

Is Hotelling's Rule Relevant to Domestic Oil Production?

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A model of oil supply from known reserves is developed to incorporate geological and engineering principles in the oil field operator's decision problem. Oil production from existing wells within an oil field is isolated from the drilling of new wells. The geo-engineering rule known as *maximum efficient recovery* (MER) is nested within the economic model to test the hypothesis that production from established fields is invariant to the price of oil. The econometric model is applied to quarterly data from seven Montana oil fields. The MER model is strongly rejected by the data, providing evidence that oil supply models should include economic as well as geo-engineering principles. © 1998 Academic Press

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1. INTRODUCTION

The US oil and gas industry is the subject of a large number of studies that model the process of petroleum supply in the United States. Petroleum supply models have received widespread attention because of their long-standing use for policy analysis. The primary concern of these models is the response of domestic oil and gas production to changes in the price received by producers. Most petroleum supply models focus on the upstream sector of the industry, where the operations designed to find and develop oil and gas reserves occur. The principal output response is generally viewed as stemming from changes in exploration and drilling activity. Therefore, much theoretical and empirical work exists on the response of exploration and reserve additions to price changes.¹

Although alterations in the level of exploratory activities affect future production, changes in output from established fields significantly impact domestic production because production from mature fields provides the vast majority of oil and gas produced. However, with the notable recent exceptions of Pesaran's [33] aggregate model of the UK Continental Shelf and Deacon's [7] calibrated production model for the U.S., most existing supply models assume that production from established fields is governed mainly by geological and engineering considerations

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¹Examples include Attanasi *et al.* [3], Epple [11], Gould [14], and MacAvoy and Pindyck [29].

and is invariant to changes in the price received by producers.² The geo-engineering framework for oil and gas production is known as the *maximum efficient recovery* (MER) rule. In this model, production per unit time declines exogenously at a rate determined by the physical characteristics of the field. The veracity of the MER hypothesis has significant policy implications for petroleum supply. For example, the MER model predicts that a severance tax will shorten the length of the production period in an oil field, but has no effect on periodic production rates over time prior to shutdown of the field. In contrast, the economic model of oil production from a heterogeneous reserve predicts that a severance tax not only lowers cumulative extraction, but decreases current production rates. The effect on production rates is precluded in the MER model.³

In this paper, we propose a nested empirical test of the geo-engineering principle of maximum efficient recovery against a specific economic alternative. The main idea is quite simple and may be summarized as follows. In the geo-engineering MER model, given an initial investment in the depth, size, spacing, and number of oil wells in an oil field, pumping costs remain constant over time once production commences, while the quantity of oil produced per period declines at a hyperbolic rate. This decline in periodic production is due to a decrease in pressure in the oil field as the cumulative amount of oil extracted increases. As a result, the time paths of current and cumulative oil production trace out a single iso-cost curve in the current-cumulative oil production plane. The shape of this iso-cost line is determined by the (field-specific) parameters of the MER model. By noting that current production is the time derivative of cumulative production and working backwards from the MER rule, we obtain a functional relationship between current and cumulative oil production that is logically equivalent to the MER rule for that field. Under reasonable conditions⁴, this functional relationship determines the structure of the pumping cost function up to an arbitrary monotonic transformation. This structure for the periodic pumping cost function is then taken to be an integral part of the producer's economic problem of maximizing the net present value of profits from oil production. The result is that the geo-engineering MER rule is nested within the economic wealth-maximization model. The hypothesis test has the simple form of two parameter restrictions for each field.

We apply this approach at the individual oil field level using quarterly data from seven Montana oil fields. This allows us to overcome the aggregation problems noted by Halvorsen and Smith [16], as well as the issues associated with measuring the prices of natural resource products as in Barnett and Morse [4], Heal and Barrow [18], Smith [36, 37], and Slade [35], rather than the opportunity cost of *in situ* stocks of the resource. Another distinction of this study from previous work is the isolation of oil production from existing wells within an oil field from the production response that is due to the drilling of additional wells.

The paper is organized as follows. Section 2 reviews models of petroleum supply, with a focus on studies that model production from known reserves. Geo-engineering principles underlying production decline models for oil and gas extraction are

²For examples, see Deacon *et al.* [8], Amit [1], Camm *et al.* [5], and Griffin and Moroney [15]. However, also see Farzin [12] for a supply model based on optimizing principles.

³In the economic model, a severance tax may lengthen or shorten the production life.

⁴Sufficient conditions for this result are that the cost function is finite and twice differentiable in current and cumulative oil production.

presented and explained. The economic theory of exhaustible resource use is adapted to formally incorporate these geo-engineering principles into the pumping cost function for a wealth-maximizing oil firm. This cost function then is included in an economic model of production from mature oil fields. The result of these steps is an economic model of oil production that contains the geo-engineering MER model as a nested restriction. In Section 3, the economic model is estimated and hypothesis tests are conducted based on a sample of seven oil fields in Montana with sixty quarterly time series observations over the period from September, 1971 through June, 1986.⁵ The final section contains a summary and conclusions.

2. MODELS OF PETROLEUM SUPPLY

The process of discovering and developing petroleum resources consists of two major components: the discovery of new reserves and production from existing reserves. New reserve additions result from the drilling of successful new wells that either discover new petroleum accumulations or extend the boundaries of known fields. Production from existing fields occurs, for the most part, from development wells drilled within known field boundaries. Oil and gas supply models treat the drilling of new wells as a process distinct from that of producing oil from established fields, segregating the petroleum exploration phase from that of reserves production. Since the first formal supply models were created in the late 1950s and early 1960s, the vast majority have focused on the process of exploring for new reserves. This is in spite of the fact that annual oil and gas production from established reserves is far greater than are reserve additions.

Deacon *et al.* [8] provide a useful survey of models in which changes in petroleum supply stem primarily from reserve additions. Relatively few models of petroleum supply explicitly include production from established fields. Of the petroleum supply models that include production from known reserves, most adopt a framework in which it is assumed that, without the drilling of new wells, production from developed fields will decline continuously over time in a manner that is determined by geological factors. In these models, production declines by a constant percentage over time, thereby following a pattern of exponential decline. Examples include the widely cited study by MacAvoy and Pindyck [29] and, more recently, models by Deacon *et al.* [8] and by Griffin and Moroney [15]. Models by Amit [1] and Camm *et al.* [5] are similar but less complete.

In the model by MacAvoy and Pindyck [29], the short-run supply function is defined by the marginal cost of developing existing reserves through the drilling and operating of new wells. An exponential decline pattern of production from reserves is assumed in developing the marginal production cost function. An increase in output price yields an increase in the production rate from established fields through the drilling of additional production wells. Because this is the only production response from existing fields, and because production from new wells is allocated to the reserve additions process, this model predicts no short-run supply response for established fields.

⁵The sample period is terminated at the end of the second quarter of 1986 because some of the oil fields included in the study were shut in shortly thereafter.

Explicit in the models of Deacon *et al.* [8], Griffin and Moroney [15], Amit [1], and Camm *et al.* [5] is the argument that a change in the price of oil alters the period of production from an oil field. In each of these models, so long as production follows an exponential decline pattern, pumping costs remain constant over time. Hence, the production rate for each well eventually declines to the point where a producer's revenue, net of taxes and royalty payments, equals the constant periodic operating cost of the well. This point is termed the *economic limit*. Given an increase in the wellhead price of oil, revenues increase proportionately in each production period. But there is no output response, so that pumping costs remain constant. Therefore, the economic limit is reached at a later date and the total amount of oil produced from the well increases. In the absence of drilling additional production wells, this is the only avenue for an economic response in exponential decline models of oil production from mature fields.

2.1. Geo-Engineering Models of Oil Production

As oil is extracted from existing wells, the energy causing oil to flow to the wellhead is depleted so that well productivity decreases with time and with the cumulative amount of oil produced [20, 22, 30]. The causes of the negative relationship between the current production rate and cumulative past production are explicit in the formulation of one of the fundamental equations of subsurface fluid flow, termed Darcy's Law.⁶ Although Darcy's Law is most often associated with ground water flow, it is easily adapted to the nonturbulent flow of any fluid through a porous medium. Following Nind [32], Darcy's equation may be written

$$\frac{F}{A} = - \left(\frac{\kappa}{\mu} \right) \frac{\partial p}{\partial l}, \quad (1)$$

where F/A is the rate of flow per unit cross-sectional area across a rock face of area A , κ is the permeability of a homogeneous rock medium, μ is the viscosity of the fluid, and $\partial p/\partial l$ is the rate of pressure drop in the overall direction of flow. As can be seen from Eq. (1), the flow rate through a petroleum reservoir will decline with either a decrease in the pressure gradient across the reservoir or an increase in the viscosity of the fluid. Both of these occur in petroleum reservoirs, to varying degrees, as a function of cumulative production.

Darcy's Law, coupled with the decline in reservoir pressure and increase in the viscosity of the remaining crude oil as a field is produced, predicts that the maximum flow rate of crude oil to a producing well will decrease with cumulative production. The simplest type of production decline results from a reservoir with no water drive, where reservoir pressure is proportional to the amount of remaining reserves, and where crude oil viscosity remains constant [32]. In such an idealized reservoir, the relationship between cumulative oil produced and pressure is linear, as is the relationship between the production rate and cumulative production.

For a reservoir with the characteristics described above, the linear relationship between the production rate, $q(t)$, and cumulative production, $Q(t) \equiv \int_0^t q(s) ds$,

⁶See Davis and DeWiest [6], Domenico [10], and Freeze and Cherry [13].

may be written as

$$q(t) = - \left(\frac{Q(t)}{a} \right) + c, \quad (2)$$

where a is a positive constant called the continuous production decline rate and $c = q_0$, the production rate in the initial period.

The rate of change in the production rate over time is found by differentiating Eq. (2) with respect to t ,

$$\frac{dq(t)}{dt} = - \frac{q(t)}{a}, \quad (3)$$

using the fact that current production is the time rate of change in cumulative production, $q(t) = dQ(t)/dt$. Separating the variables and integrating then gives

$$\log(q(t)) = - \left(\frac{t}{a} \right) + b, \quad (4)$$

where the constant of integration, b , satisfies $b \equiv \log(q_0)$. Once the production rate in the initial period is known, the production rate at any future time is determined by

$$q(t) = q_0 e^{-t/a}. \quad (5)$$

This type of production decline with time is termed *exponential decline*. It has the property that the ratio of the production rate to remaining reserves is constant throughout the production horizon. With a constant percentage change in production, the decline rate is constant for the life of the well.

The decline rate is a property that varies among producing wells and which can be estimated using simple linear regression analysis. Taking the natural log of both sides of the exponential decline equation gives

$$\log(q(t)) = \log(q_0) - (1/a) \cdot t. \quad (6)$$

With data on initial and subsequent production rates, the decline rate can be estimated using this relationship.

Most actual petroleum pools, however, do not have the idealized characteristics described above. Although exponential decline is often assumed for limited time periods due to its mathematical convenience, it has been found that, if assumed for longer periods, it leads to an underestimation of ultimate recovery [32]. One reason for this is that reservoir pressures generally decline at a slower rate as cumulative production increases [2]. This, coupled with an increase in the viscosity of the remaining oil, causes the relationship between the current production rate and cumulative production to be nonlinear. This nonlinear relationship is known as hyperbolic decline. The generalized hyperbolic decline and its special case of exponential decline are the most commonly used geo-engineering models of production decline.

With hyperbolic decline, the relationship between the rate of change in current production and time is modified to

$$\frac{dq(t)}{dt} = - \frac{q(t)}{(a + bt)}. \quad (7)$$

The production rate at any time, t , is now obtained by integrating Eq. (7),

$$q(t) = \begin{cases} q_0 e^{-t/a} & \text{if } b = 0; \\ q_0 \left(\frac{a}{a + bt} \right)^{1/b} & \text{if } b \neq 0. \end{cases} \quad (8)$$

Cumulative production up to time t is then obtained by integrating Eq. (8),

$$Q(t) = \begin{cases} aq_0(1 - e^{-t/a}) & \text{if } b = 0; \\ aq_0 \log \left(\frac{a + t}{a} \right) & \text{if } b = 1; \\ \left(\frac{a}{1 - b} \right) q_0 \left[1 - \left(\frac{a}{a + bt} \right)^{(1-b)/b} \right] & \text{if } b \neq 0, 1. \end{cases} \quad (9)$$

The special cases with $b = 0$ and $b = 1$ are known as *exponential* and *harmonic decline*, respectively. Since $Q(t) \leq R_0$ for all $t \geq 0$, while

$$\lim_{t \rightarrow \infty} Q(t) = \begin{cases} aq_0 & \text{if } b = 0; \\ \left(\frac{a}{1 - b} \right) q_0 & \text{if } 0 < b < 1; \\ \pm \infty & \text{if } b \geq 1, \end{cases} \quad (10)$$

we must have $a > 0$ and $0 \leq b < 1$ for the hyperbolic decline model to be consistent with a positive, finite initial stock of oil. Finally, we will find it useful later to combine Eqs. (8) and (9) and solve for current periodic production as a function of cumulative production to write the MER rule in the equivalent form

$$q(t) = \begin{cases} q_0 - aQ(t) & \text{if } b = 0; \\ q_0 \left[1 - \frac{(1 - b)Q(t)}{aq_0} \right]^{1/(1-b)} & \text{if } 0 < b < 1; \\ \exp \{ -Q(t)/aq_0 \} & \text{if } b = 1. \end{cases} \quad (11)$$

Petroleum engineers use equations of the form of Eqs. (8) and (9) to estimate ultimate oil recovery, future production rates, and the total length of production. The top expression in Eq. (8) is used for approximating production decline over short time intervals, while the bottom expression is used for longer periods. In the models of petroleum supply briefly reviewed above, production decline from existing wells is assumed to follow the exponential decline rule due to its convenience. However, we depart from this practice and adopt the more general hyperbolic decline when we develop a pumping cost function for oil production from mature fields. Specifically, for $b = 0$ we have $\dot{q} = -q/a$ from Eq. (7), while for $b \neq 0$ we have $1/(a + bt) = (1/a) \cdot (q/q_0)^b$ from the second line of Eq. (8).

Hence, for all values of b , we can rewrite Eq. (7) as

$$\frac{\dot{q}(t)}{q_0} = -\left(\frac{1}{a}\right)\left(\frac{q(t)}{q_0}\right)^{1+b}, \quad (12)$$

where a dot over a variable indicates the rate of change with respect to time, that is, $\dot{q}(t) = dq(t)/dt$. This is the form of the hyperbolic decline model that we employ in our empirical work discussed in Section 3.

2.2. Economic Models of Oil Production

Hotelling's rule, that price minus marginal cost rises at the rate of interest in the exploitation of a non-renewable resource [21], relies on several conditions. These include the existence of a purely competitive market, that the amount of oil in the reservoir is known with certainty, and that the costs of extraction do not depend on the amount of oil remaining in the reservoir. Further interpretations of Hotelling's rule, the implications of relaxing various assumptions in the basic model, and the empirical validity of the theoretical result are the subject of a large and well-known literature.⁷

The assumption of uniform resource quality as employed in the basic theory of the mine is not appropriate for crude oil reservoirs. Because of the decline in natural reservoir pressure over time as oil is extracted, the amount of artificial lift required to maintain a given daily production rate will increase, leading to an increase in pumping costs. While the general case of rising extraction costs has been examined in several studies,⁸ relatively few studies adapt the theory of exhaustible resource extraction to the specifics of petroleum production.⁹

With declining resource quality, the oil producer's economic problem includes the effects of increasing marginal costs of production over time. Let $R(t)$ denote the current quantity of remaining oil reserves in the field, and denote the cost function for oil extraction by $C(q(t), R(t))$. We assume that the cost function is twice differentiable, increasing in the current extraction rate,

$$\partial C(q(t), R(t)) / \partial q > 0, \quad (13)$$

decreasing in the current stock of remaining reserves, $R(t)$,

$$\partial C(q(t), R(t)) / \partial R < 0, \quad (14)$$

and convex in $q(t)$. The oil producer's economic problem is to maximize

$$\int_0^T [p(t)q(t) - C(q(t), R(t))]e^{-rt} dt, \quad (15)$$

⁷See Devarajan and Fisher [9] for a survey.

⁸See early papers by Heal [17] and Levhari and Leviatan [26] and more recent studies by Heaps [19] and Krautkramer [24].

⁹Two notable exceptions, however, are Deacon [7] and Livernois [28].

subject to

$$\int_0^T q(t) dt \leq R_0, \quad (16)$$

$$\dot{R}(t) = -q(t), \quad R(0) = R_0, \quad (17)$$

$$q(t) \geq 0, \quad \forall t \geq 0, \quad (18)$$

where r is the real discount rate, $p(t)$ is the market price of oil, and T is the final production period.

The Hamiltonian for this optimal control problem is

$$H = [pq - C(q, R)]e^{-rt} - \lambda q, \quad (19)$$

where the time argument has been suppressed for notational convenience. In this problem, $R(t)$ is the state variable, $q(t)$ is the control variable, and $\lambda(t)$ is the co-state variable or shadow price for the equation of motion for $R(t)$. For all $t \geq 0$ such that $q(t) > 0$, the first-order conditions for an optimal extraction path are

$$\frac{\partial H}{\partial q} = \left[p - \frac{\partial C(q, R)}{\partial q} \right] e^{-rt} - \lambda = 0, \quad (20)$$

$$-\frac{\partial H}{\partial R} = \frac{\partial C(q, R)}{\partial R} e^{-rt} = \dot{\lambda}, \quad (21)$$

$$\frac{\partial H}{\partial \lambda} = \dot{R} = -q, \quad R(0) = R_0. \quad (22)$$

In addition, the transversality condition determines the optimal extraction period,

$$H_T \equiv [p(T)q(T) - C(q(T), R(T))]e^{-rT} - \lambda(T)q(T) = 0 \quad (23)$$

in free terminal time problems.¹⁰ In this study, however, we are primarily concerned with the first-order conditions in Eqs. (20)–(22).

2.3. Nesting the Geo-Engineering and Economic Models

Applications of the geo-engineering MER rule to oil production assume that total per-period extraction costs are constant over time if the production rate follows a hyperbolic decline path.¹¹ The MER rule implies that the primary economic decision that the operator makes is whether to operate the field at full capacity in a given period or shut it in. If all wells in the field operate, the operator incurs a given pumping cost and obtains an output determined by the reservoir decline function. The decline function gives a relationship between the field's output and the field's reserve. This induces a specific relationship between cost,

¹⁰See Kamien and Schwartz [23] and Lambert [25].

¹¹See Deacon *et al.* [8], Amit [1], Camm *et al.* [5], and Griffin and Moroney [15] for applications of this interpretation of the MER rule.

output, and reserves. This relationship can be represented as a restriction on the conventional extraction cost function, $C(q, R)$.¹²

The notion that pumping costs are constant with hyperbolic production decline has been applied to oil fields of various sizes and depths, with different drive mechanisms, and initial production rates. Tacitly, the MER rule asserts that if production is positive, then pumping costs are constant and extraction follows a hyperbolic decline rule. If the pumping cost function also is finite, twice differentiable in current and cumulative oil production, and convex in the current output level,¹³ then by tracing out the iso-cost line associated with the MER extraction path we recover a structure for pumping costs that nests the MER model within an economic model of oil production from established fields.

Toward this end, it is at times useful to rewrite total extraction costs in terms of cumulative extraction. Recall that cumulative oil production to time t is

$$Q(t) = \int_0^t q(s) ds, \quad (24)$$

while the remaining stock at time t is defined as the initial stock minus the cumulative quantity of oil that has been extracted by then, $R(t) \equiv R_0 - Q(t)$. Hence, the periodic extraction cost function can be rewritten as

$$C(q, R) \equiv C(q, R_0 - Q) \equiv \tilde{C}(q, Q). \quad (25)$$

To provide a heuristic illustration of the main ideas behind our approach, suppose that pumping costs have the simple additive structure $\tilde{C}(q, Q) = c(q + aQ)$, where $c(\cdot)$ is an *arbitrary* increasing and convex function of a single variable. Constant pumping costs over time implies that $0 = (\partial \tilde{C} / \partial q) \cdot \dot{q} + (\partial \tilde{C} / \partial Q) \cdot \dot{Q} = c'(q + aQ) \cdot (\dot{q} + a\dot{Q})$, making use of the fact that current production is the time rate of change of cumulative production. Hence, pumping costs remain constant if and only if either $c'(q + aQ) = 0$, $\dot{q} + a\dot{Q} = 0$, or both. For *all* smooth and strictly

¹² We are extremely grateful to an anonymous reviewer for suggesting this explanation of the MER rule.

¹³ Although twice differentiability of extraction costs is a standard assumption in the economics literature on exhaustible resource exploitation, a reviewer pointed out that these conditions and the corresponding interpretation of the MER rule are far from innocuous. For, if pumping costs are smooth, then an extraction path that satisfies the MER rule implies that the oil field operator cannot be a rational economic agent. An alternative interpretation of the MER rule (suggested by the same reviewer) is that pumping costs are independent of q and R , i.e., the marginal cost of oil extraction is zero up to the rate determined by the MER rule, and infinite at all levels above that rate. This interpretation implies that a rational oil field operator would optimally pursue the MER rule as a bang-bang solution to the oil extraction problem. (In fields with multiple wells, however, pumping costs cannot be completely independent of output, since *some* pumps may be turned on but the rest turned off in any given period, reducing both output and costs).

As will be seen in the development that follows, two important parameters in our empirical model are the reciprocal of the slope of the marginal cost curve and the ratio of marginal cost to the slope of marginal cost, both evaluated along the MER path. If marginal cost is zero, but the slope is infinite at each point along the MER path, both parameters equal zero. Moreover, the parameter restrictions associated with the MER rule are the same for both interpretations. In either case, a rejection of the MER rule implies that the bang-bang solution defined by the MER extraction path is suboptimal *and* that the oil field operator responds to economic incentives. On the other hand, a failure to reject the MER rule could result either because agents are not economically rational, pumping costs are not smooth, or both (or perhaps some other reasons which we have not considered).

increasing functions $c(\cdot)$, therefore, all iso-cost lines for this cost function are defined by the exponential decline rule, $\dot{q} = -aq$.

Conversely, suppose we observe an exponential decline extraction path which satisfies $\dot{q}/q = -a$ and $\dot{C} = 0$. Since $\dot{C} = 0$ requires $\dot{q}/q = -(\partial\tilde{C}/\partial Q)/(\partial\tilde{C}/\partial q)$ for cost functions that are strictly increasing in q , we must have a constant slope to the iso-cost line along the observed oil production path. Continuous differentiability of the cost function implies that locally (i.e., in a neighborhood of every point on that particular iso-cost line), the structure of the cost function can be recovered up to a monotonic transformation. That is, $\tilde{C}(q, Q) = C^0$, a constant, if and only if

$$\frac{\partial q(Q, C^0)}{\partial Q} = -\frac{\partial\tilde{C}(q(Q, C^0), Q)/\partial Q}{\partial\tilde{C}(q(Q, C^0), Q)/\partial q}, \quad (26)$$

where $\tilde{C}(q(Q, C^0), Q) \equiv C^0$. But the right-hand side of Eq. (26) equals a constant, $-a$, so that $\partial q(Q, C^0)/\partial Q = -a$ along the iso-cost line associated with the observed exponential decline extraction path. Integrating with respect to Q then implies that $q(Q, C^0) = -aQ + g(C^0)$, where, $g(\cdot)$ is a strictly increasing, continuously differentiable, but otherwise arbitrary (and unknown) function of the constant level of pumping costs, C^0 . Inverting $g(\cdot)$ then gives $C^0 = c(q + aQ)$, say. Twice differentiability of the cost function (in fact, continuous differentiability is sufficient) implies that this additive structure for the cost function must continue to hold throughout a (possibly small) neighborhood of each point on the observed extraction path.^{14, 15}

Now, returning to the general case of hyperbolic decline, we proceed by totally differentiating extraction costs with respect to time,

$$0 = \left(\frac{\partial C}{\partial q}\right)\dot{q} + \left(\frac{\partial C}{\partial R}\right)\dot{R} = -\left(\frac{\partial C}{\partial q}\right)\left(\frac{q}{a + bt}\right) - \left(\frac{\partial C}{\partial R}\right)q, \quad (27)$$

where the right-hand side is obtained by substituting $\dot{q} = -q/(a + bt)$ and $\dot{R} = -q$. Thus, under the MER rule, the iso-cost curve has a slope equal to $-1/(a + bt)$,

$$\frac{\partial q(R, C^0)}{\partial R} = -\left(\frac{\partial C(q(R, C^0), R)}{\partial R}\right) \Big/ \left(\frac{\partial C(q(R, C^0), R)}{\partial q}\right) = \frac{1}{a + bt}, \quad (28)$$

¹⁴If costs are not smooth, but rather are discontinuous at the MER production rate, then the MER path defines the boundary between zero and infinite marginal extraction costs. This limiting case of the construction here forms the basis for the alternative interpretation of the MER rule discussed in footnote 12.

¹⁵One might think that since $\partial q/\partial Q$ is independent of q (as well as Q in this example), so that the cost function is additively separable in current and cumulative production, pumping costs must be linear in current output, i.e., $g(C^0) = \alpha + \beta C^0$ for some constants α and β . (This point was argued forcefully by a reviewer.) However, this is an incorrect conclusion. For *any* smooth, strictly monotonic transformation of $q + aQ$, say $c(q + aQ)$, including *all* those for which $c''(q + aQ) > 0$, we obtain the constant cost condition $0 = c'(q + aQ) \cdot (\dot{q} + aq)$. Hence, $c'(q + aQ) > 0$ implies $\dot{c} = 0$ if and only if $\dot{q} + aq = 0$ for *all* such cost functions $c(\cdot)$.

This result is analogous to the ordinality of utility functions, which results from the fact that we only are able to identify the marginal rates of substitution between consumption goods in consumer demand analysis. In the present case, the information transmitted by a MER extraction path only includes the structure of a single iso-cost curve. This is an insufficient amount of information to identify the shape of the marginal cost curve in addition to the shape of the iso-cost curve. Nevertheless, it is a sufficient amount of information to form the basis for an empirical test of the MER hypothesis.

where $q(R, C^0)$ is defined by the implicit function theorem,

$$C(q(R, C^0), R) \equiv C^0. \quad (29)$$

Next, by methods analogous to those commonly used to recover expenditure functions in the theory of consumer behavior and cost functions in the theory of the firm, given the MER time path relating R to t , we recover the structure of the cost function up to an arbitrary monotonic transformation. First, substitute $R - R_0 = Q$ for Q in (9) and solve for $1/(a + bt)$, which gives¹⁶

$$\frac{1}{a + bt} = \frac{1}{a} \left[1 - \left(\frac{1 - b}{a} \right) \left(\frac{R_0 - R}{q_0} \right) \right]^{b/(1-b)}. \quad (30)$$

Second, combine this with Eq. (28) to generate the partial differential equation

$$\frac{\partial q(R, C^0)}{\partial R} = \frac{1}{a} \left[1 - \left(\frac{1 - b}{a} \right) \left(\frac{R_0 - R}{q_0} \right) \right]^{b/(1-b)}. \quad (31)$$

Third, integration with respect to R implies

$$q(R, C^0) = q_0 \left[1 - \left(\frac{1 - b}{a} \right) \left(\frac{R_0 - R}{q_0} \right) \right]^{1/(1-b)} + f(C^0), \quad (32)$$

where the "constant of integration," $f(C^0)$, is an unknown, strictly increasing and twice continuously differentiable function of C^0 that does not depend on either q or R . Finally, solving Eq. (32) for C^0 by inverting $f(\cdot)$ gives the class of general solutions for the extraction cost function as

$$C(q, R) = g \left\{ q - q_0 \left[1 - \left(\frac{1 - b}{a} \right) \left(\frac{R_0 - R}{q_0} \right) \right]^{1/(1-b)} \right\}. \quad (33)$$

Note that $C(\cdot)$ is additively separable in q and $Q = R_0 - R$, hence a function of one variable only.

¹⁶The two limiting cases

$$\lim_{b \rightarrow 0} \left[1 - \left(\frac{1 - b}{a} \right) \left(\frac{R_0 - R}{q_0} \right) \right]^{b/(1-b)} = 1$$

and

$$\lim_{b \rightarrow 1} \left[1 - \left(\frac{1 - b}{a} \right) \left(\frac{R_0 - R}{q_0} \right) \right]^{b/(1-b)} = e^{-(R_0 - R)/aq_0}$$

are tacitly included in this expression.

We proceed next by exploiting the first-order conditions for an optimal economic path. First, differentiate Eq. (20) with respect to time,

$$\dot{\lambda} = \left[\dot{p} - \left(\frac{\partial^2 C(q, R)}{\partial q^2} \right) \dot{q} - \left(\frac{\partial^2 C(q, R)}{\partial q \partial R} \right) \dot{R} - r \left(p - \frac{\partial C(q, R)}{\partial q} \right) \right] e^{-rt}. \quad (34)$$

Second, equate this to Eq. (21) and eliminate $\dot{\lambda}$ and e^{-rt} ,

$$\frac{\partial C(q, R)}{\partial R} = \left[\dot{p} - \left(\frac{\partial^2 C(q, R)}{\partial q^2} \right) \dot{q} - \left(\frac{\partial^2 C(q, R)}{\partial R \partial q} \right) \dot{R} - r \left(p - \frac{\partial C(q, R)}{\partial q} \right) \right]. \quad (35)$$

Third, substitute $-q$ for \dot{R} from Eq. (22), and replace $\partial C/\partial R$ with

$$\frac{\partial C}{\partial R} \equiv -\frac{1}{a} \left[1 - \left(\frac{1-b}{a} \right) \left(\frac{Q}{q_0} \right) \right]^{b/(1-b)} \frac{\partial g}{\partial q}, \quad (36)$$

which is obtained by differentiating Eq. (33) with respect to R . Replace $\partial^2 C/\partial R \partial q$ with

$$\frac{\partial^2 C}{\partial R \partial q} \equiv -\left(\frac{1}{a} \right) \left[1 - \left(\frac{1-b}{a} \right) \left(\frac{Q}{q_0} \right) \right]^{b/(1-b)} \frac{\partial^2 g}{\partial q^2}, \quad (37)$$

which is obtained by differentiating Eq. (36) with respect to q , and replace $Q = R_0 - R$ with the right-hand side of Eq. (9). Then, after canceling common factors and solving for \dot{q}/q_0 , we have

$$\begin{aligned} \frac{\dot{q}}{q_0} = & -\left(\frac{1}{a} \right) \left(\frac{q}{q_0} \right)^{1+b} + \left(\frac{1}{q_0} \right) \left(\frac{\dot{p} - rp}{\partial^2 g/\partial q^2} \right) \\ & + \left[r + \left(\frac{1}{a} \right) \left(\frac{q}{q_0} \right)^b \right] \left(\frac{\partial g/\partial q}{\partial^2 g/\partial q^2} \right). \end{aligned} \quad (38)$$

Note that the first term on the right-hand side represents the MER effect (compare this with Eq. (12) above), while the second and third terms represent economic response effects. Finally, restate Eq. (38) in the form

$$\frac{\dot{q}}{q_0} = -\left(\frac{1}{a} \right) \left(\frac{q}{q_0} \right)^{1+b} + c \cdot (\dot{p} - rp) + d \cdot \left[r + \left(\frac{1}{a} \right) \left(\frac{q}{q_0} \right)^b \right], \quad (39)$$

where $c \equiv 1/[g''(x)q_0]$, $d \equiv g'(x)/g''(x)$, and

$$x \equiv q - q_0 \left[1 - \left(\frac{1-b}{a} \right) \left(\frac{Q}{q_0} \right) \right]^{1/(1-b)}. \quad (40)$$

Note that x remains constant along the MER path.¹⁷ Eq. (39) forms the basis for the empirical model and nested hypothesis tests presented and discussed in the next section.

¹⁷In fact, if the MER rule is correct, then combining (8) and (9) above, we have $x \equiv 0 \forall t \geq 0$.

3. THE ECONOMETRIC MODEL AND EMPIRICAL RESULTS

The Montana Department of Revenue has maintained quarterly records on total crude oil output, total revenue, and state severance tax payments for each field in the state since 1971 [31]. The sample period for this study is from September, 1971 through June, 1986. Seventy-one fields reported production throughout the sample period. Of these, eighteen fields were eliminated from the sample because they had at least two quarters of missing data. Of the remaining fields, only those which did not experience an increase in the number of producing wells during the sample period were included in the sample. The reason for this restriction is to isolate the production response of existing wells within a field from the production response resulting from the drilling of new wells and increasing the reserve base of the field. While it would be desirable to jointly model the number of producing wells per field and the production per well, the data is not available which would allow a distinction to be made between new wells designed to increase production from existing reserves and new wells designed to increase field reserves. Of the fifty-three fields with complete information over the entire sample period, eleven experienced no increase in producing wells. Four of these fields were under secondary recovery operations during the sample period. The remaining seven fields are included in the estimation. Brief descriptions of these fields are found in Table I.

In the empirical model, the independent variables involving output price are the real net wellhead price of oil and quarterly changes in that price. In determining net price, the wellhead price is adjusted to account for the presence of royalty fees and certain taxes in the following way. The producer's quarterly revenue net of royalty fees and taxes can be written

$$pq = p_w q - RP - S - W, \quad (41)$$

where q is the quarterly output of oil, p_w is the wellhead price, RP is the royalty payment, W is the windfall profit tax, and S is the state severance tax. We abstract from federal and state corporate income taxes and property taxes because the tax rates for these taxes remained essentially constant throughout the sample period.¹⁸

Wellhead prices may vary greatly across fields due to differences in the characteristics of the crude oil such as gravity and sulfur content. Quarterly total revenue and quantity data are available from the state of Montana on a field-by-field basis. This allows the wellhead price for each field to be calculated separately, which controls for quality variability across fields. This also partially controls for the effects of price controls that were in effect during 1973 through 1981. However, during this period of time, production classified as "old oil" was subject to price controls, while production classified as "new oil" was uncontrolled. Because the uncontrolled price was higher than the controlled price, the use of quarterly field-wide prices yields an average price (weighted by the production of old and new oil) rather than a marginal price. For the most part, however, the production from the sample fields was classified as old oil under the price controls. There was a small amount of production from two of the sample fields that was classified as new oil. However, the volume of new oil from these fields was small and limited to

¹⁸The top marginal federal corporate income tax rate declined from 48% to 46% in 1979. However, this minor change is ignored.

TABLE I
Oil Field Descriptions

Field 1	Name: Big Wall Field Discovered: 1948 WRP rate applied: 70% Number of production wells:	Location: T10N; R26 Musselshell County Main operator: Texaco Drive mechanism: Water drive 1971 = 19; 1986 = 11
Field 2	Name: Brush Lake Field Discovered: 1969 WPT rate applied: 70% Number of production wells:	Location: T33N; R58E Sheridan County Main operator: Chevron Drive mechanism: Solution gas/weak water drive 1971 = 7; 1986 = 5
Field 3	Name: Gas City Field Discovered: 1955 WPT rate applied: 70% Number of production wells:	Location: T13, 14N; R55E Dawson County Main operator: Shell Drive Mechanism: Not reported 1971 = 17; 1986 = 12
Field 4	Name: Glendive Field Discovered: 1951 WPT rate applied: 70% Number of production wells:	Location: T15N; R54E Dawson County Main operators: Shell, Texaco Drive mechanism: Water drive/solution gas 1971 = 15; 1986 = 12
Field 5	Name: Poplar East Field Discovered: 1952 WPT rate applied: 50% Number of production wells:	Location: T28, 29N; R50-52E Roosevelt County Main operators: Murphy Oil Drive mechanism: Water drive 1971 = 64; 1986 = 38
Field 6	Name: Red Creek Field Discovered: 1955 WPT rate applied: 50% Number of production wells:	LocationL T31N; R12W Glacier County Main operators: Rainbow Resources, Stovall Drive mechanism: Not reported 1971 = 20; 1986 = 14
Field 7	Name: Tule Creek Field Discovered: 1960 WPT rate applied: 50% Number of production wells:	Location: T30N; R47E Dawson County Main operators: PetroLewis, Murphy Drive Mechanism: Water drive 1971 = 6; 1986 = 4

Source: Data on the number of wells was obtained from Petroleum Information, Inc. The remaining information is from the Montana Geological Society [31].

only a few quarters during the sample period.¹⁹ Therefore, the effect of using an average wellhead price, rather than the appropriate marginal price, should be small.

Royalty payments are assumed to be a constant fraction of gross revenue from a field per quarter. Thus, for a royalty rate of δ , assumed to be 12.5% during the sample period, royalty payments are

$$RP = \delta p_w q. \quad (42)$$

¹⁹ From the fourth quarter of 1978 through the third quarter of 1981, an average of 26% of the production in Field One was new oil. In the second quarter of 1973 and again in the second quarter of 1974, approximately 1% of the production in Field Five was new oil. Finally, from the fourth quarter in 1976 through the fourth quarter of 1977, an average of 21% of the production in Field Five was new oil. None of the other five fields had any new oil during the sample period.

Montana state severance tax payments are assessed on gross revenues. For a severance tax rate of t_s , severance tax payments are²⁰

$$S = t_s p_w q. \quad (43)$$

The federal Windfall Profits Tax (WPT) was an excise tax on the "windfall profit" from taxable crude oil produced after February 29, 1980 and before August 23, 1988.²¹ WPT tax rates varied according to tier and type of producer. The oil produced from the sample fields falls under the Tier 1 category, which was subject to the highest WPT rates of 70% for producers identified as one of the major integrated oil companies and 50% for independent oil producers. For this study, the relevant rate was determined on a field-by-field basis, with fields operated by independent firms having the lower tax rate applied to all production from the field and fields operated by major producers having the higher rate applied.

The windfall profit on which producers were taxed was the excess of the wellhead price over the sum of the adjusted base price and a severance tax adjustment. The original base for Tier 1 oil was the wellhead price per property in March, 1979 minus 21 cents. The base price was adjusted upward by the percentage by which the quarterly implicit GNP price deflator exceeded the deflator for the quarter ending June 30, 1979. The severance tax adjustment is the amount by which any severance tax imposed exceeds the severance tax which would have been imposed if the oil had been valued at its adjusted base price. The producer is assumed liable for windfall profits taxes only on the producer's economic interest in the field because the royalty owner pays the tax on royalty production. Total windfall profits tax payments are

$$W = (1 - \delta)t_w \{p_w - [p_b + t_s(p_w - p_b)]\}q, \quad (44)$$

where t_w is the windfall profits tax rate and p_b is the adjusted base price. Combining Eqs. (41)–(44), the producer's real net price per quarter can be expressed as

$$p = (p_w(1 - t_s - \delta) - (1 - \delta)t_w \{p_w - [p_b + t_s(p_w - p_b)]\})/PPI, \quad (45)$$

where PPI is the quarterly producer price index.

For empirical estimation with discrete data, we replace the time derivatives with discrete differences, $\Delta q_t = q_t - q_{t-1}$ and $\Delta p_t = p_t - p_{t-1}$. The discrete analogue to Eq. (12) for the MER rule is modeled as

$$y_{i,t} = y_{i,t-1} - \alpha_i \cdot y_{i,t-1}^{1+b_i} + u_{i,t}, \quad (46)$$

for $i = 1, \dots, 7$ and $t = 2, \dots, 60$, where $y_{i,t} = (q_{i,t}/q_{i,0})$, $\alpha_i = 1/a_i$, $u_{i,t} = \rho_i u_{i,t-1} + \varepsilon_{i,t}$, and the vector $\varepsilon_t = [\varepsilon_{1,t} \dots \varepsilon_{7,t}]'$ is assumed to be a stationary stochastic

²⁰The severance tax rate in Montana changed from 2.5% to 5% on April 1, 1981, to 6% on April 1, 1983, and back to 5% on April 1, 1985.

²¹See Internal Revenue Code Sections 4986 through 4996 for further information on the Windfall Profit Tax.

TABLE II
MER Rule Model Estimates

	Log-likelihood function = 530.039				
	α	b	ρ	R^2	DW
Field 1	.01343 (.02159)	4.148 (8.749)	-.2348 (.1301)	.6428	2.024
Field 2	.2186 (.03291)	1.208 (.2859)	-3.638 (.1207)	.9200	2.043
Field 3	.03318 (.01519)	.6170 (1.243)	-.2435 (.1215)	.9397	1.920
Field 4	.02835 (.01910)	1.397 (2.444)	-3.897 (.1175)	.8466	2.162
Field 5	.05355 (.04347)	7.696 (3.500)	-.01540 (.2017)	.7951	2.040
Field 6	.08531 (.03710)	4.626 (3.290)	-.1116 (.1549)	.7984	1.877
Field 7	.06401 (.02102)	.8513 (.6928)	-.4016 (.1112)	.9154	2.240

process, with $E(\varepsilon_t) = \mathbf{0}$, $E(\varepsilon_t \varepsilon_t') = \Sigma$, and $E(\varepsilon_t \varepsilon_s') = \mathbf{0}$, $\forall t, \forall s \neq t$.²² The production data are normalized by the initial output level in each field to improve the numerical properties of the empirical model, which is nonlinear in the parameters. Table II presents the results for the MER model. In both Tables II and III, R^2 is the squared correlation between the observed and predicted values of y_t , DW is the Durbin–Watson statistic for remaining serial correlation in the error terms, and the estimated asymptotic standard errors are in parentheses below the parameter estimates. The tabulated results are system estimates with a separate correction for first-order autocorrelation in each equation. The estimation method is iterative nonlinear seemingly unrelated regression equations (SURE), which is equivalent to full information maximum likelihood.

²² We also considered MER models of the form

$$y_{i,t} = y_{i,t-1} - \left(\frac{y_{i,t-1}}{a_i + b_i t} \right) + \varepsilon_{i,t},$$

and of the form

$$y_{i,t} = \left(\frac{a_i}{a_i + b_i t} \right)^{1/b_i} + \varepsilon_{i,t}.$$

Based on the likelihood dominance criterion (Pollak and Wales [34]), other econometric properties of the model estimates, and the consistency of the parameter values with the *a priori* restrictions of the MER model, we selected (47) as our preferred specification. However, complete details on the alternative models are contained in an expanded version of the paper that is available upon request.

TABLE III
Hotelling's Rule Model Estimates

	Log-likelihood function = 552.771						
	α	b	c	d	ρ	R^2	DW
Field 1	.1852 (.0705)	-2.589 (.7078)	-.0023 (.0074)	.6541 (.0286)	-.1108 (.1558)	.6905	1.959
Field 2	.2224 (.0304)	.1947 (.2664)	-.0009 (.0032)	.1682 (.0198)	-.3957 (.1195)	.9237	1.945
Field 3	.0524 (.0179)	-1.402 (.6418)	-.0004 (.0062)	.3139 (.0309)	-.2826 (.1260)	.9433	1.846
Field 4	.0424 (.0204)	-2.978 (.9269)	-.0078 (.0088)	.4277 (.0225)	-.3754 (.1285)	.8592	2.116
Field 5	.2845 (.3138)	4.148 (3.036)	.0370 (.0090)	.7083 (.2242)	.1125 (.1951)	.8432	1.902
Field 6	.1939 (.1201)	2.244 (2.590)	.0028 (.0084)	.4344 (.2090)	-.1167 (.1582)	.7989	1.823
Field 7	.0710 (.0219)	-.8849 (.6333)	.0043 (.0028)	.1730 (.0325)	-.4373 (.1139)	.9169	2.200

The economic alternative to the hyperbolic decline rule can be written as

$$y_{i,t} = y_{i,t-1} - \alpha_i y_{i,t-1}^{1+b_i} + c_i [p_{i,t} - (1+r)p_{i,t-1}] + d_i [r + \alpha_i y_{i,t-1}^{b_i}] + u_{i,t}, \quad (47)$$

for $i = 1, \dots, 7$ and $t = 2, \dots, 60$ as in the MER specification. This specification for the economic alternative assumes that the net real wellhead price of oil in the current period is known to the field operator at the time that operating decisions are made.²³ The properties of the error terms are assumed to be the same as for the MER model.

Numerical difficulties were encountered when we attempted to estimate r , and the log-likelihood function displayed little dependence on the value of r . Hence, the real rate of time discount was held fixed at 2.5% per quarter (which is equivalent to a real annual discount rate of 10.4%) in all fields and the remaining parameters were estimated conditional on r for the results presented in Table III.²⁴

²³We also considered alternative forms of expectations processes, including myopic, adaptive, and rational price expectations. Detailed results for these alternative models are contained in the expanded version of the paper noted in footnote 22. None of the conclusions reached in this paper are altered by these alternative forms of price expectations. The empirical results are quite similar across the alternative forms of the econometric model, but the perfect foresight model dominates all other alternative specifications based on the likelihood dominance criterion.

²⁴The unrestricted estimate of r was -9.11×10^{-3} with an asymptotic standard error of .0771 and value of the log-likelihood function equal to 553.072. Negative values of r less than $-.01$ per quarter generated an undefined log-likelihood function. A search of the log-likelihood function over r from 0% to 10% per quarter (equivalently, 0% to 46% per year) resulted in log-likelihood values that monotonically declined from 553.068 at $r = 0$ to 552.166 at $r = 0.10$. The log-likelihood at $r = 0.025$ per quarter was 552.771. None of the results of the paper are changed over this full range of values of the real discount rate.

Treating the MER rule as the null and the economic response model as the alternative, the two hypotheses can be stated as

$$\mathbf{H}_0 : c_i = d_i = 0, \quad \forall i$$

$$\mathbf{H}_1 : c_i \neq 0 \quad \text{or} \quad d_i \neq 0 \ni i$$

Thus, it is straightforward to test H_0 against H_1 with a likelihood ratio test. The value of the likelihood ratio statistic for the results presented in Tables II and III is 45.46. This test statistic is asymptotically distributed as a $\chi^2(14)$ random variable under the null hypothesis that the MER model is true. The probability value for a $\chi^2(14)$ greater than or equal to 46.46 is 3.43×10^{-5} . Hence, we strongly reject the MER model in favor of the economic alternative.

In addition, for both the MER and economic models, we tested for the *a priori* restrictions of the parameters, in particular, $b_i \in [0, 1], \forall i$ and $c_i > 0, \forall i$. For the unrestricted MER model, five of the b_i estimates are greater than 1. Individually, none of these \hat{b}_i are significantly different from 1 at the 5% level of significance using a two-tailed test, although one is significantly greater than 1 with a one-tailed test. The likelihood ratio test statistic for the joint restriction that all five of these b_i parameters are equal to 1 is 14.64, which is asymptotically distributed as a $\chi^2(5)$ random variable. The probability value for this test result is .012 and we reject the hypothesis that $b_i \in [0, 1], \forall i$, in the MER model at the 5% level of significance but not at the 1% level.

In the unrestricted economic model, six of the b_i estimates are outside the unit interval, four negative and two greater than 1. Three (two) of the negative \hat{b}_i 's are significantly less than zero at the 5% (1%) level with a two-tailed test, while neither of the two \hat{b}_i 's that exceed 1 are statistically greater than 1 at standard significance levels. A joint test that all six of these b_i 's are in the closed unit interval generates an asymptotically $\chi^2(6)$ statistic of 30.90, which has a probability value of 2.65×10^{-5} . We therefore reject the hypothesis that they lie in the unit interval.²⁵ In addition, four of the estimated c_i are negative, although none are individually statistically different from zero at standard levels of significance. A joint test that all four are equal to 0.0001 produces an asymptotically $\chi^2(4)$ statistic equal to 0.99 with a probability value of .912 and we do not reject the hypothesis that $c_i > 0, \forall i$. The joint test that all $b_i \in [0, 1]$ and all $c_i > 0$ produces a test statistic of 22.32, which is asymptotically distributed as a $\chi^2(10)$ and has a probability value of .0135. Hence, we reject this hypothesis at the 5% level of significance, but can not reject it at the 1% level. Finally, a likelihood ratio test of the restricted MER model against the restricted economic alternative gives a test statistic of 37.74, which is approximately distributed as a $\chi^2(9)$ random variable, with a probability value of 1.94×10^{-5} . Hence, as in the case of the unrestricted versions for both models, we strongly reject the hypothesis that oil production from established fields is solely driven by the MER rule, and therefore invariant to changes in the market price of oil.

²⁵It is worth noting that this result is more damaging to the MER model than the economic alternative. The reason is that the variable x defined in (40) is strictly concave in R for all $b \in (0, 1]$, but is convex in R for $b \leq 0$. Hence, the second-order conditions for an optimal path are violated with a strictly positive b . Moreover, none of the positive \hat{b}_i 's are significantly greater than zero in the economic model.

4. CONCLUSIONS

The process of exploring for and developing a petroleum deposit encompasses a series of operations that may span several decades. To make tractable economic models of a process as complex as domestic petroleum supply, simplifying assumptions and techniques must be employed. This is especially true for the process of production from reserves. The geological and engineering constraints imposed on production, coupled with the significant empirical difficulties associated with estimating production cost functions, have fostered the use of supply functions not derived from the producer's underlying optimization problem.

One contribution of this paper is to develop a production cost function that incorporates the dynamics of oil and associated gas movement within the reservoir. The hyperbolic production decline function, applicable to a variety of drive mechanisms, is used in the specification of the effects of the current extraction rate and cumulative production on current costs. This provides a viable alternative to the calibration approach recently used by Deacon [7].

The principal use of petroleum supply models is to estimate the response of current and future output to changes in real prices. By assuming that the production rate is invariant to economic factors, most previous models constrain reserves production so that the only avenues of response to price variations are the drilling of new development wells and changing the length of the production horizon for existing wells. This can bias the estimated price elasticity of oil supply. We tested whether or not the current rate of output responds to price changes in petroleum supply. Our empirical results indicate that the economic model based on geoenvironmental considerations significantly outperforms the MER model. We therefore reject the notion that there is no economic response in the rate of production from established oil fields.

We conclude that, in addition to the possibility of new development drilling, producers are likely to change both the current production rate and the length of the production horizon for existing wells in response to a change in output price. The response of the production rate to changes in output price will be small for fields with few wells near the end of their producing life. However, the response may be large for large fields with many wells with high production rates.²⁶ In addition, we conclude that the relationship between output price and the economic life of an oil field is not unambiguously positive as has been claimed in several previous studies.

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²⁶ Price elasticities of supply cannot be estimated in general because we are only able to recover the pumping cost function, based on (33), up to an ordinal monotonic transformation. However, under the assumption that costs are approximately quadratic in the variable x defined in (40), we can use point estimates for c and d to estimate static short-run price elasticities of supply. These supply elasticities, at the mean for net price and quantity extracted, range from 0 to 0.654 and increase with both production rates and number of wells per field. The sample fields are small in terms of both production rates and number of wells. For large, highly productive fields, the elasticities will be much larger, as will the error stemming from the use of the MER model.

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