to account

for technical change.¹⁵ Using Lau's (1978) normalized quadratic profit function (e.g., Shumway 1983), the conventional normalized profit function is

$$(43) \quad \begin{aligned} \pi / p = \bar{\pi}(\mathbf{w} / p, \mathbf{z}) = & \alpha_0 + \sum_{i=1}^{K_1} \alpha_i \bar{w}_i + \sum_{j=1}^{K_2} \lambda_j z_j + \frac{1}{2} \sum_{i=1}^{K_1} \sum_{j=1}^{K_1} \beta_{ij} \bar{w}_i \bar{w}_j \\ & + \sum_{i=1}^{K_1} \sum_{j=1}^{K_2} \gamma_{ij} \bar{w}_i z_j + \sum_{i=1}^{K_2} \sum_{j=1}^{K_2} \tau_{ij} z_i z_j \end{aligned}$$

where $\bar{\mathbf{w}} = \mathbf{w} / p$, \mathbf{z} is a vector of fixed inputs, K_1 is the number of variable inputs, K_2 is the number of fixed inputs and technical change, and the t subscripts are dropped for convenience. The envelope theorem yields:

$$(44) \quad \frac{\partial \bar{\pi}}{\partial \bar{w}_i} = -x_i = \alpha_i + \sum_{j=1}^{K_1} \beta_{ij} \bar{w}_j + \sum_{j=1}^{K_2} \gamma_{ij} z_j, \quad i = 1, \dots, K_1.$$

One can also add the supply equation and/or profit function, which are both highly nonlinear in p . The simple approach here does not estimate all parameters, but estimates the factor demand parameters and elasticities consistently, as is common. Multiplying by p and adding t subscripts yields the GMM estimating equations as in (15):

$$(45) \quad -p_t x_{it} = p_t \alpha_i + \sum_{j=1}^{K_1} \beta_{ij} w_{jt} + \sum_{j=1}^{K_2} \gamma_{ij} p_t z_{jt} + \xi_{it}; \quad i = 1, \dots, K_1; \quad t = 1, \dots, T.$$

The variable inputs (\mathbf{x}) are chemicals, fuels, feeds, hired labor, and other inputs, while \mathbf{z} is composed of time ($t = 1, \dots, T$), self-employed labor, and capital. We assume regression errors have zero means, $E(\xi_{it}) = 0$, $i = 1, \dots, K_1$, $t = 1, \dots, T$, instruments have classical properties, the EIV assumptions in (4) hold, and $(1, \mathbf{w}, \mathbf{z})$ and lags on \mathbf{w} and \mathbf{z} are appropriate instruments.¹⁶ This yields many degrees of freedom as normally calculated because the system has 230 observations with 35 parameters.¹⁷ However, as noted earlier, this likely overstates the adequacy of data given that only 47 time series observations are available for each equation in the system. To account for the possibility of serially correlated errors, a Bartlett Kernel (lag 2) is used to obtain Newey-West robust estimates of \mathbf{A} .¹⁸

The second and third columns of table 4 present the GMM/IV estimates corresponding to (23). The next two columns present seemingly unrelated regression (SUR) estimates and the associated conventional estimates of standard errors, which correspond in spirit to the OLS case in (18). (The last two columns are considered later.) The purpose of comparing these two sets of estimates is not to provide convincing evidence that GMM is superior because, if EIV exists, no other studies are available with which to judge the adequacy of either model (which is one reason for the above Monte-Carlo study). Rather, the purpose is to show how different the estimates can be. The estimates differ substan-

tially.

Standard errors are, for the most part, lower for GMM for both variable input prices and fixed factors, although not always so. Both methods yield negative factor demand slopes. However, the magnitudes of the individual coefficients are vastly different. For example, chemical price has an unusually large marginal response by SUR compared to GMM. Conversely, fuel price has a much stronger marginal own price effect by GMM than SUR. Nine of ten cross-price coefficients are negative according to SUR, indicating a high degree of substitution, while only four are negative by GMM. At this level of aggregation, the substitution pattern in SUR seems unlikely. For example, chemicals and feeds (β_{13}) are likely not substitutes as estimated by SUR. In contrast, they are strong substitutes as estimated by GMM. As for fixed factor effects, chemicals and capital (γ_{12}) are estimated to be statistically significant complements by GMM, but are estimated to be substitutes by SUR. Fuels and technology (γ_{21}) are significant complements indicating that technology is fuel-using by GMM, but are estimated to be substitutes by SUR. Presumably, technical change has been fuel-using as more tasks have been mechanized and hand labor has been displaced. Finally, there are markedly more statistically significant coefficients for GMM versus SUR. This may partly reflect the attenuation bias that is likely for SUR estimates, but may also occur for the same reasons as the large mean-squared errors in table 2 for OLS with larger sample sizes or the tendency of GMM methods to over-reject in small samples.

A quantitative comparison of interest is that of estimated elasticities. The respective elasticity estimates at the sample means for chemicals, fuels, feed, other inputs and hired labor are -0.215 , -2.936 , -0.039 , -1.116 and -0.228 , respectively, by GMM, and -1.160 , -0.823 , -0.287 , -1.240 and -0.159 by SUR. Thus, GMM gives more elastic responses for fuels and hired labor but less elastic response for all other categories of inputs.

Finally, the last two columns of table 4 give parameter estimates analogous to (36) and (37), which use GMM with higher-order moments to obtain estimates of ν .¹⁹ Comparing these GMM estimates with the first two columns of table 4, the estimates with higher-order moments appear more noisy as judged by the magnitude of standard errors. The number of statistically significant coefficients, aside from the second moment of measurement error, falls from 22 to 15 but is still substantially larger than with SUR. (Recall that the size of the SUR tests are inappropriate as in table 3.) The pattern of the estimates between the two GMM cases is generally similar. All factor demands are negatively sloped.²⁰

The most interesting result is that measurement error as represented by ν is estimated to be 1.0015 with an approximate standard error of .00036.²¹ While this estimate may appear to be very small ($\nu = 1$ represents no measurement error), it implies a standard deviation of measurement error of a very plausible 3.9 percent. A test of the null hypothesis that the variance of measurement error is zero yields a t -statistic of 4.18. Hence, even at a significance level of .001, no measurement error is soundly rejected.

Conclusions

This article has proposed an approach to the estimation of systems of production equa-

tions where a numeraire price is measured with error. Although, one might suspect measurement error in all data used by economists, output price measurement error is particularly likely in agriculture where most input decisions are made before output price is known. This causes a nonlinear EIV problem because output price is typically used to deflate input prices. This problem is exacerbated if output represents an aggregate so that an output price index must be used.

This article has developed a simple way to estimate a production system where degree-zero homogeneity in prices is imposed in a linear form. The approach yields consistent estimates of the parameters without relying on extra sample data. An extension of the approach further yields a method of estimating the variance of measurement error.

Monte-Carlo simulations illustrate the viability of the approach for various sample sizes and magnitudes of measurement error. As the simulations show, with annual postwar data on a single equation one should be cautious about reaching conclusions about the magnitude of measurement error. Our results show that the approach is appropriate even in a single equation when the magnitude of the measurement error is unknown if the sample size is sufficient.

Estimation with aggregate annual agricultural data for the postwar period shows that correcting for measurement error in the output price has substantial effects on estimated coefficients and elasticities. Moreover, the magnitude of measurement error in a common aggregate price index is highly statistically significant.

Future research might consider more complete estimation of the production system. When a normalized quadratic production system is estimated, both the profit equation and the supply equation are highly nonlinear. If either of these is included in the estimated system when the price used for normalization is measured with error, the methods discussed here must be expanded. A fruitful direction may be to adapt a nonlinear two or three stage process as in Kelejian (1971) and Amemiya (1990) to address endogeneity issues. For example, if one wishes to estimate all of the parameters of the normalized profit function (e.g., the intercept), then it must be included. However, the profit function is a quadratic in the normalized prices. If the normalization is accomplished with output price as in Lau (1978), then both linear and quadratic terms in normalized prices will be measured with error. The simple approach derived here for factor demand estimation would not be directly applicable to such a system. However, an extension that applies similar principles of GMM estimation appears applicable.

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Table 1. Comparison of Estimators: Bias.

Parameter	OLS	CIV	GMM	OLS	CIV	GMM	OLS	CIV	GMM
	$T = 45, v - 1 = .01$			$T = 200, v - 1 = .01$			$T = 4500, v - 1 = .01$		
α	1.356	.209	.187	1.447	-.047	-.044	1.300	.050	.070
β_1	-.127	.001	-.030	-.148	-.041	-.021	-.148	-.036	-.008
β_2	-.085	-.011	-.013	-.069	-.001	.027	-.081	-.016	.004
γ_1	-.018	-.013	-.011	-.012	.004	.003	-.003	-.001	-.002
γ_2	-.009	-.018	.017	-.016	-.004	-.002	-.0003	-.0006	-.0008
v			.056			-.009			-.0004
	$T = 45, v - 1 = .10$			$T = 200, v - 1 = .10$			$T = 450, v - 1 = .10$		
α	11.925	.990	-2.479	13.547	-.806	-.343	11.449	.050	.079
β_1	-1.205	-.062	.251	-1.253	-.278	.008	-1.256	-.333	-.009
β_2	-.656	-.181	.123	-.697	-.161	.043	-.745	-.212	.007
γ_1	-.077	-.081	-.004	-.142	.016	.008	-.019	-.003	-.003
γ_2	-.172	-.025	.053	-.119	.015	-.004	.002	-.0007	-.007
v			-.011			-.012			-.0006
	$T = 45, v - 1 = .20$			$T = 200, v - 1 = .20$			$T = 4500, v - 1 = .20$		
α	22.698	-46.33	-11.34	24.960	-2.763	-.686	20.921	.017	.080
β_1	-2.144	1.802	1.671	-2.159	-.576	.042	-2.162	-.748	-.009
β_2	-1.223	-.219	.249	-1.274	-.363	.061	-.1.331	-.488	.009
γ_1	-.136	2.041	.117	-.266	.040	.012	-.034	-.005	.005
γ_2	-.339	-.436	-.012	-.210	-.015	.011	.004	-.0006	-.0006
v			-.070			-.005			-.0008

Note: See the text for definitions of variables and parameters of the simulations.

Table 2. Comparison of Estimators: Mean-Squared Error.

Parameter	OLS	CIV	GMM	OLS	CIV	GMM	OLS	CIV	GMM
	$T = 45, v - 1 = .01$			$T = 200, v - 1 = .01$			$T = 4500, v - 1 = .01$		
α	248.505	947.403	519.943	54.641	90.039	89.938	3.842	3.042	3.528
β_1	.950	5.207	3.759	.146	.368	.615	.027	.013	.022
β_2	.945	2.799	2.860	.134	.233	.422	.011	.008	.015
γ_1	.631	.681	.692	.141	.145	.163	.007	.007	.008
γ_2	.362	.559	.439	.068	.069	.077	.003	.003	.003
v			5.157			.002			<.0001
	$T = 45, v - 1 = .10$			$T = 200, v - 1 = .10$			$T = 4500, v - 1 = .10$		
α	431.814	1102040	932.977	246.753	151.53	118.460	133.907	5.641	4.767
β_1	2.409	4254.68	7.502	1.715	.620	.831	1.584	.128	.029
β_2	1.493	1971.87	4.525	.658	.380	.570	.561	.057	.020
γ_1	.788	296.83	1.041	.201	.254	.215	.009	.012	.010
γ_2	.441	296.77	.694	.099	.111	.102	.004	.005	.004
v			3.088			.004			.0001
	$T = 45, v - 1 = .20$			$T = 200, v - 1 = .20$			$T = 4500, v - 1 = .20$		
α	809.937	8770164	339616	686.237	356.03	154.469	20.921	.017	.08
β_1	5.278	33985.1	4652.82	4.760	1.316	1.109	-2.162	-.748	-.009
β_2	2.270	1902.02	5558.87	1.752	.769	.756	-1.331	-.488	.009
γ_1	.823	6176.02	26.71	.257	.562	.278	-.034	-.005	.005
γ_2	.534	678.70	48.45	.131	.228	.132	.004	-.0006	-.0006
v			30.68			.007			-.0008

Note: See the text for definitions of variables and parameters of the simulations.

Table 3. Size of Tests.

Parameter	OLS	CIV	GMM	OLS	CIV	GMM	OLS	CIV	GMM
	$T = 45, v - 1 = .01$			$T = 200, v - 1 = .01$			$T = 4500, v - 1 = .01$		
α	.0620	.0028	.0548	.0640	.0392	.0544	.148	.0524	.052
β_1	.0592	.0020	.0492	.0880	.0472	.0556	.620	.0524	.044
β_2	.0668	.0104	.0696	.0656	.0416	.0524	.250	.0544	.052
γ_1	.0540	.0284	.0768	.0460	.0404	.0548	.054	.0636	.049
γ_2	.0540	.0172	.0776	.0520	.0492	.0560	.052	.0520	.056
v			.0276			.0504			.048
	$T = 45, v - 1 = .10$			$T = 200, v - 1 = .10$			$T = 4500, v - 1 = .10$		
α	.0988	.0192	.0580	.3484	.0308	.0492	1.000	.0532	.0556
β_1	.3168	.0292	.0452	.9240	.1044	.0516	1.000	.682	.048
β_2	.1536	.0272	.0668	.4980	.0756	.0484	1.000	.505	.052
γ_1	.0504	.0192	.0684	.0476	.0372	.0516	.046	.052	.052
γ_2	.0420	.0180	.0636	.0536	.0360	.0544	.040	.046	.051
v			.0492			.0520			.053
	$T = 45, v - 1 = .20$			$T = 200, v - 1 = .20$			$T = 4500, v - 1 = .20$		
α	.2008	.0460	.0648	.7864	.0368	.0516	1.0000	.0572	.0584
β_1	.7284	.1112	.0536	1.0000	.2360	.0472	1.0000	.9840	.0532
β_2	.3568	.0636	.0680	.9392	.1740	.0468	1.0000	.9452	.0524
γ_1	.0360	.0060	.0612	.0456	.0312	.0504	.0276	.0528	.0524
γ_2	.0412	.0152	.0528	.0488	.0232	.0556	.0144	.0448	.0492
v			.0712			.0576			.0524

Note: See the text for definitions of variables and parameters of the simulations.

Table 4. Factor Demand Estimates for U.S. Agriculture.

Parameter	GMM w/o p_t^2 Moments		SUR Estimation		GMM with p_t^2 Moments	
	Parameter Estimates	Standard Errors	Parameter Estimates	Standard Errors	Parameter Estimates	Standard Errors
α_1	1.629	.808*	.897	1.461	3.4555	1.967*
α_2	3.772	1.615*	1.344	.956	3.977	2.244*
α_3	-.357	.440	-1.456	.698*	-1.292	.747*
α_4	3.167	1.003*	1.255	1.013	2.188	1.336
α_5	-2.581	1.553*	-1.359	1.085	-3.849	2.284*
β_{11}	2.350	4.240	12.688	20.219	4.817	3.494
β_{12}	1.216	1.407	-4.914	7.888	6.708	5.252
β_{13}	2.190	2.420	-.120	4.545	1.666	1.6851
β_{14}	.210	1.517	-.003	.032	.012	1.723
β_{15}	.825	1.830	-.121	4.545	-2.716	3.775
β_{22}	12.430	4.885*	3.483	4.581	9.102	6.366
β_{23}	-2.101	1.107*	-3.017	3.804	-5.007	2.505*
β_{24}	2.698	2.024	-1.426	5.787	-.802	3.384
β_{25}	-6.032	3.191*	-1.342	2.926	-8.845	5.460
β_{33}	.982	1.057	7.220	5.094	1.342	2.038
β_{34}	-2.197	1.123*	-.094	6.315	-3.814	1.552*
β_{35}	.532	1.355	4.936	2.597*	3.609	1.537*
β_{44}	11.461	2.346*	12.737	9.903	10.603	2.471*
β_{45}	-4.605	1.981*	-2.596	3.616	-4.464	2.441*
β_{55}	7.899	3.907*	5.486	3.525	12.429	5.459*
γ_{11}	-.039	.007*	-.003	.032	-.047	.016*
γ_{12}	-.380	.126*	.186	.505	-.595	.319*
γ_{13}	-1.471	.447*	-.623	.711	-2.574	1.186*
γ_{21}	-.028	.012*	.001	.015	-.008	.018
γ_{22}	-.516	.246*	-.083	.232	-.320	.322
γ_{23}	-2.640	.863*	-1.103	.475*	-2.817	1.282*
γ_{31}	-.007	.004*	-.019	.016	-.005	.012
γ_{32}	.006	.071	-.134	.246	.116	.198
γ_{33}	-.577	.240*	-.268	.317	-.099	.484
γ_{41}	-.037	.009*	-.020	.021	-.020	.014
γ_{42}	-1.131	.168*	-.877	.339*	-.874	.248*
γ_{43}	-1.268	.499*	-.330	.491	-.654	.631
γ_{51}	-.004	.008	-.021	.013	-.012	.014
γ_{52}	.243	.220	-.106	.189	0.203	.289
γ_{53}	.696	.871	-.410	.521	1.409	1.277
v					1.0015	.00036*

Note: Subscripts 1-5 refer to: chemicals, fuels, feeds, hired labor, and other inputs as in Ball, et al. (1997). Thus, β_{12} is the response of chemical demand to fuels price and α_1 is the intercept for the chemicals equation. All input quantities are measured as negatives as would appear directly by using the envelope theorem applied to a normalized profit function. Statistical significance at the $\alpha = .10$ level is represented by '*'. Coefficients γ_{ij} refer to the marginal response to z as represented by time, capital, and self employed labor, respectively. Standard errors are robust using the Newey-West procedure with lag length 2.

Endnotes

¹ Linear rational expectations as commonly applied yields consistent estimates as a two stage or GMM like estimator (McCallum 1976; Wickens 1982). Part of what is done in this article could be viewed in the same way as Hansen and Singleton's (1982) work on nonlinear rational expectations. There, no structure or prediction-like equations are used to develop expectations of price. Rather a valid set of instruments are derived from the first order conditions of the optimization problem. This is essentially what is proposed here after transformation of the model but the meaning is broader than rational expectations. Additionally, a non-linear higher set of moments allows identifying the variance of measurement error.

² Newey (2001) also considers the transformation used here for a Leser-type Engel curve in an EIV context. However, the functional form, context, and the means of addressing the problem are very different here.

³ Throughout, the time-series properties of data are not explicitly considered. That is, unit roots are ruled out. In part, this is due to the paucity of data for typical annual agricultural production systems. Further, Zellner, Kmenta, and Dreze (1966) and Mundlak (1996) argue that bias and inconsistency of production function estimation can be avoided where profit maximization is with respect to expected or anticipated production. We follow the dual approaches with normalized prices because they continue to be widely applied in agriculture, and in fact came into heavy use after the first of these papers. Second the problem we address does not consider stochastic production. Though not explicitly considered, many of the results in this paper also inform methodologies that do consider both random production as well as random prices when "real" prices are involved.

⁴ Thus, without going into the details about orthogonal projections and rational expectations, we merely assume that the classical EIV problem holds. If interpreted as a rational expectations model, we assume current levels of the exogenous variables are in the information set.

⁵ This may be a binding assumption. For example, if δ is normally distributed, then $1/\delta$ does not have a finite mean.

⁶ An intriguing simulation method by Carroll, Ruppert and Stefanski (1998) introduces additional measurement error and then estimates what consistent parameter estimates would be without measurement error by extrapolation. However, inference is difficult.

⁷ That is, in addition to (4) assume $\text{plim}(\frac{1}{T}\sum_t z_t \varepsilon_t) = \text{plim}(\frac{1}{T}\sum_t z_t \delta_t) = 0$. A readily available instrument for this model is simply w_t .

⁸ Share estimation in consumer demand is a case where EIV surely also applies. Because actual rather than permanent income is used to create budget shares, even if income does not appear on the right-hand side of the regression, least squares estimates will be inconsistent. A similar problem occurs in production if cost or profit is measured with error and the shares use a mis-measured variable in the denominator.

⁹ The estimators pursued at this point are properly called IV estimators, which are a special case of GMM, until the error structure is addressed in the over-identified case below.

¹⁰ However, due to the nonscalar covariance structure of the composite error term \square , valid inference requires a consistent estimate of the covariance matrix of estimated parameters. One such approach uses a general heteroskedasticity-autocorrelation consistent (HAC) estimator (Newey and West 1987).

¹¹ For example, the first term in the northwest corner of the matrix in (33) is estimated from the estimates of $\tilde{\theta}$ and data on w . Subtracting from the error covariance yields a regression to estimate σ^2 using p_t^2 as a regressor.

¹² An exception is TSP[®], which permits the user to individually specify which instruments are used in each equation.

¹³ In an empirical application, data may exhibit some autocorrelation. Thus, a consistent estimate of the covariance matrix of the parameter estimates may have to accommodate heteroskedasticity and autocorrelation. The Newey-West or some other HAC procedure is commonly used.

¹⁴ We have estimated each factor demand equation individually without cross-equation parameter restric-

tions (not shown). Each equation then has 45 observations and 14 parameters. Thus, one could view each equation as a case of single equation estimation. However, as is very common, the results are more “significant” and more useful for economic analysis (e.g., welfare analysis) when estimated as a system with cross-equation parameter constraints. For the sake of brevity, only the system results are presented.

¹⁵ The respective means for input quantities are: 0.746, 0.972, 1.478, 0.978, 1.881, and 1.013. Variable input price means are 7473 for chemicals, 4183 for fuels, 25147 for feeds, 5408 for hired labor, and 11560 for other inputs. The mean output price is 109476.

¹⁶ This system does not have the singular covariance matrix typically found when estimating share equations or systems that “add up”.

¹⁷ There are 9 instruments and thus a total of 45 moment equations in the system to identify the 35 parameters. Note that in order to use a HAC estimator, the number of moment conditions (orthogonality conditions) per equation must be less than the number of time series observations.

¹⁸ The Newey-West estimator of A with Z^* used as instruments in each equation, gives a similar structure for each variance or auto-covariance estimate. For example, the auto-covariance matrix for lag s has typical element $\hat{\Gamma}_{ij}(s) = \frac{1}{N} \sum_{t=s+1}^N e_{it} e_{jt-s} Z_t^{*\top} Z_{t-s}^*$, where i and j index equations, and the e 's are residuals. When $s=0$ and $i \neq j$, one gets the contemporaneous covariance across equations. This represents a robust consistent approach to covariance estimation. But there are limitations in small samples when a system of equations is estimated. One must account both for the own serial correlation structure in each equation and the cross serial correlation structure among the equations. Many econometricians suggest caution when using HAC methods in small samples. Nevertheless, they have become widely used in modern time series econometric analysis (e.g., Davidson and MacKinnon 1993, p. 661).

¹⁹ Note that, unlike the case of exact identification in the first two columns of table 4, the weighting matrix is included in the solution for an over-identified system in the latter two columns. Thus, the two GMM estimates differ.

²⁰ The GMM results in table 4 do not conform to every restriction implied by economic theory. In particular, one eigenvalue in the matrix of input price substitution effects is positive, although it is small.

²¹ Depending on the nature of assumptions made about v , its estimate can be as high as 1.005. But the results reported here seem to be most representative.