Prices vs. Quantities

Martin L. Weitzman

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Introduction

- What is the best way to implement control for the benefit of the organization as a whole?

- Is it better to directly administer the activity under scrutiny or to fix transfer prices and rely on self-interested profit or utility maximization to achieve the same ends in decentralized fashion?
- **Example:control certain forms of pollution by setting emission standards or by charging the appropriate pollution taxes**

- It is neither easier nor harder to name the right prices than the right quantities because in principle exactly the same information is needed to correctly specify either.

- In any particular setting there may be important practical reasons for favouring either prices or quantities as planning instruments
- Even on an abstract level, it would be useful to know how to identify a situation where employing orne mode is relatively advantageous, other things being equal.

- Amount q of a certain commodity can be produced at cost $C(q)$, yielding benefits $B(q)$.
- It is assumed that $B"(q) < 0$, $C"(q) > 0$, $B'(O) > C'(O)$, and $B'(q) < C'(q)$ for q sufficiently large.
- The planning problem is:

$$\max_q B(q) - C(q).$$

$$\begin{aligned} B'(q^*) &= C'(q^*) \\ p^* &\equiv B'(q^*) = C'(q^*) \end{aligned}$$

- Incomplete Information: $C(q, \theta)$, where θ is a disturbance term or random variable, unobserved and unknown at the present time.
- $B(q, \eta)$ η is a RV.
- θ and η are independently distributed.

- Now an ideal instrument of central control would be a contingency message whose instructions depend on which state of the world is revealed by θ and η.

$$B_1(q^*(\theta, \eta), \eta) = C_1(q^*(\theta, \eta), \theta) = p^*(\theta, \eta).$$

- Moral Hazard Problem

- The issue of prices vs. quantities has to be a "second best " problem by its very nature.

$$E[B(\hat{q}, \eta) - C(\hat{q}, \theta)] = \max_q E[B(q, \eta) - C(q, \theta)],$$

$$E[B_1(\hat{q}, \eta)] = E[C_1(\hat{q}, \theta)].$$

- When a price instrument p is announced, production will eventually be adjusted to the output level
- $$q = h(p, \theta)$$
- which maximizes profits given p and θ. Such a condition is expressed as

  $$ph(p, \theta) - C(h(p, \theta), \theta) = \max_q \; pq - C(q, \theta),$$

  implying

  $$C_1(h(p, \theta), \theta) = p.$$

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If the planners are rational, they will choose that price instrument $\tilde{p}$ which maximizes the expected difference between benefits and costs given the reaction function $h(p, \theta)$:

$$E[B(h(\tilde{p}, \theta), \eta) - C(h(\tilde{p}, \theta), \theta)] = \max_{p} E[B(h(p, \theta), \eta) - C(h(p, \theta), \theta)].$$

The solution $\tilde{p}$ must obey the first order equation

$$E[B_1(h(\tilde{p}, \theta), \eta) \,.\, h_1(\tilde{p}, \theta)] = E[C_1(h(\tilde{p}, \theta), \theta) \,.\, h_1(\tilde{p}, \theta)],$$

which can be rewritten as

$$\tilde{p} = \frac{E[B_1(h(\tilde{p}, \theta), \eta) \,.\, h_1(\tilde{p}, \theta)]}{E[h_1(\tilde{p}, \theta)]}. \qquad \ldots(3)$$

Corresponding to the optimal *ex ante* price $\tilde{p}$ is the *ex post* profit maximizing output $\tilde{q}$ expressed as a function of θ,

$$\tilde{q}(\theta) \equiv h(\tilde{p}, \theta). \qquad \ldots(4)$$

- After the quantity $\widehat{q}$ is prescribed, producers will continue to generate that assigned level of output for some time even though in all likelihood

$$B_1(\hat{q}, \eta) \neq C_1(\hat{q}, \theta).$$

In the price mode on the other hand, $\tilde{q}(\theta)$ will be produced where except with negligible probability

$$B_1(\tilde{q}(\theta), \eta) \neq C_1(\tilde{q}(\theta), \theta).$$

- Neither instrument yields an optimum ex post. The *relevant question* is which one comes closer under what circumstances

- It is natural to define the comparative advantage of prices over quantities as

$$\Delta \equiv E[(B(\tilde{q}(\theta), \eta) - C(\tilde{q}(\theta), \theta)) - (B(\hat{q}, \eta) - C(\hat{q}, \theta))].$$

- The coefficient $\triangle$ is intended to be a measure of comparative or relative advantage only.
- Let the symbol "$\circeq$ " denote an *accurate local approximation*:

$$C(q, \theta) \circeq a(\theta) + (C' + \alpha(\theta))(q - \hat{q}) + \frac{C''}{2}(q - \hat{q})^2 \qquad ...(6)$$

$$B(q, \eta) \circeq b(\eta) + (B' + \beta(\eta))(q - \hat{q}) + \frac{B''}{2}(q - \hat{q})^2. \qquad ...(7)$$

- WLG $\alpha(\theta)$ and $\beta(\eta)$ are standarized,θ

$$E[\alpha(\theta)] = E[\beta(\eta)] = 0 \text{ and } E[\alpha(\theta).E[\beta(\eta)] = 0$$

- Note that the stochastic functions

$$\begin{aligned} a(\theta) &\doteq C(\widehat{q},\theta) \\ b(\eta) &\doteq B(\widehat{q},\eta) \end{aligned}$$

- Differentiating (6) and (7) wrt q

$$C_1(q,\theta) \doteq (C' + \alpha(\theta)) + C''.(q - \hat{q}) \qquad \ldots(10)$$

$$B_1(q,\eta) \doteq (B' + \beta(\eta)) + B''.(q - \hat{q}). \qquad \ldots(11)$$

- Stochastic changes in $\alpha(\theta)$ represent pure unbiased shifts of the marginal cost function. The variance of $\alpha(\theta)$ is precisely the mean square error in marginal cost:

$$\sigma^2 \equiv E[(C_1(q,\theta) - E[C_1(q,\theta)])^2] \doteq E[\alpha(\theta)^2].$$

$$E[(B_1(q,\eta) - E[B_1(q,\eta)])^2] = E[\beta(\eta)^2].$$

- **Result of the paper** "COEFFICIENT OF COMPARATIVE ADVANTAGE"

$$\Delta \cong \frac{\sigma^2 B''}{2C''^2} + \frac{\sigma^2}{2C''}.$$

- as σ^2 shrinks to zero we move closer to the perfect certainty case where in theory the two control modes perform equally satisfactorily.
- Clearly $\triangle$ depends critically on the curvature of cost and benefit functions around the optimal output level.
- the sign of $\triangle$ simply equals the sign of $C'' + B''$
- The coefficient $\triangle$ is negative and large as either the benefit function is more sharply curved or the cost function is closer to being linear
    - Using a price control mode in such situations could have detrimental consequences
- When marginal costs are nearly flat, the smallest miscalculation or change results in either much more or much less than the desired quantity.
- The price mode looks relatively more attractive when the benefit function is closer to being linear.

- Having seen how C'' and B'' play an essential role in determining $\triangle$, it may be useful to check out a few of the principal situations where we might expect to encounter cost and benefit functions of one curvature or another.
- The amount of pollution which makes a river just unfit for swimming could be a point where the marginal benefits of an extra unit of output change very rapidly.
    - is that it doesn't pay to " fool around " with prices in such situations.
- Quantities are better signals for situations demanding a high degree of coordination.