A Solution to the Problem of Externalities When Agents Are Well-Informed

Introduction

- There is a unilateral externality
- The agents involved know the relevant technology and the tastes of all other agents.
- The "regulator" who has the responsibility for determining the final allocation, does not have this information.
- How can the regulator design a mechanism that will implement an efficient allocation?
Simple two-stage games whose subgame-perfect equilibria implement efficient allocations

- In the case of public goods, the mechanisms implement Lindahl allocations;
- in the case of a negative externality, the injured parties are compensated (compensation mechanisms)
Two agents

Firm 1 produces output $x$

- $\max \pi_1 = rx - c(x)$

Firm 2’s profits: $\pi_2 = -e(x)$ [negative externality!]

All of this information is known to both agents but is not known by the regulator

$x$ will not be efficient
There are three classic solutions to this problem of externalities:

1. Ronald Coase (1960), involves negotiation between the agents.
2. Kenneth Arrow (1970), involves setting up a market for the externality.
3. C. Pigou (1920), involves intervention by a regulator who imposes a Pigovian tax

Assume that the government has full information, then,

- Internalizing the externality..... EASY!
- \( \max_x rx - c(x) - e(x) \)
- Pigovian tax: \( P^* = e'(x^*) \)
- \( \max_x rx - c(x) - P^* x \)
However, the regulator does not know the externality cost function and cannot determine the appropriate value of \( p^* \).

Design a mechanism that induces the agents to reveal their information and achieve an efficient level of production.

Compensation mechanism that solves the regulator’s problem

- **Announcement stage.** Firm 1 and 2 simultaneously announce the magnitude of the appropriate Pigovian tax; \( p_1 \) and \( p_2 \).
- **Choice stage.** The regulator makes side-payments to the firms. The two firms face profit-maximization problems:
  - \( \pi_1 = rx - c(x) - p_2 x - \alpha_1 (p_1 - p_2)^2 \)
  - \( \pi_2 = p_1 x - e(x) \)
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A Simple Example of the Compensation Mechanism

max $\Pi_1$  max $\Pi_2$
Unique SPE of this game:

- \( p_1 = p_2 = p^* \) and \( x^* \)

Backward Induction, Firm 1 maximizes its profits,

- \( r = c'(x) + p_2 \)
- This determines the optimal choice, \( x \), as \( x(p_2) \). Note that \( x'(p_2) < 0 \)

Price-setting stage of the game:

- If firm 1 believes that firm 2 will announce \( p_2 \), then: \( p_1 = p_2 \)
Firm 2’s pricing decision.

Although firm 2’s announcement:

- No direct effect on firm 2’s profits,
- Indirect effect through the influence of \( p_2 \) on firm 1’s output choice in stage 2

\[
\pi'_2(p_2) = [p_1 - e'(x)]x'(p_2) = 0
\]

\( p_1 = e'(x) \), therefore

\( r = c'(x) + e'(x) \) which is the condition for social optimality!
For example, suppose that firm 1 thinks that firm 2 will report a large price for the externality.

Then, since firm 1 is penalized if it announces something different from firm 2
Firm 1 will also want to announce a large price

If firm 1 announces a large price, firm 2 will be "overcompensated" for the externality

But firm 2 can give firm 1 an incentive to produce a large amount of output iff it reports a small price for the externality
This contradicts the original assumption

The only equilibrium for the mechanisms occurs if firm 2 is just compensated (on the margin) for the cost that firm 1 imposes on it
Three agents

Suppose that agent 1 imposes an externality on agents 2 and 3.

$p_{ij}^k$ represents the price announced by agent $k$ that measures (in equilibrium) the marginal cost that agent $j$’s choice imposes on agent $i$.

Compensation mechanism for this problem has payments of the form:

\[ \pi_1 = rx - c(x) - [p_{21}^2 + p_{31}^3]x - \|p_{21}^1 - p_{21}^2\| - \|p_{31}^1 - p_{31}^3\| \]

\[ \pi_2 = p_{21}^1 - e_2(x) \]

\[ \pi_3 = p_{31}^1 - e_3(x) \]
If payments are distributed so as to balance the budget out of equilibrium, the payoffs become,

$$\pi_1 = rx - c(x) - [p_{21}^2 + p_{31}^3]x - ||p_{21}^1 - p_{21}^2|| - ||p_{31}^1 - p_{31}^3||$$

$$\pi_2 = p_{21}^1 x - e_2(x) + ||p_{31}^3 - p_{31}^1|| x + ||p_{31}^1 - p_{31}^3||$$

$$\pi_3 = p_{31}^1 x - e_3(x) + ||p_{21}^2 - p_{21}^1|| x + ||p_{21}^1 - p_{21}^2||$$

it is possible to verify that the unique equilibrium of this mechanism is the efficient outcome.

In fact, it is not necessary to have penalty terms when there are more than two agents

- set the penalty terms above equal to zero
- $$r - c'(x) - [p_{21}^2 + p_{31}^3] = 0$$
- $$[p_{21}^1 - e_2'(x) + p_{31}^3 - p_{31}^1]x'(p_{21}^2 + p_{31}^3) = 0$$
- $$[p_{31}^1 - e_3'(x) + p_{21}^2 - p_{21}^1]x'(p_{21}^2 + p_{31}^3) = 0.$$
Linear prices are fine in a convex environment, but if the environment is not convex, linear prices will not support efficient allocations.

Generalization of the CM:

- **Announcement stage.** Firm 1 and 2 each announce the externality costly function for firm 2: $e_1(\cdot)$ and $e_2(\cdot)$
- **Choice stage.** Firm 1 chooses $x$ and each firm receives payoffs given by:
  - $\pi_1 = rx - c(x) - e_2(x) - \|e_1 - e_2\|$
  - $\pi_2 = e_1(x) - e(x)$
  - Note that $\|e_1 - e_2\|$ represents any norm in the appropriate function space.
In equilibrium firm 1 will always want to report the same function as firm 2: $e_1(x) \equiv e_2(x)$

Maximization of profits by firm 1 implies

$$rx^* - c(x^*) - e_2(x^*) \geq rx - c(x) - e_2(x) \quad (1)$$

The equilibrium choice of $x$ must also max. firm 2’s profits:

$$e_1(x^*) - e(x^*) \geq e_1(x) - e(x) \quad (2)$$

Adding (1) and (2) and using $e_1(x) \equiv e_2(x)$

$$rx^* - c(x^*) - e(x^*) \geq rx - c(x) - e(x)$$

which shows that $x^*$ is the socially optimal amount
Conclusions

- Analyze the case of a repeated game and non-convex environment!!

- The compensation mechanism provides a simple mechanism for internalizing externalities in economic environment.

- Transfer payments can be chosen so that the compensation mechanism is balanced, and penalty payments, when they are used, can be chosen to be arbitrarily small.

- The main problem with the mechanism is that it requires complete information by the agent.