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Integrability of the linear approximate almost ideal demand system

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Abstract

Integrability of the linear approximate almost ideal demand system (AIDS) is solved, including closed form solutions for the expenditure function, and generating a new method to nest the rank and functional form of a quadratic price independent generalized linear incomplete demand system (QPIGL-IDS).

Keywords: Linear approximate almost ideal demand system; Functional form; Incomplete demand system; Rank

JEL classification: D12; E21

1. Introduction

In the more than two decades since its introduction by Deaton and Muellbauer (1980), the almost ideal demand system (AIDS) has been widely used in demand analysis. The majority of empirical applications follow Deaton and Muellbauer's lead and replace the translog price index with Stone's index to deflate income. This generates the linear approximate almost ideal demand system (LA-AIDS), which is linear in the unknown parameters and therefore simpler to estimate. Deaton and Muellbauer (1980) (pp. 317–320) cautioned against imposing symmetry on the LA-AIDS, and avoided doing so. They interpreted Stone's index as an approximation to the "true" translog index. Nevertheless, most applications of the LA-AIDS test for and impose symmetry of the matrix of log-price coefficients (e.g. Anderson and Blundell, 1983; Moschini and Meilke, 1989).[1] There really can be only one explanation

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[1] An important exception is Browning and Meghir (1991), where the non-linear AIDS is estimated using starting values from the LA-AIDS with a symmetric matrix of log-price coefficients.

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for this practice; the LA-AIDS is presumed to be the "true" model and symmetry of the matrix of log-price coefficients is presumed to be the correct way to obtain Slutsky symmetry and economic rationality of the demand equations that are estimated.

The LA-AIDS has been criticized for reasons other than its failure to be consistent with economically rational consumer choices. Eales and Unnevehr (1988) point out that budget shares appear on both sides of the regression equations, producing simultaneity problems. Pashardes (1993); Buse (1998) criticize the errors in variables problem created by using of Stone's index rather than the "true" translog price index on the right-hand-side of the regression equations. Moschini (1995) argues that Stone's index is not a proper price index at all and that without some mechanism to scale prices (e.g. at sample means), Stone's index leads to biased and inconsistent parameter estimates.

The purpose of this paper is to completely clarify Slutsky symmetry, and the structural implications of integrability, for the LA-AIDS. We identify the parameter restrictions that are consistent with integrability and integrate back to the expenditure function. Two possible cases are found: (i) the log-income coefficients vanish, so that the demands are homothetic, and the matrix of log-price coefficients is symmetric; or (ii) the matrix of log-price coefficients has rank one and is proportional to the outer product of log-income coefficients. Both cases are highly restrictive, the former with respect to income and the latter with respect to prices. Moreover, the non-homothetic case is non-linear in the parameters, eliminating the apparent advantage of the LA-AIDS in estimation.

However, both cases admit closed form solutions for the expenditure (and indirect utility) functions. In addition, previously unknown relationships among LA-AIDS and the linear incomplete demand system (LIDS) and Stone–Geary linear expenditure system (LES) are identified. Box–Cox transformations to nest rank and functional form in a quadratic price independent generalized linear incomplete demand system (QPIGL-IDS) follow immediately from these relationships.

2. Integrability of LA-AIDS

Let $\mathbf{p}$ be the n-vector of market prices for goods, let u be the utility index, let $e(\mathbf{p}, u)$ be the consumer's expenditure function, and let $\mathbf{w}$ be the n-vector of budget shares. The LA-AIDS model can be written in matrix notation as

$$\mathbf{w} = \frac{\partial \ln[e(\mathbf{p}, u)]}{\partial \ln(\mathbf{p})} = \boldsymbol{\alpha} + \mathbf{B}\ln(\mathbf{p}) + \boldsymbol{\gamma}\left[\ln[e(\mathbf{p}, u)] - \ln(\mathbf{p})' \, \frac{\partial \ln[e(\mathbf{p}, u)]}{\partial \ln(\mathbf{p})}\right] \tag{1}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ are n-vectors and $\mathbf{B}$ is an $n \times n$ matrix of parameters. It is convenient to change variables to logarithms. Let $\mathbf{x} \equiv \ln(\mathbf{p})$ and $y(\mathbf{x}, u) \equiv \ln[e(\mathbf{p}(\mathbf{x}), u)]$, with $p_i(\mathbf{x}) \equiv e^{x_i}$, $i = 1, \ldots, n$, and rewrite (1) as

$$(\mathbf{I} + \boldsymbol{\gamma}\mathbf{x}'\,)\frac{\partial y(\mathbf{x}, u)}{\partial \mathbf{x}} = \boldsymbol{\alpha} + \mathbf{B}\mathbf{x} + \boldsymbol{\gamma} y(\mathbf{x}, u). \tag{2}$$

The far left-hand-side matrix has determinant $|\mathbf{I} + \boldsymbol{\gamma}\mathbf{x}'\,| = 1 + \boldsymbol{\gamma}'\,\mathbf{x}$, and so it is non-singular and has inverse $(\mathbf{I} - \boldsymbol{\gamma}\mathbf{x}'\,)/(1 + \boldsymbol{\gamma}'\,\mathbf{x})$, if and only if $\boldsymbol{\gamma}'\,\mathbf{x} \neq -1$ (e.g. Dhrymes, 1984 pp. 38–39). Typically, $\mathbf{p}$ is a vector of price indices each normalized to one in a base period, so that x vanishes in that period. The elements of $\boldsymbol{\gamma}$ capture departures from homotheticity of the demands and usually are small (Anderson and Blundell, 1983; Browning and Meghir, 1991; Buse, 1998; Deaton and Muellbauer, 1980; Moschini, 1995; Moschini and Meilke, 1989; Pashardes, 1993). Moreover, the $(n-1)$-dimensional hyperplane, $\boldsymbol{\gamma}'\mathbf{x}=-1$, has (Lebesgue) measure zero in n-space and zero probability of being observed. In addition, $0°$ homogeneity of the demand equations requires that the elements of $\boldsymbol{\gamma}$ sum to zero, i.e. $\iota'\boldsymbol{\gamma}=0$, where $\iota=[1 \quad \cdots \quad 1]'$.

Therefore, for $\mathbf{x}=\theta\iota$ for some $\theta\in\mathbb{R}$ (including $\theta=0$), $|\mathbf{I}+\boldsymbol{\gamma}\mathbf{x}'|=1$ and the matrix $\mathbf{I}+\boldsymbol{\gamma}\mathbf{x}'$ is non-singular in an open neighborhood of this entire line. We therefore make the following assumption.

A. $1+\boldsymbol{\gamma}'\mathbf{x}\neq 0\,\forall\mathbf{x}\in\mathcal{N}\subset\mathbb{R}^n$,

where $\mathcal{N}$ is open, has a non-empty interior, and contains essentially all of the economically relevant values for the log-price variables, $\mathbf{x}$. Property **A** permits us to write the LA-AIDS as a system of linear partial differential equations,

$$\frac{\partial y(\mathbf{x},u)}{\partial\mathbf{x}}-\boldsymbol{\gamma}\frac{y(\mathbf{x},u)}{(1+\boldsymbol{\gamma}'\mathbf{x})}=\left[\mathbf{I}-\frac{\boldsymbol{\gamma}\mathbf{x}'}{(1+\boldsymbol{\gamma}'\mathbf{x})}\right](\boldsymbol{\alpha}+\mathbf{B}\mathbf{x}), \tag{3}$$

where we have used $[\mathbf{I}-\boldsymbol{\gamma}\mathbf{x}'/(\mathbf{1}+\boldsymbol{\gamma}'\mathbf{x})]\boldsymbol{\gamma}\equiv\boldsymbol{\gamma}/(\mathbf{1}+\boldsymbol{\gamma}'\mathbf{x})$. Noting that

$$\frac{\partial}{\partial\mathbf{x}}\left[\frac{y(\mathbf{x},u)}{1+\boldsymbol{\gamma}'\mathbf{x}}\right]=\left[\frac{\partial y(\mathbf{x},u)}{\partial\mathbf{x}}-\boldsymbol{\gamma}\frac{y(\mathbf{x},u)}{(1+\boldsymbol{\gamma}'\mathbf{x})}\right]\frac{1}{(1+\boldsymbol{\gamma}'\mathbf{x})} \tag{4}$$

and multiplying both sides of Eq. (3) by $1/(\mathbf{1}+\boldsymbol{\gamma}'\mathbf{x})$, we obtain an *exact* partial differential equation system. Slutsky symmetry is therefore equivalent to symmetry of the matrix

$$\begin{aligned}&\frac{\partial}{\partial\mathbf{x}'}\left\{\frac{1}{(1+\boldsymbol{\gamma}'\mathbf{x})}\left[\mathbf{I}-\frac{\boldsymbol{\gamma}\mathbf{x}'}{(1+\boldsymbol{\gamma}'\mathbf{x})}\right](\boldsymbol{\alpha}+\mathbf{B}\mathbf{x})\right\}\\&\quad=\frac{\mathbf{B}}{(1+\boldsymbol{\gamma}'\mathbf{x})}-\frac{[(\boldsymbol{\alpha}+\mathbf{B}\mathbf{x})\boldsymbol{\gamma}'+\boldsymbol{\gamma}(\boldsymbol{\alpha}'+\mathbf{x}'\mathbf{B}'+\mathbf{x}'\mathbf{B})]}{(1+\boldsymbol{\gamma}'\mathbf{x})^2}-\frac{2(\boldsymbol{\alpha}'\mathbf{x}+\mathbf{x}'\mathbf{B}\mathbf{x})\boldsymbol{\gamma}\boldsymbol{\gamma}'}{(1+\boldsymbol{\gamma}'\mathbf{x})^3}.\end{aligned} \tag{5}$$

Imposing symmetry on each of the terms associated with like powers of $\mathbf{1}+\boldsymbol{\gamma}'\mathbf{x}$ and ignoring terms that are automatically symmetric, we obtain the conditions $\mathbf{B}=\mathbf{B}'$ and $\boldsymbol{\gamma}\mathbf{x}'\mathbf{B}\equiv\mathbf{B}'\mathbf{x}\boldsymbol{\gamma}'\ \forall\ \mathbf{x}\in\mathcal{N}$. There are only two ways these conditions can be satisfied simultaneously $\forall\ \mathbf{x}\in\mathcal{N}$: (i) $\boldsymbol{\gamma}\neq 0$ and $\mathbf{B}=\beta_0\boldsymbol{\gamma}\boldsymbol{\gamma}'$ for some $\beta_0\in\mathbb{R}$ (including $\beta_0=0$); and (ii) $\boldsymbol{\gamma}=\mathbf{0}$ and $\mathbf{B}=\mathbf{B}'$.

After combining and rearranging terms, the first case gives the LA-AIDS model in the form

$$\frac{\partial y}{\partial\mathbf{x}}=\boldsymbol{\alpha}+\beta_0\boldsymbol{\gamma}\boldsymbol{\gamma}'\mathbf{x}+\boldsymbol{\gamma}\left(\frac{y-\boldsymbol{\alpha}'\mathbf{x}-\beta_0(\boldsymbol{\gamma}'\mathbf{x})^2}{1+\boldsymbol{\gamma}'\mathbf{x}}\right)=\boldsymbol{\alpha}+\boldsymbol{\gamma}\left(\frac{y-\boldsymbol{\alpha}'\mathbf{x}+\beta_0\boldsymbol{\gamma}'\mathbf{x}}{1+\boldsymbol{\gamma}'\mathrm{x}}\right). \tag{6}$$

This system of linear first-order partial differential equations is straightforward to solve. Note that

$$\frac{\partial}{\partial \mathbf{x}}\left(\frac{\boldsymbol{\alpha}'\,\mathbf{x}}{1+\boldsymbol{\gamma}'\,\mathbf{x}}\right)=\left[\boldsymbol{\alpha}-\gamma\left(\frac{\boldsymbol{\alpha}'\,\mathbf{x}}{1+\boldsymbol{\gamma}'\,\mathbf{x}}\right)\right]\frac{1}{(1+\boldsymbol{\gamma}'\,\mathbf{x})}, \tag{7}$$

$$\frac{\partial}{\partial \mathbf{x}}\left\{\beta_0\left[\ln(1+\boldsymbol{\gamma}'\,\mathbf{x})-\left(\frac{\boldsymbol{\gamma}'\,\mathbf{x}}{1+\boldsymbol{\gamma}'\,\mathbf{x}}\right)\right]\right\}=\frac{\beta_0\boldsymbol{\gamma}\boldsymbol{\gamma}'\,\mathbf{x}}{(1+\boldsymbol{\gamma}'\,\mathbf{x})^2}, \tag{8}$$

combine Eqs.(4), (6)–(8), and integrate with respect to $\mathbf{x}$ to obtain the log-expenditure function as

$$y(\mathbf{x},u)=\boldsymbol{\alpha}'\,\mathbf{x}+\beta_0\left[(1+\boldsymbol{\gamma}'\,\mathbf{x})\ln(1+\boldsymbol{\gamma}'\,\mathbf{x})-\frac{\boldsymbol{\gamma}'\,\mathbf{x}}{(1+\boldsymbol{\gamma}'\,\mathbf{x})}\right]+(1+\boldsymbol{\gamma}'\,\mathbf{x})u, \tag{9}$$

with an obvious normalization. The second case produces the homothetic AIDS and LA-AIDS,

$$\frac{\partial y}{\partial \mathbf{x}}=\boldsymbol{\alpha}+\mathbf{B}\mathbf{x}. \tag{10}$$

Direct integration with respect to $\mathbf{x}$ gives the logarithmic expenditure function as

$$y(\mathbf{x},u)=\boldsymbol{\alpha}'\,\mathbf{x}+\frac{1}{2}\mathbf{x}'\,\mathbf{B}\mathbf{x}+u, \tag{11}$$

again with an obvious normalization. This establishes the necessity of Eq.(6) or Eq. (10) for integrability.

On the other hand, Eq. (10) trivially has the LA-AIDS form, with $\boldsymbol{\gamma}=0$. To show sufficiency for Eq. (6), write

$$y-\mathbf{x}'\,\frac{\partial y}{\partial \mathbf{x}}=y-\mathbf{x}'\left[\boldsymbol{\alpha}+\beta_0\boldsymbol{\gamma}\boldsymbol{\gamma}'\,\mathbf{x}+\boldsymbol{\gamma}\left(\frac{y-\boldsymbol{\alpha}'\,\mathbf{x}-\beta_0(\boldsymbol{\gamma}'\,\mathbf{x})^2}{1+\boldsymbol{\gamma}'\,\mathbf{x}}\right)\right]=\frac{y-\boldsymbol{\alpha}'\,\mathbf{x}-\beta_0(\boldsymbol{\gamma}'\,\mathbf{x})^2}{1+\boldsymbol{\gamma}'\,\mathrm{x}}, \tag{12}$$

so that Eq. (6) has the LA-AIDS form,

$$\frac{\partial y}{\partial \mathbf{x}}=\boldsymbol{\alpha}+\beta_0\boldsymbol{\gamma}\boldsymbol{\gamma}'\,\mathbf{x}+\boldsymbol{\gamma}\left(y-\mathbf{x}'\,\frac{\partial y}{\partial \mathbf{x}}\right). \tag{13}$$

Remark 1. The homothetic, integrable AIDS (LA-AIDS) in (10) has the same functional structure as the homothetic, integrable LIDS of LaFrance (1985). For the AIDS all income elasticities are one, while for the LIDS they all vanish. If we can forego identical functional forms for the demand equations of *all* goods, which is probably a minor concern in many cases, this points to a nesting procedure for the homothetic AIDS and LIDS using a Box–Cox transformation. Denote total expenditure by m, assume that the demand model applies to n of $N \geq n+1$ goods, and define the Box–Cox transformations

$m(\lambda) \equiv (m^{\lambda} - 1)/\lambda$, $p_i(\lambda) \equiv (p_i^{\lambda} - 1)/\lambda$, and $\mathbf{p}(\lambda) \equiv [p_1(\lambda) \cdots p_n(\lambda)]'$. Finally, assume that a common deflator, $\pi(\tilde{\mathbf{p}})$, normalizes m and p, where $\pi(\tilde{\mathbf{p}})$ is a positive-valued and 1° homogeneous function of (any non-empty subset of) the prices of the other goods. Then we can write an integrable, homothetic price independent generalized linear incomplete demand system (PIGL-IDS) as

$$\mathbf{w} = m^{-\lambda}\mathbf{P}^{\lambda}[\boldsymbol{\alpha} + \mathbf{B}\mathbf{p}(\lambda)], \tag{14}$$

where $\mathbf{P}^{\lambda} \equiv \mathrm{diag}[p_i^{\lambda}]$. Direct integration shows that the expenditure function for this IDS has the form

$$e(\mathbf{p}, \tilde{\mathbf{p}}, u) \equiv \pi(\tilde{\mathbf{p}})\left\{1 + \lambda\left[\boldsymbol{\alpha}'\,\mathbf{p}(\lambda) + \frac{1}{2}\mathbf{p}(\lambda)'\,\mathbf{B}\mathbf{p}(\lambda) + \theta(\tilde{\mathbf{p}}, u)\right]\right\}^{1/\lambda}, \tag{15}$$

where $\theta(\tilde{\mathbf{p}}, u)$ is 0° homogeneous in the prices of other goods and increasing in u but otherwise not identified (Epstein, 1982; LaFrance, 1985; LaFrance and Hanemann, 1989). The demands (14) are homothetic with common income elasticity equal to $1 - \lambda\ \forall\ \lambda \in \mathbb{R}$.[2]

This technique for nesting the homothetic AIDS/LA-AIDS and the LIDS models within a particular homothetic PIGL-IDS generalizes to the non-homothetic AIDS,

$$\mathbf{w} = \boldsymbol{\alpha} + \mathbf{B}\ln(\mathbf{p}) + \boldsymbol{\gamma}[\ln(m) - \alpha_0 - \boldsymbol{\alpha}'\ln(\mathbf{p}) - 1/2\ln(\mathbf{p})'\,\mathbf{B}\ln(\mathbf{p})]. \tag{16}$$

Maintaining the above definitions for $m(\lambda)$ and $\mathbf{p}(\lambda)$, we can write an integrable, non-homothetic, rank two PIGL-IDS that is linear in the Box–Cox transformation of total expenditure and linear and quadratic in the Box–Cox transformations of prices as,

$$\mathbf{w} = m^{-\lambda}\mathbf{P}^{\lambda}\{\boldsymbol{\alpha} + \mathbf{B}\mathbf{p}(\lambda) + \boldsymbol{\gamma}[m(\lambda) - \alpha_0 - \boldsymbol{\alpha}'\,\mathbf{p}(\lambda) - 1/2\mathbf{p}(\lambda)'\,\mathbf{B}\mathbf{p}(\lambda)]\}. \tag{17}$$

For $\lambda = 0$ we obtain the non-linear (non-homothetic) AIDS, while for $\lambda = 1$ we obtain the linear-quadratic incomplete demand system (LQ-IDS) of LaFrance (1990). For all values of λ we obtain a non-homothetic, integrable, rank two PIGL-IDS with expenditure function,

$$e(\mathbf{p}, \tilde{\mathbf{p}}, u) \equiv \pi(\tilde{\mathbf{p}})\{1 + \lambda[\alpha_0 + \boldsymbol{\alpha}'\,\mathbf{p}(\lambda) + 1/2\mathbf{p}(\lambda)'\,\mathbf{B}\mathbf{p}(\lambda) + \theta(\tilde{\mathbf{p}}, u)e^{\boldsymbol{\gamma}'\,\mathbf{p}(\lambda)}]\}^{1/\lambda}. \tag{18}$$

[2] For an incomplete demand system, homotheticity is defined by equality of the income elasticities for the subset of goods of interest. It is not necessary that any other demands have this income elasticity. In particular, the common income elasticity of demand for a homothetic subset of goods is not necessarily equal to one, and may not be constant (LaFrance and Hanemann, 1989). This is one of several ways in which weakly integrable incomplete demand systems are more flexible and provide a richer class of models than complete systems.

This procedure also extends to rank three models with linear and quadratic terms in the Box–Cox transformation of total expenditure. Write a QPIGL-IDS in the Howe et al. (1979) form,

$$\mathbf{w} = m^{-\lambda}\mathbf{P}^{\lambda}\{\boldsymbol{\alpha} + \mathbf{B}\mathbf{p}(\lambda) + \boldsymbol{\gamma}[m(\lambda) - \alpha_0 - \boldsymbol{\alpha}'\,\mathbf{p}(\lambda) - 1/2\mathbf{p}(\lambda)'\,\mathbf{B}\mathbf{p}(\lambda)] + (\mathbf{I} + \gamma\mathbf{p}(\lambda)'\,)\delta[m(\lambda) - \alpha_0 - \boldsymbol{\alpha}'\,\mathbf{p}(\lambda) - 1/2\mathbf{p}(\lambda)'\,\mathbf{B}\mathbf{p}(\lambda)]^2\}, \tag{19}$$

with expenditure function equal to

$$e(\mathbf{p}, \tilde{\mathbf{p}}, u) \equiv \pi(\tilde{\mathbf{p}})\left\{1 + \lambda\left[\alpha_0 + \boldsymbol{\alpha}'\,\mathbf{p}(\lambda) + 1/2\mathbf{p}(\lambda)'\,\mathbf{p}(\lambda) - \frac{e^{\boldsymbol{\gamma}'\,\mathbf{p}(\lambda)}}{(\delta'\,\mathbf{p}(\lambda)e^{\boldsymbol{\gamma}'\,\mathbf{p}(\lambda)} + \theta(\tilde{\mathbf{p}}, u))}\right]\right\}^{1/\lambda}. \tag{20}$$

So long as $\boldsymbol{\alpha}$ and $\mathbf{B}$ do not vanish completely and simultaneously, it follows that: (a) $\boldsymbol{\gamma} \neq 0$ and $\boldsymbol{\delta} \neq 0$ are necessary and sufficient for a rank three QPIGL-IDS; (b) $\boldsymbol{\gamma} \neq \mathbf{0}$ and $\boldsymbol{\delta} = \mathbf{0}$ are necessary and sufficient for a rank two quasi-linear PIGL-IDS; (c) $\boldsymbol{\gamma} = \mathbf{0}$ and $\boldsymbol{\delta} \neq \mathbf{0}$ are necessary and sufficient for a rank two QPIGL-IDS that excludes the linear terms in superlative income; and (d) $\boldsymbol{\gamma} = \boldsymbol{\delta} = 0$ is necessary and sufficient for a rank one homothetic PIGL-IDS. Thus, we obtain a class of incomplete demand models that permits a simple and straightforward method for nesting and testing for the rank and functional form of weakly integrable IDS.

Remark 2. When the coefficients on log-income do not all vanish, the LA-AIDS and LIDS models are not nested as above. However, there is a sense in which they are nested, and a structural relationship between these models continues to exist. In one sense, this relationship is shared with the LES. In particular, the integrable, non-homothetic LA-AIDS is characterized by pairwise linear identities between *budget shares*,

$$w_i \equiv \alpha_i + (\gamma_i/\gamma_1)(w_1 - \alpha_1) \quad \forall\ i = 1, \ldots, n, \tag{21}$$

where, without loss in generality, $\gamma_1 \neq 0$; a non-homothetic LIDS is characterized by pairwise linear identities between *quantities* (LaFrance, 1985),

$$q_i \equiv \alpha_i + (\gamma_i/\gamma_1)(q_1 - \alpha_1) \quad \forall\ i = 1, \ldots, n; \tag{22}$$

and for a given vector of prices, the LES is characterized by pairwise linear identities among *expenditures*,

$$e_i \equiv \alpha_i p_i + (\gamma_i/\gamma_1)(e_1 - \alpha_1 p_1) \quad \forall\ i = 1, \ldots, n, \tag{23}$$

where $e_i \equiv p_i q_i$ is the expenditure on the ith good. In this context, the *linear* part of the LA-AIDS acronym obtains a new meaning.[3]

References

Anderson, G., Blundell, R., 1983. Testing restrictions in a flexible dynamic demand system: an application to consumer's expenditure in Canada. Review of Economic Studies 50, 397–410.

Browning, M., Meghir, C., 1991. The effects of male and female labor supply on commodity demands. Econometrica 59, 925–951.

Buse, A., 1998. Testing homogeneity in the linearized almost ideal demand system. American Journal of Agricultural Economics 80, 208–220.

Deaton, A., Muellbauer, J., 1980. An almost ideal demand system. American Economic Review 70, 312–326.

Dhrymes, P., 1984. Mathematics for Econometrics Springer, New York.

Eales, J.S., Unnevehr, L.J., 1988. Demand for beef and chicken products: separability and structural change. American Journal of Agricultural Economics 70, 521–532.

Epstein, L.G., 1982. Integrability of Incomplete systems of demand functions. Review of Economic Studies 49, 411–425.

Howe, H.R., Pollak, A., Wales, T.J., 1979. Theory and time series estimation of the quadratic expenditure system. Econometrica 47, 1231–1247.

LaFrance, J.T., 1985. Linear demand functions in theory and practice. Journal of Economic Theory 37, 147–166.

LaFrance, J.T., 1990. Incomplete demand systems and semilogarithmic demand models. Australian Journal of Agricultural Economics 34, 118–131.

LaFrance, J., Hanemann, M.J., 1989. The dual structure of incomplete demand systems. American Journal of Agricultural Economics 71, 262–274.

Moschini, G., 1995. Units of measurement and the stone index in demand system estimation. American Journal of Agricultural Economics 77, 63–68.

Moschini, G., Meilke, K., 1989. Modeling the pattern of structural change in US meat demand. American Journal of Agricultural Economics 71, 253–261.

Pashardes, P., 1993. Bias in estimating the almost ideal demand system with the Stone index approximation. Economic Journal 103, 908–915.

[3] It is worth noting that with $m(\lambda)$ and $\mathbf{p}(\lambda)$ defined as before, we can write (21) and (22) as special cases of the linear identities $\frac{\partial m(\lambda)}{\partial p_i(\lambda)} \equiv \left(\frac{p_i}{m}\right)^{1-\lambda} q_i \equiv \alpha_i + \left(\frac{\gamma_i}{\gamma_1}\right)\left[\left(\frac{p_1}{m}\right)^{1-\lambda} q_1 - \alpha_1\right] \equiv \alpha_i + \left(\frac{\gamma_i}{\gamma_1}\right)\left[\frac{\partial m(\lambda)}{\partial p_1(\lambda)} - \alpha_1\right], \forall\ i = 1, \ldots, n,$ with $\lambda = 0$ or 1 generating Eq. (21) or Eq. (22), respectively. But this is not sufficient to identify the complete set of solutions to the partial differential equations for the demands. For example, when $\lambda = 1$ the demands for the non-homothetic PIGL extension of the LA-AIDS are $\mathbf{q} = \boldsymbol{\alpha} + \boldsymbol{\gamma}' (m - \boldsymbol{\alpha}' \mathbf{p} + \beta_0 \boldsymbol{\gamma}' \mathbf{p})/(1 + \boldsymbol{\gamma}' \mathbf{p})$. These clearly differ from the demands for a non-homothetic LIDS, $\mathbf{q} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\gamma}' \mathbf{p} + \boldsymbol{\gamma} m$, with $\boldsymbol{\alpha} = \alpha_0 \boldsymbol{\gamma} - \boldsymbol{\beta}$ for some $\boldsymbol{\alpha}_0 \in \mathbb{R}$ and $\boldsymbol{\beta} \in \mathbb{R}^n$, as do the associated (quasi-)expenditure functions.