Homogeneity and Supply

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Supply functions in the ubiquitous Gorman class are examined for their homogeneity properties. Homogeneity places surprisingly strong restrictions on functional forms. These forms facilitate testing of aggregability given homogeneity or homogeneity given aggregability or testing both.

Key words: functional form, homogeneity, supply.

Economists' motivations for choice of functional form include ease of implementation and interpretation, needed flexibility, and aggregability. For demand applications, one of the late Terence Gorman's great legacies is an attractive generalization of previous work on representative consumers (Gorman 1953, 1961; Muellbauer 1975, 1976; Deaton and Muellbauer 1980a, 1980b) called a Gorman system of Engel curves or a Gorman system in short (Gorman 1981; Lewbel 1987, 1989). A Gorman system readily satisfies exact aggregation. For both consumer and producer applications, this legacy is deeply embedded in agricultural economics, as evidenced by the many applications that take the Gorman class of functions (additive in functions of the aggregated variable) as the starting point for empirical analysis (Shumway 1995; Shumway and Lim 1993; Green and Alston 1990; Chambers and Pope 1991; LaFrance et al. 2002).

Gorman class functional forms are ubiquitous because they are easily interpretable, flexible, and have the added advantage of possessing convenient aggregation properties. However, many applications to agricultural producer behavior focus on a single commodity (Adams and Behrman 1976; Aradhyula and Holt 1989; Askari and Cummings 1977; Azzam and Yanagida 1987; Braulke 1982; Brester 1996; Burt and Worthington 1988; Debertin and Pagoulatos 1992; Eckstein 1985; Griliches 1960; Hallam 1990; Holt and Moschini 1992; Kaiser, Streeter, and Liu 1988; LaFrance and Burt 1983; LaFrance and de Gorter 1985; Levins 1982; Lohr and Park 1992; Marsh 1994, 1999; Milligan 1978; Nerlove 1958, 1959, 1979; Nerlove and Addison 1958; Rucker, Burt, and LaFrance 1984; and Wickens and Greenfield 1973). In these cases, one can either employ a popular Gorman specification (without the system restrictions) or specify any appropriate representation of interest. In the former case, one might use a functional form that is unnecessarily inflexible because choosing from a system "off of the shelf" may have imbedded restrictions that come from the economic theory of systems, for example, symmetry and adding up of demand, expenditure or cost, or symmetry from profit maximization. That is, in a single equation, a researcher could not impose any of the cross-equation restrictions such as symmetry or adding up but would typically impose homogeneity (and usually expect but not impose nonnegativity and monotonicity in the own price). If one chooses an arbitrary form, the imposition of homogeneity may seem trivial by merely deflating by some price or index of prices. However, this can easily destroy aggregability (Lewbel 1989). Further, this may not allow a nested approach for testing homogeneity, which is notoriously rejected in empirical work (Deaton and Muellbauer 1980a, 1980b; Shumway 1995).

The approach that we take is to isolate the homogeneity condition within the general Gorman class of supply functions. Language and notation focus on producer supply behavior, but our results apply to a variety of other settings, for example, a factor demand equation or a normalized profit function.[1] To

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[1] All of the results apply—with appropriate changes in notation—to a single consumer demand equation and a supply or input demand equation with profit or cost replacing output price. However, we feel that the standard supply case is most useful and illuminating. Furthermore, when applying our results to a system, the flexibility in the main proposition of this article is restricted in a complete system of equations due to the additional properties of symmetry and adding up.

Amer. J. Agr. Econ. xx(x) (xx 2008): 1–7

DOI: 10.1111/j.1467-8276.2007.01130.x

maximize the empirical relevance of the analysis and reduce the notational clutter, a single supply equation is considered. In doing so, we are not advocating the single equation approach, which sacrifices attractive opportunities to model and test supply and demand symmetries, concavity, and adding up. However, because homogeneity is an equation-by-equation concept, the results transfer immediately to any number of equations with the Gorman structure. This approach has the advantage of seeing exactly what restrictions homogeneity alone places on a Gorman system.[2]

Our results are novel and, to us, they are striking. We began this inquiry conjecturing that homogeneity might not substantially restrict or guide the choice of functional form. This turns out to be far from true. We find that supply functions must include explicit sums and products of power, logarithmic, and trigonometric functions and must do so in rather explicit ways. We find that the set of possible homogeneous functional forms depends crucially on the number of functions of the aggregated variable being utilized. That said, there is a great deal of flexibility remaining to measure supply response and a great deal that can be learned from considering homogeneity in isolation from other properties of a supply function. We present a class of functional forms that are homogeneous and can be imposed directly in applied work, or that can be perturbed to test for homogeneity or aggregability.[3]

Gorman Class Functions

The rationale and approach for specifying an aggregable supply system follows Chambers and Pope (1991, 1994). By focusing on supply, q, and given heterogeneity in output price due to temporal and spatial reasons, it is reasonable to specify supply as

$$(1) \qquad q = \sum_{k=1}^{K} \alpha_k(\boldsymbol{w}) h_k(p)$$

with K smooth functions of input prices, α_k: $\mathbb{R}^n_{++} \to \mathbb{R}$, $k = 1, \ldots, K$, times K smooth functions of output price, h_k: $\mathbb{R}_{++} \to \mathbb{R}$, $k = 1, \ldots, K$.[4] We also could add fixed inputs or technical change variables. However, to avoid notational clutter, for the present such shifters can be subsumed in a and $\boldsymbol{h}$. The functional form in (1) defines the Gorman-class of functions. Given that supply functions are positive valued and increasing in the output price, we expect that $\sum_{k=1}^{K} \alpha_k(\boldsymbol{w}) h_k(p) > 0$ and that $\sum_{k=1}^{K} \alpha_k(\boldsymbol{w}) h'_k(p) > 0$.

Homogeneity (0°) of q in $(\boldsymbol{w}, p)$ is most basically described by[5]

$$(2) \qquad q(t\boldsymbol{w}, tp) = q(\boldsymbol{w}, p), \ \forall\, t > 0\,.$$

Given the smoothness assumption, homogeneity also can be written in terms of the Euler equation (Allen 1938)

$$(3)$$

$$\frac{\partial q(\boldsymbol{w}, p)}{\partial \boldsymbol{w}^{\mathrm{T}}} \boldsymbol{w} + \frac{\partial q(\boldsymbol{w}, p)}{\partial p} p = \sum_{k=1}^{K} \frac{\partial \alpha_k(\boldsymbol{w})}{\partial \boldsymbol{w}^{\mathrm{T}}} \boldsymbol{w} h_k(p) + \sum_{k=1}^{K} \alpha_k(\boldsymbol{w}) h'_k(p) p = 0.$$

One could immediately impose homogeneity by dividing by p or a w_i in (1). In the first

[2] One of the amazing, and at times frustrating, aspects of Gorman (1981) is that he simultaneously imposes symmetry, adding up, and homogeneity so that the role of each is unclear.

[3] An Appendix containing proofs of the main results is available on AgEconSearch as LaFrance and Pope (2008). This Appendix also discusses a technical property required for the supply function not to have any mathematical or empirical redundancies. Specifically, both the K input price functions, $\{\alpha_1, \ldots \alpha_k\}$, and the K output price functions, $\{h_1, \ldots, h_k\}$, must be linearly independent across the K–dimensional constants over the interior of their respective domains of definition. We also assume throughout that all of these functions are smooth, i.e., $\alpha_k, h_k \in \ell^\infty, \forall\, k = 1, \ldots, K$. Interested readers are referred to the Appendix and the references cited there for additional details.

[4] As a reviewer pointed out, if one includes the origin in the domain of $(\boldsymbol{w}, p)$, no non-constant functions exist that satisfy 0° homogeneity globally. In addition, our main result obtains log and power functions of the ratios $p/\alpha_k(\boldsymbol{w}), k = 1, \ldots, K$, with exponents that can be greater than, equal to, or less than one. Ex post, this requires that we restrict the domain of $(\boldsymbol{w}, p)$ to $\mathbb{R}^n_{++} \times \mathbb{R}_{++}$ and the range of α to $\mathbb{R}^{\mathrm{K}}_{++}$.

[5] Homogeneous functions are the subject of a longstanding analysis in the mathematical theory of Lie transformation groups (Campbell 1903; Lie 1880, English translation and commentary in Hermann 1975; and Olver 1995). The basic representation for 0° homogeneity in equation (2) defines what is known in the mathematical field of differential topology as a dilation group—i.e., the supply function is invariant to any one parameter dilation of the form $(\boldsymbol{w}, p) \to (t\boldsymbol{w}, tp), t > 0$. For example, Olver (1995, p. 46) shows that in the two-dimensional plane (i.e., where w is a scalar), the only local solutions to this problem are p/w, w/p, or some monotonic transformation. Since both of these ratios are not defined when the denominator vanishes, there is no global solution to the homogeneity problem even in the plane. However, market prices are positive, and we are free to restrict the domain of the supply function to the strictly positive $(n + 1)$ dimensional orthant to avoid this technical issue. Campbell (1903) contains a detailed analysis of the general properties of homogeneous functions using Lie group theory. While the general properties of homogeneous functions are broadly understood in this field of mathematics, this article derives the complete set of solutions to homogeneity in the Gorman class of single equation models from first principles.

case, deflation destroys aggregability, at least in the conventional sense, by creating terms that are functions of $\boldsymbol{w}/p$ (Lewbel 1989). In the latter case, the variables are p/w_i and w_j/w_i. This generally destroys independence of an aggregate output price index from the elements of $\boldsymbol{w}$. If testing homogeneity is of interest, most researchers would prefer a nested test, which is what our development provides. This is opposed to an arbitrary form in which deflated and nondeflated models are compared with a nonnested test. Thus, the next section focuses on the restrictions on each of the functions in (1) that lead to homogeneity. This requires solving the differential equation in (3).

The Main Result

In the Appendix to this article, the proof of our main result is derived by solving the Euler equation for homogeneity when the supply equation has the Gorman form. The results depend on the number, K, of functions included in the supply function. The explicit results for $K \leq 3$ are highlighted because this appears to be parsimonious and sufficient for most empirical representations.[6]

PROPOSITION 1. *(The Main Result): Let the supply function take the Gorman form,* $q = \sum_{k=1}^{K} \alpha_k(\boldsymbol{w})h_k(p)$, *with K smooth, linearly independent, functions of input prices,* $\boldsymbol{w}$, *and K smooth, linearly independent, functions of output price, p. If q is* 0° *homogeneous in* $(\boldsymbol{w}, p)$, *then each output price function is either: (i)* p^ε, *with* $\varepsilon \in \mathbb{R}$; *(ii)* $p^\varepsilon\,(\ln p)^j$, *with* $\varepsilon \in \mathbb{R}$, $j \in \{1, \ldots, K\}$; *(iii)* $p^\varepsilon \sin\,(\tau \ln\, p), p^\varepsilon \cos\,(\tau \ln\, p)$, *with* $\varepsilon \in \mathbb{R}, \tau \in \mathbb{R}_+$, *appearing in pairs with the same* $\{\varepsilon, \tau\}$ *for each pair; or (iv)* $p^\varepsilon\,(\ln\ p)^j \sin\,(\tau \ln\ p)$, $p^\varepsilon\,(\ln\ p)^j \cos\,(\tau \ln p)$, *with* $\varepsilon \in \mathbb{R}$, $j \in \{1, \ldots, [{}^1\!/\!{}_2K]\}$, $\tau \in \mathbb{R}_+$, *and* $K \geq 4$, *appearing in pairs with the same* $\{\varepsilon, j, \tau\}$ *for each pair, where* $[{}^1\!/\!{}_2K]$ *is the largest integer no greater than* ${}^1\!/\!{}_2K$. *If* $K \in \{1, 2, 3\}$, *then the supply of q can be written as:*

(a) $K = 1$ $q = [p/\alpha_1(\boldsymbol{w})]^{\varepsilon_1}$;

(b) $K = 2$

i. $q = [p/\alpha_1(\boldsymbol{w})]^{\varepsilon_1} + [p/\alpha_2(\boldsymbol{w})]^{\varepsilon_2}$;

ii. $q = [p/\alpha_1(\boldsymbol{w})]^{\varepsilon_1} \ln(p/\alpha_2(\boldsymbol{w}))$; *or*

iii. $q = [p/\alpha_1(\boldsymbol{w})]^{\varepsilon_1}[\sin(\tau \ln(p/\alpha_2(\boldsymbol{w}))) + \cos(\tau \ln(p/\alpha_2(\boldsymbol{w})))]$;

(c) $K = 3$

i. $q = [p/\alpha_1(\boldsymbol{w})]^{\varepsilon_1} + [p/\alpha_2(\boldsymbol{w})]^{\varepsilon_2} + [p/\alpha_3(\boldsymbol{w})]^{\varepsilon_3}$;

ii. $q = [p/\alpha_1(\boldsymbol{w})]^{\varepsilon_1} + [p/\alpha_2(\boldsymbol{w})]^{\varepsilon_2} \ln(p/\alpha_3(\boldsymbol{w}))$;

iii. $q = [p/\alpha_1(\boldsymbol{w})]^{\varepsilon_1}\{\alpha_2(\boldsymbol{w}) + [\ln(p/\alpha_3(\boldsymbol{w}))]^2\}$; *or*

iv. $q = [p/\alpha_1(\boldsymbol{w})]^{\varepsilon_1} + [p/\alpha_2(\boldsymbol{w})]^{\varepsilon_2}\{\sin(\tau \ln[p/\alpha_3(\boldsymbol{w})]) + \cos(\tau \ln[p/\alpha_3(\boldsymbol{w})])\}$.

In each case except (c) iii, where $\alpha_2(\boldsymbol{w})$ *is homogeneous of degree zero, each* $\alpha_i(\boldsymbol{w})$ *is positively linearly homogeneous for* $i = 1, 2, 3$.

Beginning with $K = 1$, where there is no possibility of complex roots to (3), a simple power function of p emerges. Only a single linearly homogeneous index in $\boldsymbol{w}$ is required. One might consider $p/\alpha_1(\boldsymbol{w}) > 0$ to be the "real price," in which case monotonicity of supply in the output price requires $\varepsilon_1 > 0$. If $\varepsilon_1 < 1$, then the smaller is this elasticity, the greater is the concave curvature of supply in real price, while if $\varepsilon_1 > 1$, then the larger is this elasticity, the greater is the convex curvature of supply in real price.

For $K = 2$, supply curves need not go through the origin and more flexible functional forms are introduced. A linear combination of power functions in p are obtained—each with separate exponents ((b) i). These are generalizations of the PIGL forms developed by Muellbauer (1975, 1976) and Lewbel (1987). Other generalizations include log functions included with power functions. For the log form in (b) ii, expanding the log term reveals that the two output price functions enter in a specific multiplicative way, with p^{ε_1} and $p^{\varepsilon_1}\ln(p)$ serving as the two price functions in (1). This introduces generalizations of the PIGLOG class of functions developed in Muellbauer (1975, 1976) in demand analysis. Finally, for $K = 2$ (b) iii, the possibility of complex roots occurs and trigonometric solutions are introduced with output price functions $p^{\varepsilon_1}\sin(\tau \ln(p))$ and $p^{\varepsilon_1}\cos(\tau \ln(p))$. These identify a pair of terms similar to the Fourier series approximations developed in Gallant (1984) and applied by Chalfant (1987) and Wohlgenant (1984).

Moving on, $K = 3$ adds an additional price function. Analogous to (b) i, an additional power function is possible, as noted in (c) i. A mixture of (a) and (b) ii also is possible (presented as case (c) ii), as well as the generalization of the trigonometric forms found when

[6] Non-negativity and monotonicity of supply in p are usually not imposed in the empirical implementation of single equation models, but clearly they are expected to be satisfied.

$K = 2$ as presented in (c) iv. Perhaps most surprising is the appearance of the $(\ln p)^2$ term, which generalizes (b) ii, and is presented as case (c) iii in the proposition.

All of these cases follow from a reduction of the Euler equations to an equivalent linear ordinary differential equation (see equation (A.9) in the Appendix). When $K = 3$, the specific solutions to this differential equation depend on whether there are unique real roots, a repeated real root combined with a unique real root, one real root with multiplicity 3, or a real root combined with a complex conjugate pair of roots.

A few comparisons of the forms in proposition 1 and popular forms have been noted above and there is a large set of Gorman-class forms that could be discussed. However, the conclusion is clear even from $K = 1$ that homogeneity defines the class of functions but allows a much more rich set of behaviors than were ε_1 constrained to be 1 as in conventional systems of demand analysis. Thus, we conclude that the set of functions consistent with homogeneity is flexible and appears to be empirically meaningful. Yet, it is surprising that homogeneity alone defines the class of output price functions that can be used.

Profit Functions

Even if a production or general equilibrium system is not estimated, it is useful to know the profit functions corresponding to Proposition 1, so that welfare analysis can be performed (Hausman 1981, LaFrance 1993). That is, assuming that firms solve

$$\pi(p, \boldsymbol{w}) = \max_{q,\mathbf{x}}\{pq - \boldsymbol{w}^{\mathrm{T}}\mathbf{x} : (q, \mathbf{x}) \in \mathcal{Y} \subset \mathbb{R}_+ \times \mathbb{R}^n_+\}$$

where $\mathcal{Y}$ is the production possibilities set and $\boldsymbol{x}$ is the input vector, then also assuming smoothness and integrating the supply function in each case (a)–(c) above gives the profit function, $\pi(p, \boldsymbol{w})$. Note that if the supply equation is a sum of functions, then a sufficient condition for monotonicity of the supply function in the output price is that each function is increasing in p. However, this is not necessary. Hence, it is not imposed except for $K = 1$. We present results for $K = 1$, 2, and $K = 3$ (c) iii. The results for the remaining $K = 3$ cases can be anticipated from the results for $K = 1$ and 2. As is the case for proposition 1 above, the complete proof of our next result is contained in the Appendix.

PROPOSITION 2. *Let the supply of q take the form in Proposition* 1; *then homogeneity requires profit functions of the following forms:*

(a) $K = 1\ \varepsilon_1 > 0$

$$\pi(p, \boldsymbol{w}) = \frac{p}{(1+\varepsilon_1)}\left(\frac{p}{\alpha_1(\boldsymbol{w})}\right)^{\varepsilon_1} - \beta(\boldsymbol{w});$$

(b) $K = 2$

i.a. $\varepsilon_1, \varepsilon_2 \neq -1$[7]

$$\pi(p, \boldsymbol{w}) = \frac{p}{(1+\varepsilon_1)}\left(\frac{p}{\alpha_1(\boldsymbol{w})}\right)^{\varepsilon_1} + \frac{p}{(1+\varepsilon_2)}\left(\frac{p}{\alpha_2(\boldsymbol{w})}\right)^{\varepsilon_2} - \beta(\boldsymbol{w});$$

i.b. $\varepsilon_1 \neq -1, \varepsilon_2 = -1$

$$\pi(p, \boldsymbol{w}) = \frac{p}{(1+\varepsilon_1)}\left(\frac{p}{\alpha_1(\boldsymbol{w})}\right)^{\varepsilon_1} + \alpha_2(\boldsymbol{w})\ln\left(\frac{p}{\beta(\boldsymbol{w})}\right) - \gamma(\boldsymbol{w});$$

ii.a. $\varepsilon_1 \neq -1$

$$\pi(p, \boldsymbol{w}) = \frac{p}{(1+\varepsilon_1)}\left(\frac{p}{\alpha_1(\boldsymbol{w})}\right)^{\varepsilon_1} \times \left[\ln\left(\frac{p}{\alpha_2(\boldsymbol{w})}\right) - \frac{1}{(1+\varepsilon_1)}\right] - \beta(\boldsymbol{w});$$

ii.b. $\varepsilon_1 = -1$

$$\pi(p, \boldsymbol{w}) = \frac{1}{2}\alpha_1(\boldsymbol{w})\left[\ln\left(\frac{p}{\alpha_2(\boldsymbol{w})}\right)\right]^2 - \beta(\boldsymbol{w});$$

iii.

$$\pi(p, \boldsymbol{w}) = \left(\frac{p}{(1+\varepsilon_1)^2+\tau^2}\right)\left(\frac{p}{\alpha_1(\boldsymbol{w})}\right)^{\varepsilon_1} \times \left[(1+\varepsilon_1+\tau)\sin\left(\tau\ln\left(\frac{p}{\alpha_2(\boldsymbol{w})}\right)\right) + (1+\varepsilon_1-\tau)\cos\left(\tau\ln\left(\frac{p}{\alpha_2(\boldsymbol{w})}\right)\right)\right] - \beta(\boldsymbol{w});$$

(c) $K = 3$

iii.a. $\varepsilon_1 \neq -1$

$$\pi(p, \boldsymbol{w}) = \frac{p}{(1+\varepsilon_1)^3}\left(\frac{p}{\alpha_1(\boldsymbol{w})}\right)^{\varepsilon_1} \times \left\{1 + (1+\varepsilon_1)^2\alpha_2(\boldsymbol{w}) + \left[(1+\varepsilon_1) \times \ln\left(\frac{p}{\alpha_3(\boldsymbol{w})}\right) - 1\right]^2\right\} - \beta(\boldsymbol{w});$$

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[7] Although we don't focus on monotonicity, it is clear that both exponents (ε_1 and ε_2) cannot be negative for supply to be increasing in p.

iii.b. $\varepsilon_1 = -1$

$$\pi(p, \boldsymbol{w}) = \alpha_1(\boldsymbol{w}) \left[\alpha_2(\boldsymbol{w}) \ln\left(\frac{p}{\beta(\boldsymbol{w})}\right) + \frac{1}{3}\left(\ln\left(\frac{p}{\alpha_3(\boldsymbol{w})}\right)\right)^3 \right] - \gamma(\boldsymbol{w}).$$

In each case, $\beta(\boldsymbol{w})$ *and* $\gamma(\boldsymbol{w})$ *are positively linearly homogeneous functions of* $\boldsymbol{w}$.

These results can be used to develop systems or partial systems of production behavior for applied welfare analysis. For example, one could differentiate the profit functions in proposition 2 with respect to p and $\boldsymbol{w}$ and estimate a system of supply and factor demands.[8] Some of the forms are simpler or otherwise more attractive than others. These profit functions are both necessary and sufficient in each case, and sufficiency can be shown simply by differentiating with respect to p.

We have said little about regularity conditions such as monotonicity or nonnegativity. In single equation problems as we consider here, these conditions are seldom imposed but merely checked after estimation. However, it is possible to impose local or global monotonicity in a single equation or a system with constrained estimation. In any given application, monotonicity should at least be examined. To illustrate its importance, in (b) ii, for $\alpha_1 : \mathbb{R}^n_{++} \rightarrow \mathbb{R}_+$, monotonicity of supply requires that $1 - \ln(p/\alpha_2(\boldsymbol{w})) > 0$. Thus, the practical usefulness of this case hinges both on the domain of p and $\boldsymbol{w}$, and the range of α_2.[9]

Conclusion

Measuring producer or consumer behavior requires selection of a functional form. The Gorman class of functions is flexible and aggregable and encompasses most empirical specifications in the literature. Yet, surprisingly little is known about how homogeneity of degree zero of supply and demand functions applies. We find substantial generalizations of the functional forms found in Gorman-based systems and routinely applied in empirical demand analysis, where homogeneity, adding up, and symmetry are imposed in a complete system. The functional forms in the propositions provide a coherent approach for empirical work using any supply or demand function in the general Gorman class. Finally, although the derived functional forms are substantive generalizations of those in the complete systems literature, we are struck with the following observation: homogeneity alone determines the admissible class of functional forms.

[Received June, 2007; accepted November 2007.]

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[8] Profit also can be added if one accounts for the statistical properties of the errors that are due to adding up, since profit is created from the output supply and input demands.

[9] Ignoring aggregation, fixed inputs, $\boldsymbol{z}$, can be included as $\alpha_k(\boldsymbol{w}, \boldsymbol{z})$ and/or $h_k(p, \boldsymbol{z})$ with no change in the conditions required for q to be homogeneous in $(\boldsymbol{w}, p)$.

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