16 AGGREGATION AND ARBITRAGE IN JOINT PRODUCTION

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Abstract: *Two common problems in econometric models of production are aggregation and unobservable variables. Many production processes are subject to production shocks, hence both expected and realized output is unknown when inputs are committed. Expectations processes are notoriously difficult to model, especially when working with aggregated data or risk averse decision makers. Duality methods for the incomplete systems of consumer demand equations are adapted to the dual structure of variable cost function in joint production. This allows the identification of necessary and sufficient restrictions on technology and cost so that the conditional factor demands can be written as functions of input prices, fixed inputs, and cost. These are observable when the variable inputs are chosen and committed to production, hence the identified restrictions allow ex ante conditional demands to be studied using only observable data. This class of production technologies is consistent with all von Neumann-Morgenstern utility functions when ex post production is uncertain. We then derive the complete class of input demand systems that are exactly aggregable, can be specified and estimated with observable data, and are consistent with economic theory for all von Newmann/Morgenstern risk preferences. We extend this to a general and flexible class of input demand systems that can be used to nest and test for aggregation, global economic regularity, functional form, and flexibility. The theory is applied to U.S. agricultural production and crop acreage allocation decisions by state for the years 1960-1999. Ongoing work includes applying this model to a recently updated data set created by the USDA/ERS through 2004 and estimating the intensive and extensive margin effects for state-level crop production with a stochastic dynamic programming model of risk aversion, asset management, and adjustment costs.*

Key Words: Aggregation, asset management, functional form, input demand, rank, risk

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16.1 Introduction

Analysis of multi-product behavior of firms is common in agricultural economics. Techniques of analysis might be based on the distance or production functions, or profit, revenue, or cost functions (Färe and Primont 1995; Ball 1988; Just, Zilberman, and Hochman 1988; Shumway 1983, Lopez 1983; Akridge and Hertel 1986). There is a large literature on functional structure and duality that helps guide empirical formulations and testing based on concepts of non-jointness and separability (Lau 1972, 1978; Blackorby, Primont and Russell 1977; Chambers 1984). Specifications of non-jointness generally reduce to some form of additivity (Hall 1973; Kohli 1983). Separability in some partition of inputs or outputs often results in separability in a similar partition of prices so long as aggregator functions are homothetic (e.g., Blackorby, Primont and Russell 1977; Lau 1978). Such restrictions on technology guide empiricists as they think about aggregation based on functional structure.

Two ubiquitous problems in the econometric modeling of production are aggregation and whether or not the variables are observable. Aggregation is unavoidable and useful. Many production processes are subject to production shocks. Planned output and product prices are unobservable when inputs are committed. Expectations processes are difficult to model, especially when working with aggregate data or decision makers who are averse to risk. An alternative solution that avoids this issue might prove useful in applied production analyses.

This paper presents a new class of variable input demand systems recently derived by LaFrance and Pope (2008b, 2010). The demand models in this class can be estimated with observable data, are exactly aggregable, and are consistent with economic theory for all risk preferences. LaFrance and Pope also extend this class to a general, flexible class of input demands that can be used to nest and test for aggregation, global economic regularity, functional form, and flexibility. Almost all existing input demand systems are restricted cases of this class. However, the focus of this study is on flexible, exactly aggregable full rank input demand systems.

16.2 The Production Model and Two Results

The neoclassical model of conditional demands for variable inputs with joint production, fixed inputs, and production uncertainty is

$$\mathbf{x}(\mathbf{w},\bar{\mathbf{Y}},\mathbf{z}) = \arg\min\left\{\mathbf{w}^{\top}\mathbf{x} : F(\mathbf{x},\bar{\mathbf{Y}},\mathbf{z}) \le 0\right\}, \tag{16.1}$$

where $\mathbf{x} \in \mathbb{R}_+^{n_x}$ is an n_x–vector of variable inputs, $\mathbf{w} \in \mathbb{R}_+^{n_x}$ is an n_x–vector of variable input prices, $\bar{\mathbf{Y}} \in \mathbb{R}_+^{n_y}$ is an n_y–vector of planned outputs, $\mathbf{z} \in \mathbb{R}_+^{n_k}$ is an n_z–vector of fixed inputs, and $F : \mathbb{R}_+^{n_x} \times \mathbb{R}_+^{n_y} \times \mathbb{R}_+^{n_z} \to \mathbb{R}$ is the joint production function (the boundary of a closed and convex production possibilities set with free disposal of one or more inputs and one or more outputs). The variable cost function is $c(\mathbf{w},\bar{\mathbf{Y}},\mathbf{z}) \equiv \mathbf{w}^{\top}\mathbf{x}(\mathbf{w},\bar{\mathbf{Y}},\mathbf{z})$.[1] We assume throughout that the production process is subject to supply shocks of the general form

$$\mathbf{Y} = \bar{\mathbf{Y}} + \mathbf{h}(\bar{\mathbf{Y}},\mathbf{z},\boldsymbol{\varepsilon}),\ E\left[\mathbf{h}(\bar{\mathbf{Y}},\mathbf{z},\boldsymbol{\varepsilon}) \mid \mathbf{x},\bar{\mathbf{Y}},\mathbf{z}\right] = \mathbf{0}. \tag{16.2}$$

In both static and dynamic settings, it is a simple matter to show that (16.1) is implied by (16.2) and the expected utility hypothesis for *all* von Newman-Morgenstern preferences (Pope and Chavas 1994).

Three issues are associated with the estimation of the equation system (16.1) above. First, planned/expected output is a vector of latent, unobservable variables. To estimate (16.1) directly, therefore, one needs to either identify and estimate the planning and expectations formation process or address errors in variables associated with using the observable $\mathbf{Y}$ in place of the theoretically correct unobservable $\bar{\mathbf{Y}}$ in the conditional demand equations (Pope and Chavas 1994). There also is a fairly large literature which proposes various approaches to the specification of *ex ante* cost functions when output is uncertain under potentially risk-averse behavior (e.g., Pope and Chavas 1994; Pope and Just 1998; Chambers and Quiggin 2000; Chavas 2008).

The essential problem is that if inputs are applied *ex ante* under stochastic production, then the outputs aren't observed. One straightforward solution is to make the correct assumptions such that *c* exists in some quantity (or quantities). LaFrance and Pope (2010) identify the necessary and sufficient condition to consistently estimate conditional input demands as functions of variables that are observable when the inputs are committed to production – prices of the inputs, the levels of quasi-fixed inputs, and the variable cost of production – so that the variable input demands can be written as

$$\mathbf{x}(\mathbf{w},\bar{\mathbf{Y}},\mathbf{z}) = \mathbf{g}(\mathbf{w},\mathbf{z},c(\mathbf{w},\bar{\mathbf{Y}},\mathbf{z})). \tag{16.3}$$

This is not the typical approach to formulating conditional demands. But it makes particular sense in situations like agriculture where output is observed only *ex post*. We restate the fundamental result of LaFrance and Pope (2010) on this question as follows.

Proposition 1*: The variable input demand equations have the structure* (16.3) *if and only if the variable cost function has the structure*

$$c(\mathbf{w},\bar{\mathbf{Y}},\mathbf{z}) \equiv \tilde{c}(\mathbf{w},\mathbf{z},\theta(\bar{\mathbf{Y}},\mathbf{z})), \tag{16.4}$$

[1] The nonstandard notation of a bold and capital $\bar{\mathbf{Y}}$ to denote expected/planned crop outputs is intended to distinguish this from expected crop yields per acre, $\bar{\mathbf{y}}$, which will be used later in the paper.

which in turn holds if and only if the joint production function has the structure,

$$F(\mathbf{x}, \bar{\mathbf{Y}}, \mathbf{z}) \equiv \tilde{F}(\mathbf{x}, \mathbf{z}, \theta(\bar{\mathbf{Y}}, \mathbf{z})). \tag{16.5}$$

In other words, outputs are weakly separable from the variable inputs. Although this result is somewhat restrictive in outputs,[2] it is quite flexible in the inputs. LaFrance and Pope call any such joint production process an *ex ante joint production system*.

The second issue, especially in agricultural supply modeling, is that aggregation across economic agents appears to be both an unavoidable and a useful data management strategy. For example, aggregation increases the statistical precision of economic variables like the yield per acre of a crop on a farm with several fields, in a county, state, region, or country. Aggregation from micro-level decision makers to macro-level data has been studied extensively in consumer behavior,[3] but has received little attention in the field of production economics (Chambers and Pope 1991, 1994; LaFrance and Pope 2008a, 2008b, 2010).[4]

Taking the class of input demands that can be specified in terms of observable variables as the point of departure, LaFrance and Pope (2008b) extend aggregation theory to multiple output production systems. Their main result on this issue requires the following definition. Let $\omega : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by

$$\omega(\eta(\mathbf{w}), \theta) = \begin{cases} \theta, & \text{if } K = 1, 2 \text{ or } K = 3 \text{ or } 4 \text{ and } \lambda'(s) = 0, \\ \\ \theta + \int_0^{\eta(\mathbf{w})} \left[\lambda(s) + \omega(s, \theta)\right] ds, & \text{if } K = 3 \text{ or } 4 \text{ and } \lambda'(s) \neq 0, \end{cases} \tag{16.6}$$

subject to $\omega(0, \theta) = \theta$ and $\partial\omega(0, \theta)/\partial s = \lambda(0) + \theta^2$, for some $\eta : \mathbb{R}_+^{n_x} \to \mathbb{R}$, and some

[2] Among other things it implies that marginal rates of product transformation are independent of the variable inputs and factor intensities. Of course, if these restrictions are deemed too strong, then an alternative approach to formulating the variable cost function becomes necessary.

[3] An important subset of the literature on this topic includes: Gorman (1953, 1961, 1965, 1981); Burt and Brewer (1971); Diewert (1971); Phlips (1971); Muellbauer (1975, 1976); Cicchetti, Fisher and Smith (1976); Howe, Pollak and Wales (1979); Deaton and Muellbauer (1980); Jorgenson, Lau and Stoker (1980, 1981, 1982); Russell (1983, 1996); Jorgenson and Slesnick (1984, 1987); Lewbel (1987a, 1988; 1989a, 1989b, 1990, 1991, 2003, 2004); Jorgenson (1990); Diewert and Wales (1987, 1988); Blundell (1988); Wales and Woodland (1988); Brown and Walker (1989); van Daal and Merkies (1989); Jerison (1993); Russell and Farris (1993, 1998); and Banks, Blundell, and Lewbel (1997), LaFrance, Beatty, Pope and Agnew (2000, 2002), Beatty and LaFrance (2005), LaFrance, Beatty and Pope (2005, 2006), and LaFrance and Pope (2009). The focus in the literature has been interior solutions and smooth demands. We remain faithful to this approach throughout the present paper.

[4] This is somewhat surprising in light of the fact that agricultural input demand and output supply data is generally only available at the county- or state-level of aggregation, with few exceptions.

$\lambda:\mathbb{R}\to\mathbb{R}$. With this mathematical device, LaFrance and Pope (2008b) characterize the class of full rank exactly aggregable ex ante joint production systems in the sense of Gorman (1981), Lau (1982), and Lewbel (1987a) with the following result.

> ***Proposition 2*: *Let*** $\pi:\mathbb{R}_+^{n_x}\to\mathbb{R}_{++}$ *be increasing, concave, and positively linearly homogeneous in* **w**; *let* $\eta:\mathbb{R}_+^{n_x}\to\mathbb{R}$, *be homogeneous of degree zero in* **w**; *let* $\alpha,\beta,\gamma,\delta:\mathbb{R}_+^{n_x}\to\mathbb{C}=\{a+\iota b,\ a,b\in\mathbb{R}\}$, *where* $\iota=\sqrt{-1}$, *the* $\alpha,\beta,\gamma,\delta\in\mathcal{C}^\infty$ *are homogeneous of zero degree in* **w** *and they satisfy* $\alpha\delta-\beta\gamma\equiv 1$; *and let* $f:\mathbb{R}_+\times\mathbb{R}_+^{n_z}\to\mathbb{C}$ *satisfy* $\partial f(c/\pi,\mathbf{z})/\partial(c/\pi)\neq 0$. *Then the variable cost function for any full rank, exactly aggregable, ex ante joint production system is a special case of*:

$$f\left(\frac{c(\mathbf{w},\bar{\mathbf{Y}},\mathbf{z})}{\pi(\mathbf{w})},\mathbf{z}\right)=\frac{\alpha(\mathbf{w})\omega(\eta(\mathbf{w}),\theta(\bar{\mathbf{Y}},\mathbf{z}))+\beta(\mathbf{w})}{\gamma(\mathbf{w})\omega(\eta(\mathbf{w}),\theta(\bar{\mathbf{Y}},\mathbf{z}))+\delta(\mathbf{w})}. \tag{16.7}$$

It is instructive to consider the structure of the input demand functions implied by (16.7). This can be accomplished by differentiating with respect to **w** and then applying Hotelling's/Shephard's lemma. To make the notation as compact as possible, let a bold subscript **w** denote a vector of partial derivatives with respect to the variable input prices and suppress the arguments of the functions $\{\alpha,\beta,\gamma,\delta,\eta,\pi\}$ to yield (after considerable straightforward but tedious algebra):

$$\begin{aligned}\mathbf{x}=\pi_{\mathbf{w}}\left(\frac{c}{\pi}\right)+\pi\Bigg\{&\left[\alpha\beta_{\mathbf{w}}-\beta\alpha_{\mathbf{w}}+(\alpha^2\lambda+\beta^2)\eta_{\mathbf{w}}\right]\left(\frac{1}{f_{c/\pi}}\right)\\&+\left[\beta\gamma_{\mathbf{w}}-\gamma\beta_{\mathbf{w}}+\delta\alpha_{\mathbf{w}}-\alpha\delta_{\mathbf{w}}-2(\alpha\gamma\lambda+\beta\delta)\eta_{\mathbf{w}}\right]\left(\frac{f}{f_{c/\pi}}\right)\\&+\left[\gamma\delta_{\mathbf{w}}-\delta\gamma_{\mathbf{w}}+(\alpha^2\lambda+\delta^2)\eta_{\mathbf{w}}\right]\left(\frac{f^2}{f_{c/\pi}}\right)\Bigg\}.\end{aligned} \tag{16.8}$$

Note that this system of demand equations has the finitely additive and multiplicatively separable structure that is required for exact aggregation over $(c/\pi,\mathbf{z})$ across agents (Lau 1982). Also note that there are up to four linearly independent functions of variable cost and fixed inputs and up to four linearly independent vectors of input price functions. This is the maximum rank for any exactly aggregable demand system (Lewbel 1987, LaFrance and Pope 2009).

A third issue with estimating conditional input demands is that quasi-fixed inputs, planned outputs, total variable cost, and variable input prices are jointly determined with the input demands. Consistent estimation requires methods that address this simultaneity. We address this in the empirical application to state-level demands for agricultural inputs.

16.3 Econometric Structure

Let $i=1,\cdots,I$ index states, $j=1,\cdots,N$ index variable inputs, and $t=1,\cdots,T$ index time. In general, the state-level variable input demand equations can be written as

$$x_{ijt} = f_{ij}(\mathbf{w}_{it}, k_{it}, c_{it}, t; \boldsymbol{\theta}) + u_{ijt},\ i=1,\cdots,I,\ j=1,\cdots,N,\ t=1,\cdots,T, \tag{16.9}$$

where $\mathbf{w}_{it}$ is the N×1 vector of (normalized) input prices, k_{it} is capital per acre, c_{it} is (normalized) variable cost per acre, $\boldsymbol{\theta}$ is a K×1 vector of parameters to be estimated, and u_{ijt} is a mean zero random error term. Suppose the errors are intertemporally correlated,

$$u_{ijt} = \sum_{j'=1}^{N} \phi_{jj'} u_{ij't-1} + v_{ijt},\ i=1,\cdots,I,\ j=1,\cdots,N,\ t=1,\cdots,T, \tag{16.10}$$

while the mean zero random variables v_{ijt} are uncorrelated across time, but correlated across inputs within each state, $E(\mathbf{v}_{i\bullet t}\mathbf{v}_{i\bullet t}^{\mathsf{T}}) = \boldsymbol{\Sigma}_i$, $\mathbf{v}_{i\bullet t} = [v_{i1t} \cdots v_{iNt}]^{\mathsf{T}}$. Let $\boldsymbol{\Sigma}_i^{-1} = \mathbf{L}_i\mathbf{L}_i^{\mathsf{T}}$ be a lower triangular Choleski factorization of the i^{th} state's covariance matrix. Then the typical element of $\boldsymbol{\varepsilon}_{i\bullet t} = \boldsymbol{\Sigma}_i^{-1/2}\mathbf{v}_{i\bullet t} = \mathbf{L}_i^{\mathsf{T}}\mathbf{v}_{i\bullet t}$ is $\varepsilon_{ijt} = \sum_{j'=1}^{N} \ell_{ijj'} v_{ij't}$. The mean zero, unit variance random variables, ε_{ijt}, now are uncorrelated across inputs and time, but are assumed to be correlated across space, $E(\varepsilon_{ijt}\varepsilon_{i'jt}) = \rho(d_{ii'}),\ j=1,\cdots,N,$ where $d_{ii'}$ is the geographic distance between states i and i', $\rho(0)=1$. The I×I matrix,

$$\mathbf{R} = \begin{bmatrix} 1 & \rho(d_{12}) & \cdots & \rho(d_{1I}) \\ \rho(d_{12}) & 1 & \cdots & \rho(d_{2I}) \\ \vdots & \vdots & \ddots & \vdots \\ \rho(d_{1I}) & \rho(d_{2I}) & \cdots & 1 \end{bmatrix}, \tag{16.11}$$

is symmetric, positive definite, and for simplicity, we assume $\mathbf{R}$ is constant across j.

16.4 Consistent Estimation and Inferences with Semi-Parametric GMM

Let $\mathbf{Z}_i$ denote the matrix of instruments for state i and let $\mathbb{N}_i = \mathbf{Z}_i(\mathbf{Z}_i^{\mathsf{T}}\mathbf{Z}_i)^{-1}\mathbf{Z}_i^{\mathsf{T}}$ the associated projection matrix.[5] Let $\boldsymbol{\tau} = [1 2 \cdots T]^{\mathsf{T}}$, and stack equation (16.9) by inputs and time. First, we use nonlinear two-stage least squares (NL2SLS) to estimate $\boldsymbol{\theta}$ consistently,

[5] In the empirical application, we use the same instruments for all states, so that $\mathbb{N}_i \equiv \mathbb{N}\ \forall\ i=1,\cdots,I$.

$$\hat{\boldsymbol{\theta}}_{2SLS} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \sum_{i=1}^{I} \left[\mathbf{x}_{i\bullet\bullet} - \mathbf{f}_{i\bullet}(\mathbf{w}_{i\bullet\bullet}, \mathbf{k}_{i\bullet}, \mathbf{c}_{i\bullet}, \tau; \boldsymbol{\theta}) \right]^{\mathsf{T}} \left(\mathbb{N}_i \otimes \mathbf{I}_N \right) \left[\mathbf{x}_{i\bullet\bullet} - \mathbf{f}_{i\bullet}(\mathbf{w}_{i\bullet\bullet}, \mathbf{k}_{i\bullet}, \mathbf{c}_{i\bullet}, \tau; \boldsymbol{\theta}) \right]. \quad (16.12)$$

This consistent estimator of $\boldsymbol{\theta}$ is then used to generate consistent estimates of the errors,

$$\hat{u}_{ijt} = x_{ijt} - f_{ij}(\mathbf{w}_{it}, k_{it}, c_{it}, t; \hat{\boldsymbol{\theta}}_{2SLS}),\ i = 1, \cdots, I,\ j = 1, \cdots, N,\ t = 1, \cdots, T. \quad (16.13)$$

Second, for $t = 2, \cdots, T$, we estimate the $N \times N$ intertemporal correlation matrix, $\boldsymbol{\Phi}$, by linear seemingly unrelated regressions (SUR) methods,

$$\hat{\boldsymbol{\Phi}} = \arg\min_{\boldsymbol{\Phi}} \left\{ \sum_{i=1}^{N} \sum_{t=2}^{T} (\hat{\mathbf{u}}_{i\bullet t} - \boldsymbol{\Phi}\hat{\mathbf{u}}_{i\bullet t-1})^{\mathsf{T}} \hat{\boldsymbol{\Sigma}}_i^{-1} (\hat{\mathbf{u}}_{i\bullet t} - \boldsymbol{\Phi}\hat{\mathbf{u}}_{i\bullet t-1}) \right\}. \quad (16.14)$$

One can complete this stage of estimation with $\hat{\boldsymbol{\Sigma}}_i = \mathbf{I}_N\ \forall\ i$ and iterate once on the state-specific cross-equation covariance matrices. Alternatively, one can start with the $\hat{\boldsymbol{\Sigma}}_i$ s calculated from the NL2SLS estimates for $\boldsymbol{\theta}$ with $\boldsymbol{\Phi} = [\mathbf{0}]$. Either approach gives consistent estimates for the elements of $\boldsymbol{\Phi}$ since the weight matrix does not affect consistency. The first method is robust to departures from the assumed covariance structure. The second method can be more efficient if the model is correct. We apply the first method in the empirical application.

Third, we construct consistent estimates of the spatially correlated error terms,

$$\hat{\varepsilon}_{ijt} = \sum_{j'=1}^{N} \hat{\ell}_{ijj'} \hat{v}_{ij't}, \quad (16.15)$$

where $\hat{v}_{ijt} = \hat{u}_{ijt} - \sum_{j'=1}^{N} \hat{\phi}_{jj'} \hat{u}_{ij't-1}$ and $\hat{\mathbf{L}}_i = [\hat{\ell}_{ijj'}]_{j,j'=1,\cdots N}$ satisfies $\tilde{\boldsymbol{\Sigma}}_i^{-1} = \hat{\mathbf{L}}_i \hat{\mathbf{L}}_i^{\mathsf{T}}$. We then calculate consistent sample estimates for the cross-state spatial correlations as,

$$\hat{\rho}_{ii'} = \sum_{j=1}^{N} \sum_{t=2}^{T} \hat{\varepsilon}_{ijt} \hat{\varepsilon}_{i'jt} \big/ N(T-1),\ i, i' = 1, \cdots, I. \quad (16.16)$$

We then use the $½I(I-1)$ spatial correlations to estimate the relationship between the spatial correlations and the geographic distance between states using robust nonlinear least squares to obtain $\hat{\mathbf{R}} = \left[\hat{\rho}(d_{ii'}) \right]$. In this study, we use a third-order exponential specification for the spatial correlation function,

$$\rho(d_{ii'}) = \exp\left\{ \eta_0 + \sum_{k=1}^{3} \eta_k d_{ii'} \right\}. \quad (16.17)$$

Fourth, let $\mathbf{R}^{-1} = \mathbf{Q}\mathbf{Q}^{\mathsf{T}}$, where $\mathbf{Q}$ is a lower triangular Choleski factorization of the inverse spatial correlation matrix, and write $\omega_{ijt} = \sum_{i'=1}^{I} q_{ii'} \varepsilon_{i't}$. Now, the random variables ω_{ijt} are mean zero, unit variance, and uncorrelated across inputs, states, and time. Replacing the unknown parameters and error terms with the consistent estimates developed with the above estimation steps, and substituting backwards recursively, we have

$$\begin{aligned}\hat{\omega}_{ijt} &= \sum_{i'=1}^{I} \hat{q}_{ii'} \hat{\varepsilon}_{i'jt} \\ &= \sum_{i'=1}^{I} \hat{q}_{ii'} \sum_{j'=1}^{N} \hat{\ell}_{i'jj'} \hat{v}_{i'j't} \\ &= \sum_{i'=1}^{I} \hat{q}_{ii'} \sum_{j'=1}^{N} \hat{\ell}_{i'jj'} \left(\hat{u}_{i'j't} - \sum\nolimits_{j''=1}^{N} \hat{\phi}_{j'j''} \hat{u}_{i'j''t-1} \right) \\ &\xrightarrow{P} \omega_{ijt},\end{aligned} \tag{16.18}$$

with $E(\omega_{ijt}) = 0,\ E(\omega_{ijt}^2) = 1,\ E(\omega_{ijt}\omega_{i'j't'}) = 0,\ (i, j, t) \neq (i', j', t')$. A final NL3SLS step of the form,

$$\hat{\boldsymbol{\theta}}_{3SLS} = \arg\min_{\boldsymbol{\theta}} \left\{ \sum_{i=1}^{I} \hat{\boldsymbol{\omega}}_{i\bullet\bullet}(\boldsymbol{\theta})^{\top} \left(\mathbb{N}_i \otimes \mathbf{I}_N \right) \hat{\boldsymbol{\omega}}_{i\bullet\bullet}(\boldsymbol{\theta}) \right\}, \tag{16.19}$$

gives consistent, efficient, asymptotically normal estimates of $\boldsymbol{\theta}$. White's heteroskedasticity consistent covariance estimator can be used for robustness to heteroskedasticity beyond the state-specific input demand covariance matrices.

16.5 Econometric Model, Data and Empirical Results

We apply this model of exactly aggregable demands for variable inputs that can be estimated with observable data to state-level data on farm labor, fuels and energy, agricultural chemicals, and materials for the period 1960-1999. This data has been compiled by the USDA/ERS and is described in detail in Ball, Halahan, and Nehring (2004). Land and capital are quasi-fixed inputs, and we include a time trend to proxy for technological change and other nonstationary economic forces.

The specific specification is a full rank three model adapted from LaFrance and Pope (2009) to variable costs in joint production,

$$f\left(c_t / w_{n,t}\right) = \alpha\left(\tilde{\mathbf{w}}_t / w_{n,t}, k_t, t\right) - \left[\frac{\beta\left(\tilde{\mathbf{w}}_t / w_{n,t}\right)}{\delta\left(\tilde{\mathbf{w}}_t / w_{n,t}\right) + \sqrt{\beta\left(\tilde{\mathbf{w}}_t / w_{n,t}\right)}\theta(\bar{\mathbf{Y}}_t, \mathbf{a}_t, k_t)} \right], \tag{16.20}$$

where $\quad f(x) = (x^{\kappa} + \kappa - 1)/\kappa,\ \kappa \in \mathbb{R}_+,$

$\alpha\left(\tilde{\mathbf{w}}_t / w_{n,t}, k_t, t\right) = \alpha_{n0} + \alpha_{n1}k_t + \alpha_{n2}t + (\boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 k_t + \boldsymbol{\alpha}_2 t)^{\top} \mathbf{g}\left(\tilde{\mathbf{w}}_t / w_{n,t}\right),$

with $\quad \mathbf{g}(\mathbf{x}) = [g(x_1) \cdots g(x_{n-1})]^{\top},\ \ g(x_j) = (x_j^{\lambda} + \lambda - 1)/\lambda,\ \lambda \in \mathbb{R}_+,\ \forall\ j,$

$$\beta\left(\tilde{\mathbf{w}}_t / w_{n,t}\right) = \mathbf{g}\left(\tilde{\mathbf{w}}_t / w_{n,t}\right)^{\intercal} \mathbf{B}\mathbf{g}\left(\tilde{\mathbf{w}}_t / w_{n,t}\right) + 2\boldsymbol{\gamma}^{\intercal}\mathbf{g}\left(\tilde{\mathbf{w}}_t / w_{n,t}\right) + 1,$$

and
$$\delta\left(\tilde{\mathbf{w}}_t / w_{n,t}\right) = \delta_n + \boldsymbol{\delta}^{\intercal}\mathbf{g}\left(\tilde{\mathbf{w}}_t / w_{n,t}\right),$$

The elements of the vector $\tilde{\mathbf{w}}_t = [w_{1,t}\, w_{2,t} \cdots w_{n-1,t}]^{\intercal}$ are the first $n-1$ variable input prices. The n^{th} variable input is farm labor. We treat this input asymmetrically from the other inputs in both the conditional mean and the stochastic part of the model. The translated Box-Cox functions f and g are observationally equivalent to the standard Box-Cox transformations. If $\kappa = 1$, then we have $f(x) = x$, while if $\kappa = 0$, then we have $f(x) = 1 + \ln x$. The same results apply to $g(x)$ for $\lambda = 1$ or 0, respectively. For all other values of $(\kappa, \lambda) \in \mathbb{R}_+^2$, we have functional forms of the PIGL class in input prices and cost, allowing us to nest this class of demand models with a rank three generalized translog and a rank three generalized quadratic production model. To conserve on and simplify the notation from this point forward, we drop the ~ over the first n–1 input prices and omit the ratio notation for cost and input prices by defining $N = n-1$.

Applying Hotelling's/Shephard's lemma to (16.20) gives the variable input demands for energy, chemicals, and materials in per acre expenditures for state i in year t as

$$\begin{aligned}\tilde{\mathbf{e}}_{i,t} = c_{i,t}^{1-\kappa}\Delta\left(w_{i,j,t}^{\lambda}\right)\Bigg\{&\boldsymbol{\alpha}_{0,i} + \boldsymbol{\alpha}_{1,i}k_t + \boldsymbol{\alpha}_{2,i}t + \left[\frac{f(c_{i,t}) - \alpha_i\left(\mathbf{w}_{i,t}, k_{i,t}, t\right)}{\beta\left(\mathbf{w}_{i,t}\right)}\right]\left[\mathbf{B}\mathbf{g}\left(\mathbf{w}_{i,t}\right) + \boldsymbol{\gamma}\right] \\ &+ \left[\mathbf{I}_N - \frac{\mathbf{B}\mathbf{g}\left(\mathbf{w}_{i,t}\right)\mathbf{g}\left(\mathbf{w}_{i,t}\right)^{\intercal}}{\beta\left(\mathbf{w}_{i,t}\right)}\right]\boldsymbol{\delta}\frac{\left[f(c_{i,t}) - \alpha_i\left(\mathbf{w}_{i,t}, k_{i,t}, t\right)\right]^2}{\beta\left(\mathbf{w}_{i,t}\right)}\Bigg\} + \mathbf{u}_{i,t},\; i = 1, \cdots . I,\; t = 1, \cdots, T.\end{aligned} \tag{16.21}$$

As discussed in the previous section, due to the 3-dimensional nature of the error covariance matrix, estimation is by a four-stage GMM procedure. The instruments we choose are the national averages of cost per acre, capital per acre, and normalized variable input prices lagged two periods, and the following general economy variables: real per capita disposable personal income; the unemployment rate; the real rate of return on AAA corporate 30-year bonds; the real manufacturing wage rate; the real index of prices paid by manufacturers for materials and components; and the real index of prices paid by manufacturers for fuel, energy and power. Per capita disposable personal income is deflated by the consumer price index for all items, while the aggregate wholesale price variables are deflated by the implicit price deflator for gross domestic product.

There are far too many parameters to present and discuss in detail in this paper. Thus, we will focus on a relatively small number of parameters of interest. We first present and discuss the properties of the error terms. The estimated 3×3 intertemporal autocorrelation matrix, with White/Huber robust asymptotic standard errors in parentheses, is:

$$\hat{\mathbf{\Phi}} = \begin{bmatrix} .464 & .110 & -.055 \\ (.054) & (.176) & (.070) \\ .0095 & .700 & .030 \\ (.0101) & (.047) & (.026) \\ .022 & -.116 & .711 \\ (.013) & (.062) & (.044) \end{bmatrix}. \qquad (16.22)$$

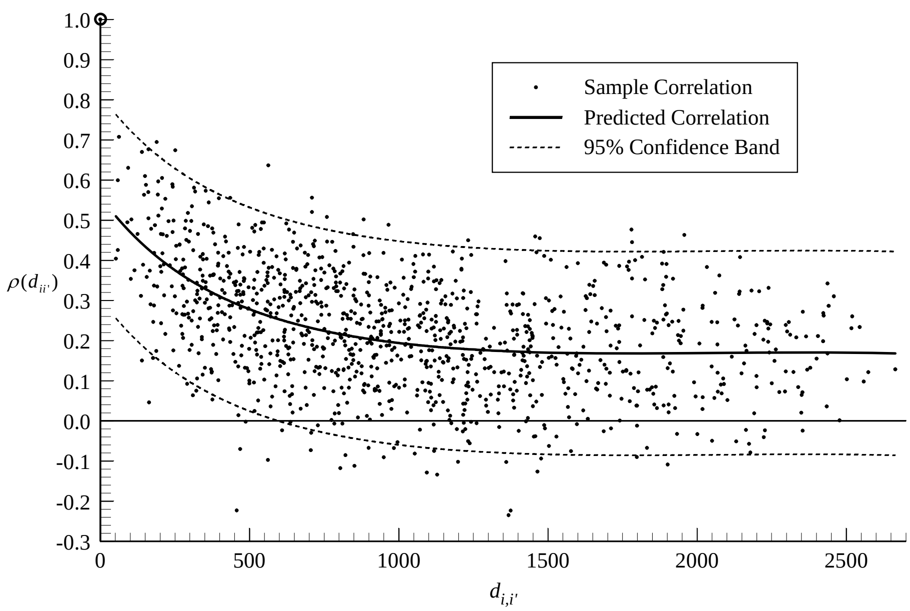
For all four variable inputs, the implied dynamics are stable, with the largest Eigen value of the 4×4 difference equation equal to 0.7, indicating no evidence of nonstationarity. In addition, the estimated error terms $\varepsilon_{i,j,t}$ from equation (16.15) above show no statistical evidence of further serial correlation.

The estimated spatial correlation function, with White/Huber robust standard errors in parentheses, is:

$$\hat{\rho}(d_{ii'}) = \exp\left\{ -\underset{(.066)}{.583} - \underset{(2.49\times10^{-4})}{1.80\times10^{-3}}\, d_{ii'} + \underset{(2.48\times10^{-7})}{8.86\times10^{-7}}\, d_{ii'}^2 - \underset{(6.81\times10^{-11})}{1.42\times10^{-10}}\, d_{ii'}^3 \right\}. \qquad (16.23)$$

A 2-dimensional plot of the empirical data, estimated correlation function, and 95% confidence band are presented in figure 1.

Figure 16.1 Spatial Correlation across the 48 Contiguous States.



There is no statistical evidence of remaining spatial correlation or heteroskedasticity in the cross-state error terms. Hence, we conclude that this estimation procedure reasonably captures the properties of the spatial/temporal error terms. One interesting property is that the spatial correlation is very flat from a distance of approximately 800 miles out to 2,500 miles, so that the error of states as far apart as Washington and Florida or Maine and California remain positively correlated. Failing to account for this property would lead to biased and inconsistent statistical inferences.

We turn next to a subset of the parameter estimates for the structural model. Table 1 presents the estimates of the parameters in the functions $\beta(\mathbf{w})$ and $\delta(\mathbf{w})$, along with the Box-Cox parameters, (κ, λ).

Table 16.1. Estimated Coefficients and Robust Asymptotic Standard Errors.

Coefficient	Estimate	Standard Error	T-Ratio	P-Value
β_{11}	.14118	.04877	2.89	.004
β_{12}	$-.113\times10^{-2}$	.00191	-.595	.552
β_{13}	$-.300\times10^{-2}$	.00350	-.856	.392
β_{14}	.38002	.06544	5.81	.000
β_{22}	$-.406\times10^{-5}$	$.386\times10^{-4}$	-.105	.916
β_{23}	$-.497\times10^{-5}$	$.723\times10^{-4}$	-.069	.945
β_{24}	$-.355\times10^{-2}$	$.509\times10^{-4}$	-.696	.486
β_{33}	$.182\times10^{-3}$	$.146\times10^{-3}$	1.25	.212
β_{34}	$-.822\times10^{-2}$	$.968\times10^{-2}$	-.849	.396
δ_1	$.714\times10^{-3}$	$.540\times10^{-4}$	1.32	.186
δ_2	$-.984\times10^{-5}$	$.119\times10^{-4}$	-.825	.409
δ_3	$-.197\times10^{-4}$	$.219\times10^{-4}$	-.901	.368
δ_4	$.170\times10^{-2}$	.00120	1.417	.158
κ	.327	.0320	10.233	.000
λ	.409	.0346	11.824	.000

It is clear from the table that the additional flexibility of the power functions in prices and variable cost due to the Box-Cox transformation is very important. Indeed, the industry standards of logarithmic or linear transformations of both of these sets of variables are rejected at any reasonable significance level. This is consistent with results we have found in the area of consumer choice behavior. On the other hand, rank three appears to seriously overfit this data, as evidenced by the insignificance of all of the δ_j parameter estimates. Indeed, a Wald test for the joint significance of these coefficients produces an asymptotically $\chi^2(4)$ test statistic of 2.11 with an implied probability value of

0.716. As a result, we currently are analyzing the structure of the simpler full rank two models in this general class.

16.6 Ongoing Work: Crops, Acres, and Capital Investment Decisions

Although the organizational form of farms can vary widely, a recent report by Hoppe and Banker (2006) finds that 98 percent of U.S. farms remained family farms as of 2003. In a family farm, the entrepreneur controls the means of production and makes investment, consumption, and production decisions. In this section, we develop and analyze a model of the intertemporal nature of these decisions. The starting point is a model similar in spirit to Hansen and Singleton's (1983), but generalized to include consumption decisions and farm investments as well as financial investments and production decisions. The additional variable definitions required for this are as follows:

W_t = beginning-of-period total wealth,
b_t = current holding of bonds with a risk free rate of return r_t,
f_t = current holding of a risky financial asset,
$p_{F,t}$ = beginning-of-period market price of the financial asset,
$\rho_{F,t+1}$ = dividend plus capital gains rate on the financial asset,
$a_{i,t}$ = current allocation of land to the i^{th} crop, $i = 1,\ldots,n_Y$,
A_t = total quantity of farm land,
$p_{L,t}$ = beginning-of-period market price of land,
$\rho_{L,t+1} = (p_{L,t+1} - p_{L,t}) / p_{L,t}$ = capital gain rate on land,
$\bar{y}_{i,t}$ = expected yield per acre for the i^{th} crop, $i = 1,\ldots,n_Y$,
$y_{i,t+1}$ = realized yield of the i^{th} crop,
$p_{Y_i,t+1}$ = end-of-period realized market price for the i^{th} farm product,
$\mathbf{q}_t$ = vector of quantities of consumption goods,
$\mathbf{p}_{Q,t}$ = vector of market prices for consumer goods,
m_t = total consumption expenditures,
$u(\mathbf{q}_t)$ = periodic utility from consumption.

As with all discrete time models, timing can be represented in multiple ways. In the model used here, all financial returns and farm asset gains are assumed to be realized at the end of each time period (where depreciation is represented by a negative asset gain). Variable inputs are assumed to be committed to farm production activities at the beginning of each decision period and the current period market prices for the variable inputs are known when these use decisions are made. Agricultural production per acre is realized stochastically at the end of the period such that

$$y_{i,t+1} = \bar{y}_{i,t}(1+\varepsilon_{i,t+1}), \; i = 1,\ldots,n_Y \,, \tag{16.24}$$

where $\varepsilon_{i,t+1}$ is a random output shock with $E(\varepsilon_{i,t+1}) = 0$. Consumption decisions are made at the beginning of the decision period and the current market prices of consumption good are known when these purchases are made. Utility is assumed to be strictly increasing and concave in $\mathbf{q}_t$. The total beginning-of-period quantity of land is $A_t = \boldsymbol{\iota}^\top \mathbf{a}_t$, with $\boldsymbol{\iota}$ denoting an n_Y–vector of ones. Homogeneous land is assumed with a scalar price, $p_{L,t}$.

We require two somewhat unusual pieces of matrix notation for this section. First, we define the $n\times n$ diagonal matrix $\Delta(x_j)$ such that x_j as the j^{th} main diagonal element for each $j = 1,\cdots,n.$ Second, the Hadamard/Schur product of two $n\times m$ matrices **A** and **B** is the matrix whose elements are element-by-element products of the elements of **A** and ***B***, $\mathbf{A}\bullet\mathbf{B} = \mathbf{C} \Leftrightarrow c_{ij} = a_{ij}b_{ij}\ \forall i, j.$ There are three ways to write the Hadamard/Schur product of two vectors, $\mathbf{x}\bullet\mathbf{y} = \Delta(\mathbf{x})\mathbf{y} = \Delta(\mathbf{y})\mathbf{x}.$

Revenue at $t + 1$ is the random price times production

$$R_{t+1} = \sum_{i=1}^{n_Y} (p_{Y_i,t+1}\overline{y}_{i,t}a_{i,t}(1+\varepsilon_{i,t+1})) \equiv (\mathbf{p}_{Y,t+1}\bullet\mathbf{a}_t\bullet\overline{\mathbf{y}}_t)^\top(\boldsymbol{\iota}+\boldsymbol{\varepsilon}_{t+1}). \tag{16.25}$$

Wealth is allocated at the beginning of period t to investments, the variable cost of production, and consumption,

$$W_t = b_t + f_t + p_{L,t}A_t + K_t + c_t(\mathbf{w}_t, \mathbf{a}_t, K_t, \overline{\mathbf{y}}_t) + m_t. \tag{16.26}$$

Although some costs occur at or near harvest (near $t + 1$), we include all costs in (16.26) at time t because they are incurred before revenues are received. Consumer utility maximization yields the quasi-convex indirect utility function conditioned on consumer good prices and expenditures,

$$\upsilon(\mathbf{p}_{Q,t}, m_t) \equiv \max_{\mathbf{q}\in R_+^{n_Q}} \left\{u(\mathbf{q}) : \mathbf{p}_{Q,t}^\top\mathbf{q} = m_t\right\}. \tag{16.27}$$

Realized end of period wealth is

$$W_{t+1} = (1+r_t)b_t + (1+\rho_{F,t+1})f_t + (1+\rho_{L,t+1})p_{L,t}A_t + (\mathbf{p}_{Y,t+1}\bullet\mathbf{a}_t\bullet\overline{\mathbf{y}}_t)^\top(\boldsymbol{\iota}+\boldsymbol{\varepsilon}_{t+1}). \tag{16.28}$$

Thus, the decision maker's wealth is increased by net returns on assets and farm revenue. The owner/operator decision maker's intertemporal utility function is assumed to be

$$U_T(\mathbf{q}_1,...,\mathbf{q}_T) = \sum_{t=0}^{T}(1+\rho)^{-t}u(\mathbf{q}_t). \tag{16.29}$$

The producer is assumed to maximize Von Neumann-Morgenstern expected utility of the discounted present value of the periodic utility flows from goods consumption.

By Euler's theorem, constant returns to scale implies linear homogeneity of the variable cost function in capital, land, and output. For the variable cost function derived and estimated in this paper, this implies

$$
\begin{aligned}
c_t(\mathbf{w}_t, \mathbf{a}_t, A_t, K_t, \bar{\mathbf{Y}}_t) \equiv & \frac{\partial c_t(\mathbf{w}_t, \mathbf{a}_t, A_t, K_t, \bar{\mathbf{Y}}_t)}{\partial \mathbf{a}_t^{\top}} \mathbf{a}_t + \frac{\partial c_t(\mathbf{w}_t, \mathbf{a}_t, A_t, K_t, \bar{\mathbf{Y}}_t)}{\partial A_t} A_t \\
& + \frac{\partial c_t(\mathbf{w}_t, \mathbf{a}_t, A_t, K_t, \bar{\mathbf{Y}}_t)}{\partial K_t} K_t \frac{\partial c_t(\mathbf{w}_t, \mathbf{a}_t, A_t, K_t, \bar{\mathbf{Y}}_t)}{\partial \bar{\mathbf{Y}}_t^{\top}} \bar{\mathbf{Y}}_t.
\end{aligned} \tag{16.30}
$$

The vector of expected crop outputs satisfies

$$
\bar{\mathbf{Y}}_t = \bar{\mathbf{y}}_t \bullet \mathbf{a}_t, \tag{16.31}
$$

where $\bar{y}_{j,t}$ is the expected yield per acre and $a_{j,t}$ is the number of acres planted for the j^{th} crop. The variable cost function might depend on time due to technological change or other dynamic forces, and the subscript t indicates this possibility. To distinguish quasi-fixed from variable inputs and to account for the possibility of hysteresis in agricultural investments, we allow for adjustment costs for total farmland and capital,

$$
C_{Adj}(A_t - A_{t-1}, K_t - K_{t-1}) = \tfrac{1}{2}\gamma_A (A_t - A_{t-1})^2 + \tfrac{1}{2}\gamma_K (K_t - K_{t-1})^2, \tag{16.32}
$$

with $\gamma_A, \gamma_K \geq 0$.

This problem is solved by stochastic dynamic programming working backwards recursively from the last period in the planning horizon to the first. In the last period, the optimal decision is to invest or produce nothing and consume all remaining wealth, i.e., $m_T = W_T$. Denote the last period's optimal value function by $v_T(W_T, A_{T-1}, K_{T-1})$. Then $v_T(W_T, A_{T-1}, K_{T-1}) = \upsilon(\mathbf{p}_{Q,T}, W_T)$ is the optimal utility for the terminal period. For all other time periods, stochastic dynamic programming using (16.26)-(16.29) to optimize agricultural production, asset ownership and net investment decisions in each period yields the (Bellman) backward recursion problem for arbitrary $t < T$, in this stochastic dynamic programming decision problem is

$$
\begin{aligned}
\ell_t = & \upsilon(\mathbf{p}_{Q,t}, m_t) + (1+r)^{-1} E_t \Big\{ V_{t+1} \Big[(1+r) b_t + (1+\rho_{F,t+1}) f_t \\
& + p_{L,t+1} A_t + (1+\rho_{K,t+1}) K_t + (\mathbf{p}_{Y,t+1} \bullet \bar{\mathbf{y}}_t \bullet \mathbf{a}_t)^{\top} (\boldsymbol{\iota} + \boldsymbol{\varepsilon}_{t+1}), A_t, K_t \Big] \Big\} \\
& + \lambda_t \Big\{ W_t - m_t - b_t - f_t - p_{L,t} A_t - K_t + c_t(\mathbf{w}_t, \mathbf{a}_t, A_t, K_t, \bar{\mathbf{y}}_t \bullet \mathbf{a}_t) \\
& - \tfrac{1}{2}\gamma_A (A_t - A_{t-1})^2 - \tfrac{1}{2}\gamma_K (K_t - K_{t-1})^2 \Big\} + \mu_t (A_t - \boldsymbol{\iota}^{\top} \mathbf{a}_t),
\end{aligned} \tag{16.33}
$$

where $E_t(\bullet)$ is the conditional expectation at the beginning of period t given information available at that point in time, $\rho_{K,t+1}$ is the proportional rate of change in the value of the capital stock K_t from the beginning of period t to the beginning of period t+1, λ_t is the shadow price for the beginning-of-period wealth allocation constraint, and μ_t is the shadow price for the land allocation constraint. The first-order, necessary and sufficient Kuhn-Tucker conditions are the two constraints and the following:

$$\frac{\partial \ell_t}{\partial m_t} = \frac{\partial \upsilon_t}{\partial m_t} - \lambda_t \le 0,\ m_t \ge 0,\ m_t \frac{\partial \ell_t}{\partial m_t} = 0; \tag{16.34}$$

$$\frac{\partial \ell_t}{\partial b_t} = E_t\left(\frac{\partial V_{t+1}}{\partial W_{t+1}}\right) - \lambda_t \le 0,\ b_t \ge,\ b_t \frac{\partial \ell_t}{\partial b_t} = 0; \tag{16.35}$$

$$\frac{\partial \ell_t}{\partial f_t} = (1+r)^{-1} E_t\left[\frac{\partial V_{t+1}}{\partial W_{t+1}}(1+\rho_{F,t+1})\right] - \lambda_t \le 0,\ f_t \ge 0,\ f_t \frac{\partial \ell_t}{\partial f_t} = 0. \tag{16.36}$$

$$\begin{aligned} \frac{\partial \ell_t}{\partial A_t} &= \left(1+r\right)^{-1} E_t\left(\frac{\partial V_{t+1}}{\partial W_{t+1}} p_{L,t+1} + \frac{\partial V_{t+1}}{\partial A_t}\right) \\ &-\lambda_t\left[p_{L,t} + \frac{\partial c_t}{\partial A_t} + \gamma_A(A_t - A_{t-1})\right] + \mu_t \le 0,\ A_t \ge 0,\ A_t \frac{\partial \ell_t}{\partial A_t} = 0; \end{aligned} \tag{16.37}$$

$$\begin{aligned} \frac{\partial \ell_t}{\partial K_t} &= (1+r)^{-1} E_t\left[\frac{\partial V_{t+1}}{\partial W_{t+1}}(1+\rho_{K,t+1}) + \frac{\partial V_{t+1}}{\partial K_{t+1}}\right] \\ &-\lambda_t\left[1 + \frac{\partial c_t}{\partial K_t} + \gamma_K(K_t - K_{t-1})\right] \le 0, K_t \ge 0, K_t \frac{\partial \ell_t}{\partial K_t} = 0; \end{aligned} \tag{16.38}$$

$$\begin{gathered} \frac{\partial \ell_t}{\partial \mathbf{a}_t} = \left(1+r\right)^{-1} E_t\left[\frac{\partial V_{t+1}}{\partial W_{t+1}}(\mathbf{p}_{Y,t+1} \bullet \bar{\mathbf{y}}_t) \bullet (\boldsymbol{\iota} + \boldsymbol{\varepsilon}_{t+1})\right] - \lambda_t \frac{\partial c_t}{\partial \bar{\mathbf{Y}}_t} \bullet \bar{\mathbf{y}}_t - \mu_t \boldsymbol{\iota} \le 0, \\ \mathbf{a}_t \ge \mathbf{0},\ \mathbf{a}_t^{\mathsf{T}} \frac{\partial \ell_t}{\partial \mathbf{a}_t} = 0; \end{gathered} \tag{16.39}$$

$$\begin{gathered} \frac{\partial \ell_t}{\partial \bar{\mathbf{y}}_t} = (1+r)^{-1} E_t\left[\frac{\partial V_{t+1}}{\partial W} \mathbf{p}_{Y,t+1} \bullet \mathbf{a}_t \bullet (\boldsymbol{\iota} + \boldsymbol{\varepsilon}_{t+1})\right] - \lambda_t \frac{\partial c_t}{\partial \bar{\mathbf{Y}}_t} \bullet \mathbf{a}_t \le \mathbf{0}, \\ \bar{\mathbf{y}}_t \ge \mathbf{0},\ \bar{\mathbf{y}}_t^{\mathsf{T}} \frac{\partial \ell_t}{\partial \bar{\mathbf{y}}_t} = 0. \end{gathered} \tag{16.40}$$

We also have the following implications of the envelope theorem:

$$\frac{\partial V_t}{\partial W_t} = \lambda_t;$$

$$\frac{\partial V_t}{\partial A_{t-1}} = \lambda_t \gamma_A (A_t - A_{t-1}); \tag{16.41}$$

$$\frac{\partial V_t}{\partial K_{t-1}} = \lambda_t \gamma_K (K_t - K_{t-1});$$

where the variables $\{\lambda_t, A_t, K_t\}$ are all evaluated at their optimal choices.

Combining the Kuhn-Tucker conditions with the results of the envelope theorem and assuming an interior solution for consumption, bonds, and risky financial assets, we obtain the standard Euler equations for smoothing the marginal utility of consumption and wealth,

$$\frac{\partial \upsilon_t}{\partial m_t} = E_t\left(\frac{\partial \upsilon_{t+1}}{\partial m_{t+1}}\right) = \frac{\partial V_t}{\partial W_t} = E_t\left(\frac{\partial V_{t+1}}{\partial W_{t+1}}\right) = \lambda_t = E_t(\lambda_{t+1}), \tag{16.42}$$

and the standard arbitrage condition for excess returns to risky financial assets,

$$E_t\left[(\rho_{F,t+1} - r)\frac{\partial V_{t+1}}{\partial W_{t+1}}\right] = 0. \tag{16.43}$$

From complementary slackness of the Kuhn-Tucker condition (16.40), for each crop we obtain the supply condition under risk,

$$E_t\left[\frac{\partial V_{t+1}}{\partial W_{t+1}}\left(p_{Y_i,t+1} - (1+r)\frac{\partial c_t}{\partial \bar{Y}_{i,t}}\right)\bar{Y}_{i,t}\right] = 0,\ i = 1,\cdots,n_y. \tag{16.44}$$

Hence, for each crop that is produced in positive quantity, this reduces to the well-known result that the conditional covariance between the marginal utility of future wealth and the difference between the ex post realized market price the marginal cost of production must vanish. The multiplicative factor $1+r$ is multiplied by ex ante marginal cost so that these two economic values are measured at a common point in time – in the present case at the end of the production period.

To obtain the arbitrage condition for the level of investment in agriculture, we combine the positive linear homogeneity property of the variable cost function in $(\mathbf{a}_t, A_t, K_t, \bar{\mathbf{Y}}_t)$ from equation (16.30) with the complementary slackness properties of the Kuhn-Tucker conditions (16.37)-(16.39),

$$0 = \frac{\partial \ell_t}{\partial \mathbf{a}_t^\top}\mathbf{a}_t + \frac{\partial \ell_t}{\partial A_t}A_t + \frac{\partial \ell_t}{\partial K_t}K_t, \tag{16.45}$$

which, after considerable rearranging and combining of terms, gives

$$\begin{aligned} E_t \Big\{ \partial V_{t+1} / \partial W_{t+1} \Big[& s_{K,t}(\rho_{K,t+1} - r) + s_{L,t}(\rho_{L,t+1} - r) + \pi_{t+1} \\ & + s_{K,t}\gamma_K \left(K_{t+1} - (2+r)K_t + (1+r)K_{t-1} \right) \\ & + s_{A,t}\gamma_A \left(A_{t+1} - (2+r)A_t + (1+r)A_{t-1} \right) \Big] \Big\} = 0, \end{aligned} \tag{16.46}$$

where $\rho_{L,t+1} = (p_{L,t+1} - p_{L,t}) / p_{L,t}$ is the proportional rate of change in the market value of farmland over period *t*, $s_{K,t} = K_t / (p_{L,t}A_t + K_t)$ is capital's share of the value of the total investment in agriculture in period *t*, $s_{L,t} = p_{L,t}A_t / (p_{L,t}A_t + K_t)$ is farmland's share of the value of the total investment in agriculture in period *t*, $s_{A,t} = A_t / (p_{L,t}A_t + K_t)$ is the ratio of the quantity of farmland to the market value of the investment in agriculture at the beginning of the production period, and

$$\pi_{t+1} = R_{t+1} - (1+r)c_t \tag{16.47}$$

is the ex post net return to crop production over the variable cost of production. The first three terms inside of the square brackets of equation (16.46) represent the total sum of the excess returns to agriculture, including this net return. The last two terms in square brackets capture the effects of adjustment costs for farm capital and farmland.

To implement this system of Euler equations, we assume that the indirect utility function for consumption goods is a member of the certainty equivalent class,

$$\upsilon(\mathbf{p}_{Qt}, m_t) = \frac{m_t}{\pi_C(\mathbf{p}_{Qt})} - ½\beta \left(\frac{m_t}{\pi_C(\mathbf{p}_{Qt})} \right)^2, \tag{16.48}$$

where $0 \le \beta < \pi_C(\mathbf{p}_{Qt}) / m_t \ \forall\ t$ and $\pi_C(\mathbf{p}_{Qt})$ is the consumer price index (CPI) for all items. Then the marginal utility of money in each period is

$$\lambda_t = \frac{1 - \beta \left[m_t / \pi_C(\mathbf{p}_{Qt}) \right]}{\pi_C(\mathbf{p}_{Qt})}. \tag{16.49}$$

This allows us to identify the effects of risk aversion separately from those of adjustment costs and hysteresis in agricultural investment decisions. Our current research effort focuses on choices for the aggregator, θ_t to implement this model and estimate equations with both national- and state-level data, which will allow us to draw coherent inferences on economic responses of agricultural producers and investors to input and output prices, risk, agricultural policies, and adjustment costs.

16.7 Conclusions

Common reasons for the choice of functional form for empirical demand analysis include parsimony, ease of estimation and interpretation, generality, flexibility, aggregation, and consistency with economic theory. This paper presents and applies a new, highly flexible structural model of micro-level production behavior that is exactly aggregable across cost differences between producers. We applied this model to a panel of state-level data on variable input choices in U.S. agriculture with a 3-dimensional semi-parametric version of the generalized method of moments. We also develop a framework to incorporate the results of the model for variable input demands within a general life cycle model of investment and agricultural asset management under risk.

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