Prices vs. Quantities

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Introduction

- What is the best way to implement control for the benefit of the organization as a whole?

- Is it better to directly administer the activity under scrutiny or to fix transfer prices and rely on self-interested profit or utility maximization to achieve the same ends in decentralized fashion?

- **Example:** control certain forms of pollution by setting emission standards or by charging the appropriate pollution taxes

- It is neither easier nor harder to name the right prices than the right quantities because in principle exactly the same information is needed to correctly specify either.
In any particular setting there may be important practical reasons for favouring either prices or quantities as planning instruments.

Even on an abstract level, it would be useful to know how to identify a situation where employing one mode is relatively advantageous, other things being equal.
Amount $q$ of a certain commodity can be produced at cost $C(q)$, yielding benefits $B(q)$.

It is assumed that $B''(q) < 0$, $C''(q) > 0$, $B'(0) > C'(0)$, and $B'(q) < C'(q)$ for $q$ sufficiently large.

The planning problem is:

$$\max_q B(q) - C(q).$$

$$B'(q^*) = C'(q^*)$$

$$p^* \equiv B'(q^*) = C'(q^*)$$

Incomplete Information: $C(q, \theta)$, where $\theta$ is a disturbance term or random variable, unobserved and unknown at the present time.

$B(q, \eta)$ $\eta$ is a RV.

$\theta$ and $\eta$ are independently distributed.
Now an ideal instrument of central control would be a contingency message whose instructions depend on which state of the world is revealed by $\theta$ and $\eta$.

$$B_1(q^*(\theta, \eta), \eta) = C_1(q^*(\theta, \eta), \theta) = p^*(\theta, \eta).$$

Moral Hazard Problem

The issue of prices vs. quantities has to be a "second best " problem by its very nature.

$$E[B(\hat{q}, \eta) - C(\hat{q}, \theta)] = \max_q E[B(q, \eta) - C(q, \theta)],$$

$$E[B_1(\hat{q}, \eta)] = E[C_1(\hat{q}, \theta)].$$
When a price instrument $p$ is announced, production will eventually be adjusted to the output level

$$q = h(p, \theta)$$

which maximizes profits given $p$ and $\theta$. Such a condition is expressed as

$$ph(p, \theta) - C(h(p, \theta), \theta) = \max_q pq - C(q, \theta),$$

implying

$$C_1(h(p, \theta), \theta) = p.$$
If the planners are rational, they will choose that price instrument $\bar{p}$ which maximizes the expected difference between benefits and costs given the reaction function $h(p, \theta)$:

$$E[B(h(\bar{p}, \theta), \eta) - C(h(\bar{p}, \theta), \theta)] = \max_p E[B(h(p, \theta), \eta) - C(h(p, \theta), \theta)].$$

The solution $\bar{p}$ must obey the first order equation

$$E[B_1(h(\bar{p}, \theta), \eta) . h_1(\bar{p}, \theta)] = E[C_1(h(\bar{p}, \theta), \theta) . h_1(\bar{p}, \theta)],$$

which can be rewritten as

$$\bar{p} = \frac{E[B_1(h(\bar{p}, \theta), \eta) . h_1(\bar{p}, \theta)]}{E[h_1(\bar{p}, \theta)]}. \quad \text{...(3)}$$

Corresponding to the optimal \textit{ex ante} price $\bar{p}$ is the \textit{ex post} profit maximizing output $\bar{q}$ expressed as a function of $\theta$,

$$\bar{q}(\theta) \equiv h(\bar{p}, \theta). \quad \text{...(4)}$$
After the quantity \( \hat{q} \) is prescribed, producers will continue to generate that assigned level of output for some time even though in all likelihood

\[
B_1(\hat{q}, \eta) \neq C_1(\hat{q}, \theta).
\]

In the price mode on the other hand, \( \tilde{q}(\theta) \) will be produced where except with negligible probability

\[
B_1(\tilde{q}(\theta), \eta) \neq C_1(\tilde{q}(\theta), \theta).
\]

Neither instrument yields an optimum ex post. The relevant question is which one comes closer under what circumstances
It is natural to define the comparative advantage of prices over quantities as

$$\Delta \equiv E[(B(\tilde{q}(\theta), \eta) - C(\tilde{q}(\theta), \theta)) - (B(\hat{q}, \eta) - C(\hat{q}, \theta))].$$

The coefficient $\Delta$ is intended to be a measure of comparative or relative advantage only.

Let the symbol "$\cong$" denote an accurate local approximation:

$$C(q, \theta) \cong a(\theta) + (C' + a(\theta))(q - \hat{q}) + \frac{C''}{2} (q - \hat{q})^2 \quad \ldots(6)$$

$$B(q, \eta) \cong b(\eta) + (B' + b(\eta))(q - \hat{q}) + \frac{B''}{2} (q - \hat{q})^2. \quad \ldots(7)$$

WLG $\alpha(\theta)$ and $\beta(\eta)$ are standarized, $\theta$

$$E[\alpha(\theta)] = E[\beta(\eta)] = 0 \text{ and } E[\alpha(\theta)E[\beta(\eta)] = 0$$
Note that the stochastic functions

\[ a(\theta) \equiv C(\hat{q}, \theta) \]
\[ b(\eta) \equiv B(\hat{q}, \eta) \]

Differentiating (6) and (7) wrt q

\[ C_1(q, \theta) \equiv (C' + \alpha(\theta)) + C''(q - \hat{q}) \] \hspace{1cm} \ldots(10)
\[ B_1(q, \eta) \equiv (B' + \beta(\eta)) + B''(q - \hat{q}) \] \hspace{1cm} \ldots(11)

Stochastic changes in \( \alpha(\theta) \) represent pure unbiased shifts of the marginal cost function. The variance of \( \alpha(\theta) \) is precisely the mean square error in marginal cost:

\[ \sigma^2 \equiv E[(C_1(q, \theta) - E[C_1(q, \theta)])^2] \equiv E[\alpha(\theta)^2]. \]
\[ E[(B_1(q, \eta) - E[B_1(q, \eta)])^2] = E[\beta(\eta)^2]. \]
Result of the paper "COEFFICIENT OF COMPARATIVE ADVANTAGE"

\[ \Delta \equiv \frac{\sigma^2 B''}{2C''^2} + \frac{\sigma^2}{2C''}. \]

as \( \sigma^2 \) shrinks to zero we move closer to the perfect certainty case where in theory the two control modes perform equally satisfactorily.

Clearly \( \Delta \) depends critically on the curvature of cost and benefit functions around the optimal output level.

the sign of \( \Delta \) simply equals the sign of \( C'' + B'' \)

The coefficient \( \Delta \) is negative and large as either the benefit function is more sharply curved or the cost function is closer to being linear

Using a price control mode in such situations could have detrimental consequences

When marginal costs are nearly flat, the smallest miscalculation or change results in either much more or much less than the desired quantity.

The price mode looks relatively more attractive when the benefit function is closer to being linear.
Having seen how $C''$ and $B''$ play an essential role in determining $\Delta$, it may be useful to check out a few of the principal situations where we might expect to encounter cost and benefit functions of one curvature or another.

The amount of pollution which makes a river just unfit for swimming could be a point where the marginal benefits of an extra unit of output change very rapidly.

- is that it doesn’t pay to "fool around" with prices in such situations.

Quantities are better signals for situations demanding a high degree of coordination.