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Agricultural arbitrage and risk preferences

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A structural intertemporal model of agricultural asset arbitrage equilibrium is developed and applied to agriculture in the North Central region of the US. The data are consistent with a unifying level of risk aversion. The levels of risk aversion are more plausible than previous estimates for agriculture. However, the standard arbitrage equilibrium is rejected; perhaps, this is due to the period and the shortness of the period studied.

1. Introduction

Many studies have applied portfolio theory (Sharpe, 1970) to explain acreage allocation in production agriculture (e.g., Behrman, 1968; Estes et al., 1981; Just, 1974; Lin et al., 1974; Lin, 1977). These applications and many since have been primarily applied to crop acreage decisions assuming linear technology and most are in a static setting. This literature, which grew out of Nerlovian models of supply response (Nerlove, 1956), is generally based upon adaptive interpretations of risk, and has evolved into a more rational risk approach (Holt and Aradhyula, 1998; Saha et al., 1994; Holt and Moschini, 1992). The general finding of this, by now, large literature is that the allocation of total acreage to specific production activities is significantly influenced by risk as generally modeled with variances and covariances. The most common finding is that an increase in the own variance of price or revenue reduces the acreage allocated to that activity. This is generally interpreted as the impact of risk aversion.[1]

From the perspective of more recent developments in portfolio theory, two general findings beg application in this empirical agricultural risk literature. First, explicit attempts to measure risk aversion structurally such as those in the equity premium puzzle are preferred (Mehra and Prescott, 1985). Only this way will researchers be able to distinguish risk aversion from other behaviors. Second, the structural approach provides a way to determine whether estimated risk aversion is credible (Siegel and Thaler, 1997). Thus, we argue that the structural approach is a sensible way to proceed at least at this stage in the development of risk literature in agriculture.

Specifically, this literature suggests advantages for a more integrative examination of the broader portfolio problem in agriculture that includes consumption, investment, and other risk sharing activities as well as production. Modern agriculture is characterized by much off-farm investment (Mishra and Morehart, 2001). At the very least, reduced form production- or acreage-oriented models may misinterpret the level of risk aversion (Just and Peterson, 2003). Worse, parameters can be biased if relevant variables are omitted. For example, if markets are incomplete, Fisher separation may not hold implying inconsistent estimation of parameters (Saha et al., 1993). A third issue concerns the advantages and disadvantages of using typical Euler equation representations of intertemporal arbitrage. Euler equations may yield important information from which to identify parameters, but imply that the dynamics must be properly specified (Carroll, 2001). For example, one must choose between the non-expected utility model of Kreps and Porteus (1978) and standard model of discounting with additive preferences (Laibson, 1997).

After building a dynamic model of consumption, investment, and production, we obtain fundamental arbitrage equations that govern allocations of wealth to financial assets and agricultural

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[1] Some authors have modeled production with explicit substitution among inputs along with risk aversion over profit or initial wealth (e.g., Saha et al., 1994; Love and Buccola, 1991; Antle, 1987).

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capital as well as the allocation of acreage. This enables econometric choice from a larger set of first-order (arbitrage) conditions in order to estimate risk preferences. We develop general conditions and then adapt them for empirical use with the available data. The crucial variable of interest driving decisions is consumption, which is facilitated by accumulation of net worth (wealth).[2] For agricultural households, these both are notoriously difficult to measure.[3]

After developing the arbitrage conditions, empirical estimates are obtained by generalized method of moments (GMM) for eight states in the North Central region of the US using stock market returns, bonds, and agricultural land allocations.[4] For these eight states in the period 1991–2000, reasonably good measurements of wealth are available which are essential for our approach.[5] To the extent that we measure the impact of policy, it must be found in the distribution of crop returns, which include government payments. While this is a relatively short and somewhat anomalous time period when compared with typical studies in finance, we suggest this comprehensive approach to arbitrage structure can be beneficial when compared with typical incomplete approaches to the estimation of risk behavior in agriculture. Using contemporaneous arbitrage equations implied by Euler conditions, an econometric model is specified over future wealth and excess returns conditional on the current information set. In spite of limited data, we find evidence of aggregate risk aversion that is rationalized by a single set of representative consumer preferences using an unconventional but reasonable specification.

2. Variable definitions and timing for a micro-model of farm behavior

Although the organizational form of farms varies, a recent report by Hoppe and Banker (2006) finds that 98% of US farms remained family farms as of 2003. In a family farm, the entrepreneur controls the means of production and makes investment, consumption, and production decisions. We begin by modeling the intertemporal interactions of these decisions. The starting point is a model similar in spirit to Hansen and Singleton's (1983) but generalized to include consumption decisions and farm investments as well as financial investments and production decisions. Variable definitions are as follows, where t denotes the time period:

W_t = beginning-of-period total wealth,
B_t = current holding of bonds with a risk free rate of return r_t,
$f_{i,t}$ = current holding of the ith risky financial asset, $i = 1, \ldots, n_F$,
$\delta_{i,t+1}$ = dividend rate on the ith risky financial asset,
$p_{F_i,t}$ = beginning-of-period market price of the ith risky financial asset,
$\gamma_{i,t+1} = (p_{F_i,t+1} - p_{F_i,t})/p_{F_i,t}$ = capital gain rate on the ith risky financial asset,
$a_{i,t}$ = current allocation of land to the ith crop, $i = 1, \ldots, n_Y$,
A_t = total quantity of farm land,
$p_{L,t}$ = beginning-of-period market price of land,
$\psi_{t+1} = (p_{L,t+1} - p_{L,t})/p_{L,t}$ = capital gain rate on land,
$\boldsymbol{x}_{i,t}$ = vector of input quantities employed in producing the ith crop,
$\boldsymbol{w}_t$ = vector of market prices for the inputs,
$\bar{y}_{i,t}$ = expected yield per acre for the ith crop, $i = 1, \ldots, n_Y$,
$y_{i,t+1}$ = realized yield of the ith crop,
$p_{Y_i,t+1}$ = end-of-period realized market price for the ith farm product,
$\boldsymbol{q}_t$ = vector of quantities of consumption goods,
$\boldsymbol{p}_{Q,t}$ = vector of market prices for consumer goods,
m_t = total consumption expenditures,
$u(\boldsymbol{q}_t)$ = periodic utility from consumption.

As with all discrete time models, timing can be represented in multiple ways. In the model used here, all financial returns and farm asset gains are assumed to be realized at the end of each time period (where depreciation is represented by a negative asset gain). Variable inputs are assumed to be committed to farm production activities at the beginning of each decision period and the current period market prices for the variable inputs are known when these use decisions are made. Agricultural production per acre is realized stochastically at the end of the period such that

$$y_{i,t+1} = \bar{y}_{i,t}(1 + \varepsilon_{i,t+1}), \quad i = 1, \ldots, n_Y,$$

where $\varepsilon_{i,t+1}$ is a random output shock with $E(\varepsilon_{i,t+1}) = 0$. Consumption decisions are made at the beginning of the decision period and the current market prices of consumption goods are known when these purchases are made. Utility is assumed to be strictly increasing and concave in $\boldsymbol{q}_t$.

The total beginning-of-period value of financial assets, and land are, respectively,

$$F_t = \sum_{i=1}^{n_F} p_{F_i,t} f_{i,t} \equiv \boldsymbol{p}_{F,t}^{\mathrm{T}} \boldsymbol{f}_t, \quad \text{and}$$

$$L_t = p_{L,t} A_t \equiv p_{L,t} \iota^{\mathrm{T}} \boldsymbol{a}_t$$

where the total beginning-of-period quantity of land is $A_t = \iota^{\mathrm{T}} \boldsymbol{a}_t$, with ι denoting an n-vector of ones and, throughout, bolded notation represents the vector form of its unbolded and i-subscripted counterpart. Note that homogeneous land is assumed with a scalar price, $p_{L,t}$. For an arbitrary n-vector $\boldsymbol{z}$, denote the $n \times n$ diagonal matrix whose typical diagonal element is z_i by $\Delta(\boldsymbol{z})$.

3. Behavior and constraints

We assume non-joint crop production with constant returns to scale so that the production function for the ith crop in per acre terms is

$$\bar{y}_{i,t} = g_{i,t}(\boldsymbol{x}_{i,t}), \quad i = 1, \ldots, n_Y. \tag{1}$$

For each crop, the cost function per acre satisfies

$$c_{i,t}(\boldsymbol{w}_t, \bar{y}_{i,t}) \equiv \min_{\boldsymbol{x}_{i,t}} \left\{ \boldsymbol{w}_t^{\mathrm{T}} \boldsymbol{x}_{i,t} : \bar{y}_{i,t} = g_{i,t}(\boldsymbol{x}_{i,t}) \right\},$$

$$i = 1, \ldots, n_Y, \tag{2}$$

and total cost across all crops is additively separable (Hall, 1978; Muellbauer, 1974; Samuelson, 1966),[6]

$$c_t(\boldsymbol{w}_t, \bar{\boldsymbol{y}}_t, \boldsymbol{a}_t) = \boldsymbol{c}_t(\boldsymbol{w}_t, \bar{\boldsymbol{y}}_t)^{\mathrm{T}} \boldsymbol{a}_t. \tag{3}$$

Revenue at $t+1$ is the random price times production[7]

$$R_{t+1} = \sum_{i=1}^{n_Y} (p_{Y_i,t+1} \bar{y}_{i,t} a_{i,t} (1 + \varepsilon_{i,t+1})) \equiv (\iota + \varepsilon_{t+1})^{\mathrm{T}} \Delta(\boldsymbol{p}_{Y,t+1}) \Delta(\boldsymbol{a}_t) \bar{\boldsymbol{y}}_t. \tag{4}$$

2 Net worth and wealth are used here interchangeably. For proprietorships, it is especially difficult to measure and untangle the contributions of human and physical capital.

3 Lence (2000) apparently had reasonable success calculating aggregate consumption by agricultural households in the US.

4 Difficulty in measuring agricultural capital services at the state level led us to omit this arbitrage equation.

5 These are the wealth data used by Lin and Dismukes (2007) in a recent Economic Research Service (ERS) study of the USDA.

6 Although $\boldsymbol{x}_t$ has a t subscript, this is a typical simplification for problems without intra-seasonal states.

7 A futures market activity can also be added for each output. In this case,

$$\pi_{t+1} = \sum_{i=1}^{n_y} [p_{h_i,t} h_{i,t} + p_{Y_i,t+1}(\bar{y}_{i,t}(1 + \varepsilon_{i,t+1}) - h_{i,t})] - (1 + r_t) c(\boldsymbol{w}_t, \bar{\boldsymbol{y}}_t, \boldsymbol{a}_t),$$

where $\boldsymbol{h}_t$ is the vector of hedging activities with associated forward or futures price $\boldsymbol{p}_{h,t}$.

Wealth is allocated at the beginning of period t to assets, production costs, and consumption, satisfying

$$W_t = B_t + F_t + L_t + c_t(\boldsymbol{w}_t, \boldsymbol{a}_t, \bar{\boldsymbol{y}}_t) + m_t = B_t + \boldsymbol{p}_{F,t}^{\mathrm{T}}\boldsymbol{f}_t + p_{L,t}\iota^{\mathrm{T}}\boldsymbol{a}_t + \boldsymbol{c}_t(\boldsymbol{w}_t, \bar{\boldsymbol{y}}_t)^{\mathrm{T}}\boldsymbol{a}_t + m_t. \quad (5)$$

Thus, beginning-of-period wealth consists of holdings of bonds B_t with numeraire price 1, risky financial assets F_t, and land L_t, plus cash on hand used to finance production costs and consumption throughout the period. Although some costs occur at or near harvest (near $t+1$), we include all costs in (5) at time t because they are incurred before revenues are received.

Consumer utility maximization yields the quasi-convex indirect utility function conditioned on consumer good prices and expenditures,

$$\upsilon(\boldsymbol{p}_{Q,t}, m_t) \equiv \max_{\boldsymbol{q}\in R_+^{n_Q}} \left\{u(\boldsymbol{q}) : \boldsymbol{p}_{Q,t}^{\mathrm{T}}\boldsymbol{q} = m_t\right\}. \quad (6)$$

Realized end-of-period wealth is

$$W_{t+1} = (1+r_t)B_t + (\iota + \delta_{t+1} + \gamma_{t+1})^{\mathrm{T}}\Delta(\boldsymbol{p}_{F,t})\boldsymbol{f}_t + (1+\psi_{t+1})p_{L,t}\iota^{\mathrm{T}}\boldsymbol{a}_t + (\iota + \varepsilon_{t+1})^{\mathrm{T}}\Delta(\boldsymbol{p}_{Y,t+1})\Delta(\boldsymbol{a}_t)\bar{\boldsymbol{y}}_t. \quad (7)$$

Thus, the decision maker's wealth is increased by the return on assets including interest, dividends, asset appreciation (less depreciation), and farm revenue.

The decision maker's intertemporal utility is assumed to follow

$$U_T(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_T) = \sum_{t=0}^{\mathrm{T}} (1+\rho)^{-t} u(\boldsymbol{q}_t). \quad (8)$$

The producer is assumed to maximize Von Neumann–Morgenstern expected utility of the discounted present value of periodic utility flow from consumption.

4. A solution approach

The problem is solved by stochastic dynamic programming, treating T as fixed and finite, and working backwards from the last period in the planning horizon to the first. In the last period of the planning horizon, the decision is simply to invest or produce nothing and consume all remaining wealth, $m_T = W_T$. Denote the last period's optimal value function by $\upsilon_T(W_T)$. Then, $\upsilon_T(W_T) = \upsilon(\boldsymbol{p}_{Q,T}, W_T)$ is the optimal utility for the terminal period. For other time periods, stochastic dynamic programming using (6)–(8) to optimize agricultural production, asset ownership and net investment decisions in each period yields the (Bellman) backward recursion problem for arbitrary $t < T$,

$$\begin{aligned}\upsilon_t(W_t) \equiv \max_{(m,B,\boldsymbol{f},\boldsymbol{k},\boldsymbol{a},\bar{\boldsymbol{y}})} \Big\{ & \upsilon(\boldsymbol{p}_{Q,t}, m) + (1+\rho)^{-1} \\ & \times E_t\Big[\upsilon_{t+1}((1+r_t)B + (\iota + \delta_{t+1} + \gamma_{t+1})^{\mathrm{T}}\Delta(\boldsymbol{p}_{F,t})\boldsymbol{f} \\ & + (1+\psi_{t+1})p_{L,t}\iota^{\mathrm{T}}\boldsymbol{a} + (\iota + \varepsilon_{t+1})^{\mathrm{T}}\Delta(\boldsymbol{p}_{Y,t+1})\Delta(\bar{\boldsymbol{y}})\boldsymbol{a})\Big] : \\ & W_t = B + \boldsymbol{p}_{F,t}^{\mathrm{T}}\boldsymbol{f} + p_{L,t}\iota^{\mathrm{T}}\boldsymbol{a} + \boldsymbol{c}_t(\boldsymbol{w}_t, \bar{\boldsymbol{y}})^{\mathrm{T}}\boldsymbol{a} + m\Big\}. \end{aligned} \quad (9)$$

The associated Lagrangean is

$$\begin{aligned} L = {} & \upsilon(\boldsymbol{p}_{Q,t}, m) + (1+\rho)^{-1} \\ & \times E_t\Big[\upsilon_{t+1}\Big((1+r_t)B + (\iota + \delta_{t+1} + \gamma_{t+1})^{\mathrm{T}}\Delta(\boldsymbol{p}_{F,t})\boldsymbol{f} \\ & + (1+\psi_{t+1})p_{L,t}\iota^{\mathrm{T}}\boldsymbol{a} + (\iota + \varepsilon_{t+1})^{\mathrm{T}}\Delta(\boldsymbol{p}_{Y,t+1})\Delta(\bar{\boldsymbol{y}})\boldsymbol{a}\Big)\Big] \\ & + \lambda_t\left(W_t - B - \boldsymbol{p}_{F,t}^{\mathrm{T}}\boldsymbol{f} - p_{L,t}\iota^{\mathrm{T}}\boldsymbol{a} - \boldsymbol{c}_t(\boldsymbol{w}_t, \bar{\boldsymbol{y}})^{\mathrm{T}}\boldsymbol{a} - m\right). \end{aligned} \quad (10)$$

We assume that bonds and consumption expenditures are positive and that the wealth constraint holds with equality. Hence, $\lambda_t > 0$ and

$$\frac{\partial \upsilon(\boldsymbol{p}_{Q,t}, m_t)}{\partial m_t} = \left(\frac{1+r_t}{1+\rho}\right) E_t\left[\upsilon'_{t+1}(W_{t+1})\right] = \lambda_t. \quad (11)$$

Substituting the middle term for the Lagrange multiplier, λ_t, the remainder of the Kuhn–Tucker conditions for an optimal solution can be written as

$$\frac{\partial L}{\partial \boldsymbol{f}} = \Delta(\boldsymbol{p}_{F,t})E_t\left[\upsilon'_{t+1}(W_{t+1})(\delta_{t+1} + \gamma_{t+1} - r_t\iota)\right] \le 0, \quad \boldsymbol{f} \ge 0,\ \boldsymbol{f}^{\mathrm{T}}\frac{\partial L}{\partial \boldsymbol{f}} = 0 \quad (12)$$

$$\begin{aligned}\frac{\partial L}{\partial \boldsymbol{a}} = E_t\Big[& \upsilon'_{t+1}(W_{t+1})(\Delta(\boldsymbol{p}_{Y,t+1})\Delta(\bar{\boldsymbol{y}})(\iota + \varepsilon_{t+1}) \\ & + (\psi_{t+1} - r_t)p_{L,t}\iota - (1+r_t)\boldsymbol{c}_t)\Big] \le 0, \quad \boldsymbol{a} \ge 0,\ \boldsymbol{a}^{\mathrm{T}}\frac{\partial L}{\partial \boldsymbol{a}} = 0; \end{aligned} \quad (13)$$

$$\begin{aligned}\frac{\partial L}{\partial \bar{\boldsymbol{y}}} = E_t\Big[& \upsilon'_{t+1}(W_{t+1})(\Delta(\boldsymbol{p}_{Y,t+1})\Delta(\boldsymbol{a})(\iota + \varepsilon_{t+1}) \\ & - (1+r_t)\frac{\partial \boldsymbol{c}_t^{\mathrm{T}}}{\partial \bar{\boldsymbol{y}}}\boldsymbol{a})\Big] \le 0, \quad \bar{\boldsymbol{y}} \ge 0,\ \bar{\boldsymbol{y}}^{\mathrm{T}}\frac{\partial L}{\partial \bar{\boldsymbol{y}}} = 0. \end{aligned} \quad (14)$$

Eq. (11) is the fundamental consumption smoothing equation. By the envelope theorem, $\lambda_t = \upsilon'_t(W_t)$, so that this Euler equation can be represented equivalently either in terms of the marginal utility of consumption or of wealth. The conditions in (12) represent financial asset arbitrage. Agricultural asset and production choices are found similarly from (13) and (14). The marginal net benefit of holding land in (13) includes future appreciation (or depreciation) $\psi_{t+1}p_{L,t}$, the marginal impact on costs of production, and the opportunity cost, $r_t p_{L,t}$. The marginal net benefit of expected crop yields in (14) includes random marginal revenue and marginal cost.

Assuming an interior solution and dividing (13) by $p_{L,t}$ yields

$$E_t\left[\upsilon'_{t+1}(W_{t+1})\left((\psi_{t+1} - r_t)\iota + \pi_{t+1}/p_{L,t}\right)\right] = 0, \quad (15)$$

where π_{t+1} is a vector of short run per acre profits from crop production, defined as $\pi_{t+1} = \Delta(\boldsymbol{p}_{Y,t+1})\boldsymbol{y}_{t+1} - (1+r_t)\boldsymbol{c}_t$. Further defining the excess return rate for the ith crop, $e_{i,t+1} = \psi_{t+1} + (\pi_{i,t+1}/p_{L,t}) - r_t$, Eq. (15) can be expressed as

$$E_t\left[\upsilon'_{t+1}(W_{t+1})e_{i,t+1}\right] = 0, \quad i = 1, \ldots, n_Y. \quad (16)$$

Eq. (16) thus resembles other asset equations with dividends, where the dividend for production is the per acre profit from land relative to land prices.

5. Aggregation across choices and households

Data sets that contain all required farm financial and production data for implementation of the above model at the farm household level are lacking. Although a few surveys of farm households are available, they have serious shortcomings for this application. For example, the Agricultural Resource Management Survey conducted by the National Agricultural Statistics Service (NASS) for the ERS is periodic but is not a panel as necessary for estimating dynamic relationships. On the other hand, other micro-level data such as the Kansas State KMAR data are focused on production. Financial variables occur as holdings or expenditures without identifying whether an increase is due to asset appreciation or additional investment.

Alternatively, aggregation both over households and observable asset categories facilitates application with available aggregate data. To illustrate the implied arbitrage equation for aggregate financial assets, consider (12). Kuhn–Tucker conditions imply that

$$\boldsymbol{f}^{\mathrm{T}}\frac{\partial L}{\partial \boldsymbol{f}} = \boldsymbol{f}^{\mathrm{T}}\Delta(\boldsymbol{p}_{F,t})E_t\left[v'_{t+1}(W_{t+1})(\delta_{t+1}+\gamma_{t+1}-r_t\iota)\right]$$
$$= E_t\left\{v'_{t+1}(W_{t+1})\sum_{i=1}^{n_f}[F_{i,t}(\delta_{i,t+1}+\gamma_{i,t+1}-r_t)]\right\} = 0 \quad (17)$$

where $F_{i,t} = p_{F_{i,t}}f_{i,t}$. Dividing this expression by F_t, assuming $F_t > 0$, obtains the financial arbitrage condition,

$$E_t\left[v'_{t+1}(W_{t+1})e_{F,t+1}\right] = 0, \quad (18)$$

where $e_{F,t+1} = \sum_{i=1}^{n_f}(F_{i,t}/F_t)(\delta_{i,t+1}+\gamma_{i,t+1})-r_t$ is excess return on the weighted average of financial investments. Eq. (18) is a crucial equation for our application because it contributes to the identification of properties of $v'_{t+1}(W_{t+1})$, which contributes to the identification of (15).

Another important issue of aggregation occurs across consuming units. Our data are available at the state level and are thus an aggregation across micro units. Aggregation usually washes out some variation in data so that perceived variability at the micro-level is inconsistent with variation reflected in aggregate data. Although many of the variables may be subject to heterogeneity, the most serious heterogeneity likely occurs in wealth, the production disturbance, and cost.

We illustrate briefly the difficulty with heterogeneous production disturbances. Adding h subscripts to represent households and j subscripts to represent states, (16) can be expressed in the form $E_t[v'_{t+1}(W_{h,j,t+1})e_{i,h,j,t+1}] = 0$. Summing over all H_j households in state j yields the implications of first-order conditions at the state level,

$$E_t\left[\sum_{h=1}^{H_j}\frac{1}{H_j}v'_{t+1}(W_{h,j,t+1})e_{i,h,j,t+1}\right] = 0. \quad (19)$$

This is not the condition imposed by a representative household approach with average state-level data, $E_t[v'_{t+1}(\bar{W}_{j,t+1})\bar{e}_{i,j,t+1}] = 0$, where overbars denote averaging.

Techniques popularized by Aczel (1966) could be used to obtain restrictions on (19) required for exact aggregation. Alternatively, suppose state-level excess returns are related to individual excess returns by $e_{i,h,j,t+1} = \bar{e}_{i,j,t+1} + u_{i,h,j,t+1}$, and that the individual marginal utility of wealth is related to the marginal utility of wealth at the state-level average wealth by $v'_{t+1}(W_{h,j,t+1}) = v'_{t+1}(\bar{W}_{j,t+1}) + \varsigma_{h,j,t+1}$. Then, using state-level data yields

$$\begin{aligned}
&E_t[v'_{t+1}(W_{h,j,t+1})e_{i,h,j,t+1}] \\
&\quad = E_t\left\{[v'_{t+1}(\bar{W}_{j,t+1}) + \varsigma_{h,j,t+1}][\bar{e}_{i,j,t+1} + u_{i,h,j,t+1}]\right\} \\
&\quad = E_t[v'_{t+1}(\bar{W}_{j,t+1})\bar{e}_{i,j,t+1}] \\
&\qquad + E_t[v'_{t+1}(\bar{W}_{j,t+1})u_{i,h,j,t+1}] + E_t[\varsigma_{h,j,t+1}\bar{e}_{i,j,t+1}] \\
&\qquad + E_t[\varsigma_{h,j,t+1}u_{i,h,j,t+1}]. \quad (20)
\end{aligned}$$

None of the latter three terms of (20) need vanish in general. In particular, because $u_{i,h,j,t+1}$ is a deviation in return realized at $t+1$ and $\varsigma_{h,j,t+1}$ depends on wealth at $t+1$, which includes this realized return, these two terms are likely to be jointly determined. Therefore, as in a typical setting with state-level panel data, we consider fixed state effects in the form

$$E_t[v'_{t+1}(\bar{W}_{j,t+1})\bar{e}_{i,j,t+1}] - (\alpha_i + \phi_j) = 0, \quad (21)$$

where α_i and ϕ_j are parameters to be estimated, $i = 1, \ldots, n_Y$, $j = 1, \ldots, 8$, and n_Y represents the number of moment equations. These parameters appear here with a minus sign so that positive estimates will correspond to overshooting of an arbitrage condition. Time effects can also be added, $t = 1, \ldots, T$, but because of the relatively short time period used in our data set, time effects do not appear useful. As usual, an idiosyncratic disturbance with zero expectation is added later for econometric purposes.

6. Identification of utility in arbitrage conditions

To model a set of endogenous choices, a system of equations that determines all choice variables is preferred. This requires as many equations as decisions. However, any identified equation can be helpful to answer a particular question. For example, a single arbitrage condition such as (21) may serve to identify preference parameters. However, potential endogeneity issues must be addressed. We apply GMM with instruments to correct for endogeneity.

When compared with the Euler formulations of arbitrage conditions, however, conditions in the form of (16) and (18) can present a numerical challenge for identification of utility. For econometric purposes, the marginal utility of wealth is typically formulated as $v'_{t+1}(W_{t+1}, \beta)$, where β is a parameter or set of parameters to be estimated. Letting β be a scalar for illustration, $v'_{t+1} : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}_+$ where $W_{t+1} \in \mathbb{R}_+$ and $\beta \in \mathbb{R}$. However, the model is not identified in the domain $\mathbb{R}$ of the parameter space in a way that makes economic sense. For example, consider the usual GMM estimator for an arbitrage condition defined by

$$\begin{aligned}
g(\boldsymbol{Y}_t, \beta) &= E_t[\boldsymbol{Z}_t^{\mathrm{T}}v'_{t+1}(W_{t+1}, \beta)\boldsymbol{e}_{t+1}] \\
&= E_t[\boldsymbol{Z}_t^{\mathrm{T}}\Phi(W_{t+1}, \boldsymbol{e}_{t+1}; \beta)] = 0, \quad (22)
\end{aligned}$$

where W_t represents wealth, $e_{i,t+1}$ is excess returns, β is the preference parameter to be estimated, Φ represents the arbitrage conditions associated with first-order conditions, $\boldsymbol{Z}_t$ represents a set of instruments in the information set that forms expectations at time t, and $\boldsymbol{Y}_t = (W_{t+1}, \boldsymbol{e}_{t+1}, \boldsymbol{Z}_t)$ represents all the data collectively. Iterated expectations rationalizes the use of a sample moment of the form $\bar{\boldsymbol{m}}(\beta) = \sum_{t=1}^{\mathrm{T}}\boldsymbol{Z}_t^{\mathrm{T}}\Phi(W_{t+1}, \boldsymbol{e}_{t+1}; \beta)/T$ to minimize

$$\hat{\beta} = \arg\min\left(\bar{\boldsymbol{m}}(\beta)^{\mathrm{T}}\boldsymbol{A}\bar{\boldsymbol{m}}(\beta)\right), \quad (23)$$

where $\boldsymbol{A}$ is the positive definite weighting matrix. If there exists $\beta_0 \in \mathbb{R}$ such that marginal utility is (numerically indistinguishable from) zero, $v'_{t+1}(W_{t+1}, \beta_0) = 0$ for all t, which implies that $E_t\Phi(W_{t+1}, \boldsymbol{e}_{t+1}; \beta_0) = 0$ and $\bar{\boldsymbol{m}}(\beta_0) = 0$, then $\hat{\beta} = \beta_0$ solves (23) regardless of the true parameter. Unfortunately, this problem occurs with many common utility functional forms. For example, marginal utility under constant relative risk aversion (CRRA), $v'_{t+1}(W_{t+1}) = W_{t+1}^{-\beta}$, implies that the minimum of (23) will occur for β large enough to be make this marginal utility numerically zero. A similar result occurs for constant absolute risk aversion (CARA) and generalizations of these two forms in common use (e.g., see Meyer and Meyer, 2006).

Several approaches can be taken to solve this problem. Among them is restriction of the domain of β to make utility strongly monotonic. This is not easy because the domain of positive marginal utility depends on the arbitrage equation and the data. Instead, we consider a Taylor series approximation of the utility function. With a small number of terms as required for practical application, the accuracy of the approximation is less, but the parameters are estimable.

In the consumption based Euler equations of Hansen and Singleton, a typical CRRA equation would be of the form

$$E_t\left[\left(\frac{q_{t+1}}{q_t}\right)^{-\beta}\left(\frac{\tilde{R}_{t+1}}{1+\rho}\right)\right] - 1 = 0 \quad (24)$$

where q is the single consumption good, $\tilde{R}$ is one plus the rate of return on the asset including dividends, and β is the Arrow–Pratt measure of relative risk aversion. No parametric restriction on β for all values of the data implies that the expectation is one for all values of the data.

A Taylor series approximation of marginal utility possesses this same virtue for the arbitrage conditions estimated here. To illustrate, the marginal utility for a CARA utility function can be written as $e^{-\beta W_{t+1}}$ and the arbitrage condition is $E_t\left(\boldsymbol{e}_{t+1}/e^{\beta W_{t+1}}\right) = 0$. Using GMM with this utility function would choose β large enough to make the moment condition identically zero at least numerically. However, the arbitrage conditions in (16) and (18) with a second-order Taylor series approximation of $e^{-\beta W_{t+1}}$ around $e^{-\beta W_t}$ can be stacked and parameterized as

$$\Phi(W_{t+1}, \boldsymbol{e}_{t+1}; \beta) = e^{-\beta W_t} E_t\Big[(1 - \beta(W_{t+1} - W_t) + 1/2\beta^2(W_{t+1} - W_t)^2)\boldsymbol{e}_{t+1}\Big] = 0. \tag{25}$$

Presuming that $e^{-\beta W_t} \neq 0$ for the true β, the root of (25) is such that the term in parenthesis is not zero.[8] That is, no $\beta \in \mathbb{R}$ exists such that the approximated marginal utility is zero for all values of wealth. One can easily expand this argument to higher order approximations.

7. Data and estimation strategy

No carefully constructed publicly available panel of agricultural data including farm and off-farm decisions and wealth variables exists. The periodic Survey of Consumer Finances and the Panel Study of Income Dynamics has too little farm information to give a very complete picture of decisions and representation of farm households. The best available data on wealth are found in the Agricultural Resource Management Survey and the US Census of Agriculture, which are conducted by NASS. For reasons explained above, this survey does not suffice for application of our model at the micro level. However, data from this survey have been used within ERS to estimate average farm household net worth (wealth) by state for the period 1991–2001. These data include eight states in the North Central Region of the US: Illinois, Indiana, Iowa, Michigan, Minnesota, Missouri, Ohio, and Wisconsin.

These data are not without issues but seem to be the best available source for net worth and are actively used by government personnel in the ERS for research. Alternative data (such as land value) would omit non-farm assets, which are a substantial portion of farm households' net worth (Mishra and Morehart, 2001) and are intended as a key source of identification for this study. Although the time-period is short and in some ways atypical due to the run up of the stock market in the 1990s, this variation is ideal for identifying the arbitrage effects on agriculture of the returns to financial assets. This period also has the advantage that the impact of government policy on crop substitution is relatively reduced and less complicated. For example, the Freedom to Farm Act of 1996 culminated a growing effort to decouple farm subsidies from acreage allocation decisions.

For each state, net returns and acres allocated to corn, soybeans, and wheat are from NASS (http://www.nass.usda.gov/index.asp), as are per acre production costs by crop (http://www.ers.usda.gov/data/costsandreturns/testpick.htm). The rate of return on bonds is the annualized return on 90-day treasury bills in secondary markets from the Federal Reserve (http://www.federalreserve.gov/releases/h15/data/Annual/H15_TB_M3.txt). The return on financial assets is the share-weighted rate of return on the S&P from Shiller (1992) as updated (http://www.econ.yale.edu/~shiller/data/chapt26.xls). The average per acre value of farmland and buildings by state is obtained from the "Farm resources, income, and expenses" chapter of the NASS publication *Agricultural Statistics* published annually from 1995 to 2005. Where discrepancies exist from one year's publication to the next, the most recently published data are used because NASS updates estimates as further information becomes available. All monetary variables (asset values and net farm returns) are deflated by the GDP implicit price deflator. The nominal rate of return for agricultural assets is calculated as the percentage change in nominal annual average value per acre of farmland and buildings. Wealth varies from \$ 221,665 to \$ 778,139 with a mean wealth of \$ 413,855. Excess returns for the market vary from −.1351 to .1319 with a mean of .1072. Excess rates of returns for the crops vary from a low of −.1354 for Corn (mean = .0268) to a high of .1585 for soybeans (mean = .0639). The excess return that seems to be particularly low is wheat, which has a mean across all states of near zero. Clearly, however these returns vary by state. For instance, Iowa has very low returns to wheat yielding second-degree stochastic dominance and very little is grown. The standard deviations are \$ 112,507 for wealth and, respectively, for the four arbitraged goods: .1319, .0532, .0447, .0978 for the market, corn, soybean and wheat land, respectively.

8. Econometric specification

Thus, our estimated arbitrage conditions include (18) and land asset arbitrage equations as in (16) for the three crops. These arbitrage conditions are parameterized by the additional α_i and ϕ_j parameters as defined in (21) to account for the use of aggregated state-level rather than micro-level data. These parameters allow testing for departures from full arbitrage, but also represent heterogeneity within states and crops as explained above.

Given the nature of the data, we suspected that there might be a difficulty in measuring precisely the curvature of risk aversion. However, in order to explore this issue, consider the marginal utility function $v'(W_t) = e^{-\beta f(W_t)}$, where $f(W_t)$ is parametrized as $(W_t^{\kappa} + \kappa - 1)/\kappa$. CARA is represented by $\kappa = 1$, while CRRA is given by $\kappa = 0$. Absolute risk aversion, $\beta W^{\kappa-1}$, is decreasing if $\kappa < 1$ while relative risk aversion, βW^{κ}, is increasing if $\kappa > 0$. In the interval $0 \leq \kappa \leq 1$, in our later empirical model, we could not reject CARA and indeed found that the lowest value of the criterion function occurs at $\kappa = 1$.[9] Hence, CARA (25) is used throughout the analysis. Since the time period is short, a quadratic approximation of the utility function is believed to adequately capture the curvature of utility as wealth changes (estimation with a cubic approximation showed that the estimate of risk aversion was only altered by less than 2%).

Specifically, the four estimated equations are

$$E_t[v'_{t+1}(W_{j,t})e_{i,t+1}] - \alpha_i - \varphi_j = \zeta_{i,j,t+1},\ E(\zeta_{i,j,t+1}) = 0,$$
$$i = F, C, S, W, j = 1, \ldots, 8, t = 1, \ldots, 9, \tag{26}$$

where subscripts F, C, S, and W refer to financial assets, corn, soybeans, and wheat, respectively, v'_{t+1} is approximated as in (25) with state per farm wealth $W_{j,t}$ (dropping the overbars for convenience), and $\zeta_{i,j,t+1}$ is an idiosyncratic disturbance added for econometric purposes. Only 9 time series observations are available for estimation because the tenth year is used to represent time $t+1$ for year 9.

Neither the selection of states nor crops is conceptualized as a random draw from among states and crops, respectively. Hence, equations are modeled as having common fixed effects, where α_i is the overall intercept for the ith moment equation. This model is referred to as Model 1. Since Wisconsin is omitted, its fixed effects are included in the equation-specific constants so each ϕ_j can be interpreted as relative to Wisconsin. In addition, we assume that $\zeta_{i,j,t+1}$ is potentially serially correlated, and correlated across equations and heteroskedastic. Due to the paucity of data, we present the Newey–West procedure and use a Bartlett Kernel with lag 1.

8 Indeed, one can then divide by $e^{-\beta W_t}$ to eliminate it.

9 For example, corresponding to the first column in Table 1, the objective function in (29) is .8872 for $\kappa = 1$ and.8901 when $\kappa = 0$ with a constant covariance matrix.

There are other ways to introduce heterogeneity and/or think about $\alpha_i + \phi_j + \zeta_{i,j,t+1}$ in (26). To the extent that ζ has systematic time effects, α_i represents the mean of the regression disturbance leaving ζ to have mean zero. That is, the equation intercepts may be interpreted as including an average time effect to the extent that time effects cause a deviation from full arbitrage on average over the period of estimation. For example, the volatility coupled with rapid growth in the stock market during 1991–2000 may have caused larger estimated deviations from full arbitrage in the form of larger covariances as represented in (21). A second way to introduce heterogeneity into our model is to have state-level utility functions. In this case, the utility function parameter is represented as

$$\beta = \beta_1 + \sum_{j=2}^{8} \beta_j d_j. \tag{27}$$

This is called the "heterogeneous preference" model or Model 2. Although combining this specification with (21) may be useful conceptually, our data do not appear to allow estimation with both the state effects in (21) and the state-level utility functions implied by (27) simultaneously. Hence, we do not estimate fixed state effects in the heterogeneous preference model.

As in Lence (2000), timing issues in the net worth data are problematic. Apparently, from all econometric tests and analysis, returns are not added into net worth until the following year. Thus, excess returns at time t are added to W_{t+1}. Since our approximation of utility uses W_t as the mean of wealth at $t+1$, we have chosen to use wealth at $t-1$ rather than at time t as an instrument. Specifically, we use W_{t-1} and W_{t-1}^2 along with state dummies, unemployment at t, and the GDP growth rate at t as the instruments that comprise Z_t. Thus, only eight observations on the eight states are available for estimation.

Stacking the equations in (26) yields the initial moment conditions for GMM, which are represented compactly as

$$\boldsymbol{g}(\boldsymbol{Y}_t, \theta) = \boldsymbol{Z}_t^{\mathrm{T}} \Phi(W_{t+1}, \boldsymbol{e}_{t+1}; \theta)/T = \boldsymbol{Z}_t^{\mathrm{T}} \zeta_{t+1}/T, \tag{28}$$

where T is the number to observations, Φ is as specified in (25) with additional parameters included as specified in (21), (26), and (27) to account for the use of aggregate data, $\boldsymbol{g}_t$ is vector-valued with the four equations in (26), and $\theta = (\beta, \alpha, \phi) \in \Theta$ where β represents the risk aversion parameter including parameters reflecting state-level variations of it, α represents the vector of wedges in arbitrage conditions in (26) associated with source of returns, and ϕ represents the vector of state fixed effects. Since, we have a fixed effects model, we invoke the usual regularity conditions with pooled asymptotics (as opposed to panel asymptotics, which concern whether time or the size of the cross-section grows larger or faster) (e.g., Hall, 2005 or Newey and McFadden, 1994). Thus, for simplicity of notation, we will use T to represent the number of observations across states and years.

The standard GMM estimator for our problem is

$$\hat{\theta} = \arg\min \left\{ Q(\theta) = \boldsymbol{g}_T^{\mathrm{T}} \boldsymbol{A}_T \boldsymbol{g}_T \right\} \tag{29}$$

where $\boldsymbol{g}_T = \sum_{t=1}^{\mathrm{T}} \boldsymbol{g}(\boldsymbol{Y}_t, \theta)/T$ and $\boldsymbol{A}_T$ is positive definite and converges to a positive definite constant matrix. Under these assumptions, the estimator $\hat{\theta}$ is consistent, $\hat{\theta} \to^P \theta$. Furthermore,

$$\begin{aligned} \sqrt{T}(\hat{\theta} - \theta_0) \xrightarrow{D} N(0,\ & (\nabla \boldsymbol{g}_T^{\mathrm{T}} \boldsymbol{A} \nabla \boldsymbol{g}_T)^{-1} \\ & \times \nabla \boldsymbol{g}_T^{\mathrm{T}} \boldsymbol{ASA}^{\mathrm{T}} \nabla \boldsymbol{g}_T (\nabla \boldsymbol{g}_T^{\mathrm{T}} \boldsymbol{A} \nabla \boldsymbol{g}_T)^{-1}), \end{aligned} \tag{30}$$

where $\nabla \boldsymbol{g}_T \equiv \partial \boldsymbol{g}_T(\hat{\theta})/\partial \theta^{\mathrm{T}}$, assuming that $E(\boldsymbol{Z}^{\mathrm{T}} \zeta \zeta^{\mathrm{T}} \boldsymbol{Z})$ is finite and positive definite, and that $E[\lim_{T\to\infty} \mathrm{Var}(\sqrt{T} \boldsymbol{g}_T(\theta_0))] = \boldsymbol{S}$ is a finite, positive definite matrix.

With $\boldsymbol{A}$ efficiently chosen as proportional to $\boldsymbol{S}^{-1}$ in (30), the covariance matrix estimate reduces to $[T \nabla \boldsymbol{g}_T^{\mathrm{T}} \boldsymbol{S} \nabla \boldsymbol{g}_T]^{-1}$. However, we estimate $\boldsymbol{S}$ by HAC with a Barlett Kernel,

$$\hat{\boldsymbol{S}} = \hat{\boldsymbol{s}}(0) + \sum_{k=1}^{K} \left(1 - \frac{k}{K+1}\right) \left(\hat{\boldsymbol{s}}(k) + \hat{\boldsymbol{s}}(k)^{\mathrm{T}}\right), \tag{31}$$

where $\hat{\boldsymbol{s}}(k)$ is the sample autocovariance of the errors of the moment conditions in (28) (e.g., Newey and West, 1987, Hall, 2005, p. 127).

9. Empirical results

We first estimate Model 1—the model in (26) using fixed effects as in (21). The states are: 1—Illinois, 2—Indiana, 3—Iowa, 4—Michigan, 5—Minnesota, 6—Missouri, 7—Ohio, 8—Wisconsin. Clearly, both the equation intercept α_i and the coefficients of the dummy variables $\phi_1, \ldots, \phi_8$ cannot be identified without some restriction. One common restriction is $\alpha_i = 0$, where all eight fixed effects are estimated. Alternatively, the average of the state effects can be zero (an averaging analogue to the random effects assumption). This can be done in Model 1 by including dummy variables for each of seven states and having an intercept. In what follows, the omitted reference state is Wisconsin. Thus, for example, $\alpha_1 + \phi_1$ is the coefficient for Illinois in the financial arbitrage equation. Model 2 uses the heterogeneous preference specification in (27) with Wisconsin also omitted as the reference state.

Table 1 presents the GMM estimates for the fixed effects model (Model 1) with heteroskedastic corrections as well as HAC estimators of the covariance matrix. Standard errors in both cases are similar in magnitude for most parameters. However, the standard error for the risk aversion parameter is considerably lower with the HAC estimator. Both present strong evidence against risk neutrality in favor of risk aversion. Given these estimates for $\alpha_F - \alpha_W$, the evidence is also convincing that the simple aggregate versions of the representative arbitrage conditions do not hold. In each case, the standard first-order conditions based on aggregate data overshoot zero leaving a positive and significant intercept α_i, $i = F, C, S, W$. In the ith equation and the jth state, the estimated intercept is $\hat{\alpha}_i + \hat{\phi}_j$. Using the first column to calculate average intercepts, they are .0860 for financial assets, .0215 for corn acreage, .0453 for soybean acreage, and .0039 for wheat acreage. The arbitrage condition with the largest mean deviation is for the stock market (financial assets) while wheat has the lowest. The same qualitative conclusion holds with Newey–West standard errors.

These results are unsurprising given the period under consideration. However, for a few states and assets, the result differs substantially from the average. For example, for wheat in Illinois, $\alpha_W = .0215$ and the fixed effect coefficient is $\phi_1 = -.0266$, indicating that the wheat arbitrage equation underestimates full arbitrage for Illinois by .0051 (a similar condition holds for Missouri and Minnesota wheat). The strong overshooting of the arbitrage conditions for other crops likely drives this. For example, for Illinois corn and soybeans, the wedges are .0139 and .0391, respectively. The states with the largest average deviations include Michigan, Ohio and Indiana.

Another issue is whether one should consider separate risk preferences for financial and cropland investors, which is in contrast to the model. This would involve specification of separate βs for each equation in (26). Though not reported, the computed quasi-likelihood statistic (which is asymptotically distributed as a $\chi^2(3)$ random variable under the null hypothesis, $H_0 : \beta_F = \beta_C = \beta_S = \beta_W$) is 1.27 with a p-value of .75. Hence, the evidence supports the conclusion of an integrated investor as modeled in (10).

Fixed effects often can be usefully ignored by differencing the data. Due to the small sample size in this case, differencing by equation rather than temporally appears more desirable. This has the advantage of eliminating possible misspecification due to the

Table 1
Estimated arbitrage conditions with fixed effects.

Parameter	Model 1 Fixed effects, Panel White Standard errors[a]	Model 1 Fixed effects, Panel Newey–West Standard errors
α_F	.1040 (.0127)*	.1053 (.0102)*
α_C	.0395 (.0059)*	.0405 (.0050)*
α_S	.0633 (.0083)*	.0657 (.0073)*
α_W	.0219 (.0051)*	.0215 (.0047)*
β	.08504 (.0283)*	.0705 (.0241)*
ϕ_1	−.0271 (.0074)*	−.0266 (.0057)*
ϕ_2	−.0069 (.0078)	−.0069 (.0064)
ϕ_3	−.0288 (.0093)*	−.0031 (.0093)*
ϕ_4	−.0001 (.0051)	.0026 (.0041)
ϕ_5	−.0291 (.0083)*	−.0304 (.0076)*
ϕ_6	−.0244 (.0063)*	−.0244 (.0054)*
ϕ_7	−.0075 (.0061)	−.0068 (.0052)
J-Statistic	57.3634 (.037)[b]	33.3908 (.761)

[a] The reported coefficients for risk aversion, the β's, and their standard errors are scaled up by 10^5. The omitted state in the fixed effects model is Wisconsin. See the text for the pairing of α's with states. Durbin–Watson statistics are very similar for the two columns. For the second column, the statistics are: 1.594, .936, .999 and 1.353, respectively, for the four equations (financial, corn, soybeans, and wheat).
[b] For the J-test, p-values are reported in parentheses.
* Denotes statistical significance at the .01 level.

Table 2
Difference in excess returns under homogeneous preferences.

Coefficient	Differenced Model 1[a] Fixed effects, Panel White Standard errors	Differenced Model 1 Fixed effects, Panel Newey–West Standard errors
β	.000000865 (.000000562)	.000001108 (.000000510)
$\alpha_{C,F}$	−.0737* (.0209)	−.05947 (.0150)
$\alpha_{S,F}$	−.0468* (.0145)	−.0368 (.0105)
$\alpha_{W,F}$	−.0879* (.02294)	−.0728 (.0173)
J-Statistic	42.7996[b] (.000)	27.1752 (.018)

[a] Durbin–Watson's are very similar for the two columns. For the second column, they are 1.412, 1.4777, and 1.6874, respectively, for the corn, soybean, and wheat equations.
[b] For the J-test, p-values are reported in parentheses.
* Denotes statistical significance at the .01 level.

Table 3
Estimated arbitrage conditions with heterogeneous preferences.

Parameter	Model 2[a] Panel White Standard errors	Model 2 Panel Newey–West Standard errors
α_F	.0991 (.0194)*	.0941 (.0129)*
α_C	.0280 (.0061)*	.0259 (.0042)*
α_S	.0575 (.0083)*	.0552 (.0089)*
α_W	.0050 (.0031)	.0039 (.0022)
β	. 1236 (.0474)*	.1334 (.0369)*
β_1	−.1534 (.0256)*	−.1514 (.0204)*
β_2	−.1132 (.0028)*	−.1239 (.0270)*
β_3	−.0189 (.0263)	−.0329 (.0222)
β_4	−.0828 (.0183)*	.0998 (.0126)*
β_5	−.0830 (.0266)*	−.0864 (.0274)*
β_6	−.1005 (.0242)*	−.0926 (.0169)*
β_7	−.1541 (.0329)*	−.1519 (.0310)*
J-Statistic	52.0327 (.041)[b]	31.0758 (.702)

[a] The reported coefficients for risk aversion, the β's, and their standard errors are scaled up by 10^5. The omitted state in the heterogeneous risk aversion model is Wisconsin. Durbin–Watson statistics for the two columns are very similar. For column two, they are 1.5997, .9332, 1.0083 and 1.332, respectively, for the financial, corn, soybean, and wheat equations.
[b] For the J-test, p-values are reported in parentheses.
* Denotes statistical significance at the .01 level.

included state effects. Subtracting the excess return for financial assets from the three crops defines excess returns in terms of the difference of returns on crops compared to the stock market. Given the unusual stock market period, we might expect negative intercepts. Table 2 presents the estimates for the fixed effects model with differencing. The estimated risk aversion values are similar but higher for HAC covariance estimates. However, $.1108 \times 10^{-5}$ is not statistically different from $.7050 \times 10^{-6}$; the t-value is 1.1649 under the null hypothesis of equality of the coefficients. Moreover, all the intercepts in Table 2 are negative, indicating that the arbitrage condition on average undershoots zero. Taking the absolute value of the coefficients, the largest is wheat with soybeans slightly smaller than corn. Thus, the most significant negative departure from the arbitrage conditions is for wheat, which has significantly lower risk-adjusted returns than the market portfolio. Durbin–Watson statistics are also considerably improved.[10] In summary, the estimates of the more parsimonious parametric structure in Table 2 seem commensurate with the results of fixed effects found in Table 1.

Turning to the heterogeneous preference structure of Model 2, Table 3 presents results for both White and Newey–West covariance estimation. Wisconsin is again omitted and all states are measured with β plus the state-level coefficient for the other shifters.

10 Durbin–Watson statistics are computed as $d_i = \frac{\sum_{j=1}^{8}\sum_{t=2}^{9}(e_{i,j,t}-e_{i,j,t-1})^2}{\sum_{j=1}^{8}\sum_{t=1}^{8}e_{i,j,t}^2}$, $i = F, C, S, W$.

Thus, using the first column, for example, risk aversion is measured for Illinois as .000001236–.000001534 which would suggest risk-loving preferences. Similarly, Ohio is estimated to have a negative level of risk aversion. However, the corresponding t-values for Illinois and Ohio, respectively, for a null hypothesis of risk neutrality using the White standard errors are −.5451 and −.5083. Hence, no clear evidence of risk preference is indicated for these states.

Consistent with Table 1, the arbitrage conditions tend to overshoot zero. All the α coefficients are positive; however, the wheat alpha coefficient provides relatively weak evidence that it is not zero. Yet, the standard error is much lower under Newey–West standard errors with a p-value of .074.

Performing a nested test for homogeneity of risk preferences (with the null hypothesis $H_0 : \beta_2 = \beta_3 = \cdots = \beta_8 = 0$ against a two-sided alternative) using a quasi-likelihood ratio test yields a statistic of 1.68 for which $TQ(\hat{\theta}) \xrightarrow{D} \chi^2(7)$. This provides virtually no support for rejecting the hypothesis of a single risk preference parameter (the upper tail p-value is approximately .95). Hence, little evidence supports further pursuit of Model 2 (although no completely correct nested or non-nested test of Model 1 versus Model 2 was done due to the paucity of the data).

Another issue of model performance is the incidence of positive marginal utilities. For Table 1, marginal utilities ranged from approximately .80 to 1.20 with a mean of approximately .98. The sample standard deviation was approximately .088. Thus, marginal utilities were positive as required with considerable variation. Similar conclusions emerge from Table 2 as well.

Finally, we consider whether our estimates of risk aversion are credible in magnitude. Estimates of .00000071–.00000085 in Table 1 represent a lower level of risk aversion than measured by many studies. However, the level of wealth in our model is different than in many studies. Saha et al. (1994) in their Table 2 report several estimates of absolute risk aversion in and out of agriculture ranging from 0 to 14.75 with all but one study finding estimates above .0012. Many of these seem implausibly large and are certainly large in comparison with our estimates. For example, using the methodology of Rabin (2000), Just and Peterson (2003) have found the estimates of Saha, Shumway, and Talpaz, ranging from .0045 to .0083, to be implausibly high.

To illustrate simply by equating $E(e^{-\beta X}) = e^{-\beta c}$, where X is a random payoff and c is a certain payoff, a person with absolute risk aversion equal to .2 would reject a bet with a 99% chance of \$ 1000,000 and 1% chance of nothing in favor of a certain \$ 24. Or, a person with absolute risk aversion of .01 would reject the bet in favor of a certain \$ 461. Each of these has a risk premium over \$ 989,000 on a nearly sure gain of \$ 1000,000. These are levels commonly estimated but appear to represent unreasonably cautious behavior. By comparison, our estimate of .00000085 in the first column of Table 1 compares to a certain \$ 985,527 and a risk premium of \$ 5659 for this bet. This absolute risk aversion implies relative risk aversion from .188 to .661 with relative risk aversion .352 at mean wealth. This is lower than most estimates in other settings. However, we believe these results are more plausible given the nature of agricultural risk and the self-selected choices of producers to enter agriculture. We suggest that an advantage of our arbitrage model is to avoid over-attribution of behavior to risk aversion, a proven weakness of some preceding approaches.

10. Conclusions

This article has considered asset choice for a rational, expected utility maximizing, and forward-looking producer/investor/consumer. After deriving the optimal conditions for choice by backward recursion, Euler equations for consumption and asset accumulation are derived. Of particular interest are the arbitrage conditions for a point in time involving expected future returns on investment multiplied by the marginal utility of wealth. These conditions do not require the measurement of consumption but require reasonably good wealth measurements.

The model is estimated using state per-farm aggregates for net worth, and a decade of data from the 1990s for the North Central region of US crop production as well as market and bond returns. Due to the relatively short time series, CARA risk preferences are specified. The risk preference parameter is statistically different from zero and estimated to be positive indicating risk aversion. There is no evidence for segmented risk preferences where the risk preference parameter differs by asset. Further, the hypothesis of homogeneity of risk preferences against the hypothesis of state differences in risk preferences cannot be rejected. Thus, a single risk preference parameter rationalizes the data.

However, evidence suggests that the standard arbitrage equilibrium does not hold, either due to aggregation errors or the short anomalous time period. Fixed effects are added to the econometric model. These are statistically significant deviations from the standard model of arbitrage equilibrium and may prove useful in other settings.

Due to the annualized nature of agricultural crops, long time series comparable to off-farm financial returns are unlikely to be available for future research. However, a longer series of net worth or consumption at the disaggregated level or development of a continuous-time production approach, such as with some types of livestock, may offer fruitful possibilities to continue and improve on this line of research.

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