A Simple Auction Mechanism for the Optimal Allocation of the Commons

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Introduction

- Regulatory authorities generally find that part of the information they need for implementing an efficient regulation is in the hands of those who are to be regulated.
- Propose a simple mechanism that implements the first-best for any number of incompletely informed firms:
  - A uniform-price sealed-bid auction of an endogenous number of licenses with a fraction of the revenues given back to firms
  - in order to guarantee the efficient outcome, he introduces two key ingredients:

1. The total number of licenses be endogenous to the demand schedules submitted by firms.
2. Rebates or paybacks.

- The supply curve of licenses reflects the cost to society (other than firms) of allocating these licenses to firms.
- Paybacks are such that the total payment for licenses of each firm is exactly equal to the “damage” it exerts upon all the other agents.
Consider $n > 1$ firms to be regulated

Inverse demand functions for pollution of the form $P_i(x_i)$ with $P_i'(x_i) < 0$

The social damage caused by pollution $x$ is $D(x)$ with $D(0) = 0$, $D'(x) > 0$, and $D''(x) \geq 0$

$D'(x)$ can be interpreted as the regulator’s supply function for licenses $S(p)$, where $D'(S(p)) = p$.

$D(x)$ is not necessary publicly known

Firm $i$’s cost of reducing emissions from $x_i^0$ to some level $x_i < x_i^0$, is

$$C_i(x_i) = \int_{x_i}^{x_i^0} P_i(z_i) \, dz$$

The regulator’s objective is $\min C(x) + D(x)$

Therefore, the socially optimal or first-best pollution level $x^* < x^0$ satisfies $P(x^*) = D'(x^*) = P_i(x_i^*)$ for all $i = 1, \ldots, n$

But the regulator cannot directly implement the first-best allocation because he does not know the demand functions
Kwerel’s Scheme

- Kwerel’s scheme is a two-stage mechanism based on the combination of two instruments:
  1. an allocation of a total of $l$ transferable licenses and
  2. a subsidy of $s$ per license to be paid to any firm holding licenses in excess of its emission

- **In the first (or reporting) stage:** the regulator asks firms to report their demand curves (i.e., types) after they are informed that the parameters $l$ and $s$ are to be set according to $s = \hat{P}(l) = D'(l)$

- **In the second (or allocation) stage:** the $l$ licenses are allocated to firms

- Assuming that the market for licenses is perfectly competitive, it must hold in equilibrium that $-C_i'(x_i) = P_i(x_i) = p$

- Kwerel argues that truth-telling is a Bayesian Nash equilibrium.
Kwerel’s Scheme

- Kwerel’s argument can be easily explained with the aid of Figure 1

![Figure 1A. Incentives to Over-report](image-url)
Kwerel’s Scheme

- Kwerel’s logic holds as long as all licenses are auctioned off and the (uniform-price) auction is competitive.
- If firms anticipate a competitive equilibrium at the auction, it is a Nash equilibrium for them to report their true types in the first stage.
- The problem is that there are many other (inefficient) equilibria that are more profitable for firms.

**PROPOSITION 1:** The unique (Nash-equilibrium) outcome in Kwerel’s scheme under a free allocation of licenses is for firms to over-report their demand curves to ensure the maximum possible number of licenses and subsidy level.
The Auction Mechanism-Single Firm

The auction scheme operates as follows:

1. The firm is informed in advance about the auction rules
2. The firm is asked to bid a nonincreasing inverse demand schedule $\hat{P}(x)$
3. With this information, the auctioneer/regulator clears the auction (i.e., determines $p$ and $l$) according to: $p = \hat{P}(l) = D'(l)$
4. The firm receives $l$ licenses and pays $p$. Soon after, the firm gets back a fraction $\alpha(l)$ of the auction revenues
5. Proceeds by backward induction.
6. Firm’s minimization problem

\[
\min C(l) + pl - \alpha(l)pl
\]

\[
\alpha'(l) + \alpha(l) \left( \frac{D''(l)l + D'(l)}{D'(l)l} \right) = \frac{D''(l)}{D'(l)}.
\]
Relationship to the VCG-DHM Mechanism

- Notwithstanding that the auction mechanism follows a VCG payoff rule, it is structurally different from the VCG tax mechanism of DHM.

\[
T_i(x_i, \hat{\theta}_i, \hat{\theta}_{-i}) = D(x_i + \sum_{j \neq i} x_j^*(\hat{\theta}_i, \hat{\theta}_{-i})) + \sum_{j \neq i} C(x_j^*(\hat{\theta}_i, \hat{\theta}_{-i}), \hat{\theta}_j) - A_i(\hat{\theta}_{-i}),
\]

- In DHM, the regulator uses the information provided by firms to compute the “overall” first-best equilibrium and make each firm bear the full social cost.
- In fact, making \(A_i(\hat{\theta}_{-i}) = 0\), firm \(i\)’s total compliance cost becomes, in equilibrium, \(C_i(x_i) + T_i(x_i) = C(x) + T(x)\). The constant term \(A_i\) is then used as a lump-sum instrument.
- In the auction mechanism, the regulator uses the information provided by firms to compute a “residual” equilibrium for each firm, simultaneously determining the firm’s equilibrium number of licenses and payments.
Consider two dates, \( t = 1, 2 \), and \( n \geq 1 \) firms
assume no discounting
Firm \( i \)'s abatement costs at date 1 are \( C(x_i) \) but at cost \( I_i \) incurred at date 1
it can reduce its abatement costs at date 2 to \( C(x_i, I_i) \)
Demand schedules for periods 1 and 2 are, respectively,
\[
P_i(x_i) = -C'(x_i) \quad \text{and} \quad P_i(x_i) = \frac{-\partial C(x_i, I_i)}{\partial x_i}
\]
The damage function in each period is \( D(x) \)
First-period social optimum is well known:
\[
D'(x) + C'(x_i) = 0
\]
The social optimum for second-period pollution and first-period R&D is
\[
\frac{\partial C_i(x_i, I_i)}{\partial x_i} + D'(x) = 0; \quad \quad (10)
\]
\[
\frac{\partial C_i(x_i, I_i)}{\partial I_i} + 1 = 0 \quad \text{for all} \quad i = 1, \ldots, n. \quad \quad (11)
\]
Most regulations, whether command-and-control or market-based, will fail to yield (10)–(11) because of incomplete information.

The auction mechanism is immune to such problems.

The regulator must run two separate auctions: for period 1 licenses and for period 2 licenses.

These bids are then used by the regulator to clear the auction.

Note, however, that the regulator does not need to later verify these investments.

In deciding whether to submit its true demand schedule $P_i(x_i, l_i)$, firm $i$ solves the problem

$$
\min_{l_i, I_i} C_i(l_i(l_i), I_i) + D_i(l_i(l_i); I_{-i}) + I_i,
$$
The author discusses how the auction scheme proposed in this paper performs under collusive behavior and whether it requires any adjustment in order to preserve its first-best properties.

Cartel firms need both to coordinate their bidding schedules and agree on the procedure for sharing the cartel profits.

It must induce its members to truthfully reveal their private information (internal mechanism design problem).
A good collusive agreement would then be for plants to coordinate as if they were acting as a single entity.

**PROPOSITION 4:** The optimal collusive agreement for a cartel of \( m \leq n \) firms is to submit only one serious bid with the true aggregate demand curve of the cartel, say \( P_c(x_c) \). One cartel member submits the serious bid while all the other members submit empty demand schedules. The optimal collusive agreement delivers the first-best allocation.

firms can behave as a single entity at the auction and then proceed with license transfers as required by the collusive agreement

three interrelated reasons:

1. paybacks for any given level of licenses are largest
2. clean-up costs for any given level of licenses are lowest
3. the level of licenses that minimizes overall costs is the first-best level
Implementing the Collusive Agreement

- the cartel has to induce its members to reveal truthfully their individual demand curves.
- Collusive profits have to be shared among the members.
- Consider a potential strong cartel of $m \leq n$ members indexed as $j = 1, \ldots, m$.
- In implementing the optimal collusive agreement of Proposition 4, the cartel organization must solve two problems:

  1. it must put in place an internal scheme that induces firms to reveal truthfully their individual demand curves to the cartel organization.
  2. the cartel must ensure obedience to the cartel mechanism.
Implementing the Collusive Agreement

- Consider the following cartel mechanism:
- Prior to the official auction, cartel members first agree on how to divide cartel profits by determining shares $w_j > 0$, where $\sum_{j=1}^{m} w_j = 1$
- After $w_j$’s are set, cartel members report their demand schedules $\tilde{P}_j(x_j)$ to the cartel mechanism.
- Let $\tilde{P}_c(x_c)$ denote the aggregate demand curve reported by cartel members.
- The cartel mechanism selects an arbitrary member to be the serious bidder, which bids $\tilde{P}_1(x_1) = \tilde{P}_c(x_1)$
- Remaining cartel members bid $\tilde{X}_j(p) = 0$ for all $j = 2, \ldots, m$.
- The cartel mechanism also establishes the way licenses and payments are transferred across cartel members after the auction.
- The cartel mechanism establishes that each cartel member $j$ will receive a number of licenses exactly equal to what he would have individually obtained.
- Member $j$’s total payment for the $l_j$ licenses will be $D_j(l_j) - w_j$. 

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PROPOSITION 5: Assuming that the cartel members can agree on the $w_j$’s, it is a dominant strategy Nash equilibrium for them to report truthfully to the cartel mechanism, i.e., $\tilde{P}_j(x_j) = P_j(x_j)$ for all $j = 1, \ldots, m \leq n$.

- It has the advantage that, by sharing the format of the auction mechanism, it makes it easier for members to understand its workings.
- the cartel mechanism (whether the one proposed here or any other) must be executed in its entirety prior to the official auction.
- the serious bidder must be informed of $\tilde{P}_c(x_c)$ before coming to the official auction.
Future research is the case in which a privately informed agent is simultaneously on the demand and supply side of the auction.

Another aspect not treated in the paper is the possibility that a firm’s pollution (or resource use, more generally) cannot be perfectly monitored.

In addition to the adverse selection problem of not observing a firm’s type (e.g., abatement costs), the regulator must now overcome the moral hazard problem of not perfectly observing the firm’s action.