Don’t Forget to Protect Abundant Resources

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Abstract

We examine an entry-deterrence model with multiple incumbents who strategically increase their individual appropriation in order to prevent entry. We show that entry deterrence yields a welfare improvement, relative to contexts of unthreatened entry, if firms exploit a relatively scarce resource. When incumbents compete for an abundant commons, however, their exploitation becomes large, and welfare losses can arise. Hence, our paper suggests that policies that protect the commons from further entry are actually more necessary when the resource is abundant than when it is scarce.

Keywords: Entry Deterrence; Multiple Incumbents; Common Pool Resources.
JEL classification: L12, Q20, D62.
1 Introduction

Common pool resources (CPRs) are often exploited by several firms. Examples include fishing grounds, forests and oil and gas reservoirs. In the forest industry, for instance, Associated Oregon Loggers reports that 65 logging companies currently operate in the Willamina/Tillamook region.\(^1\) In the case of fishing grounds, 300 boats have been fishing for tuna in the nearby Indian islands of Agatti, Suheli, Minicoy, Bitra and Androth for the past 15 years.\(^2\) Oil drilling also exhibits examples of several companies exploiting the same area. For instance, the U.S. Energy Information Administration reports that 26 drilling companies operate in the Houston and San Antonio areas in Texas.\(^3\) These firms usually face entry threats from potential entrants who also seek to benefit from the exploitation of the resource. Mason and Polasky (1994) examine how a single incumbent might strategically increase its exploitation of the CPR in order to prevent entry. Most commons are, however, exploited by multiple firms and, as this paper demonstrates, their incentives to prevent entry become especially strong when the resource is abundant. Our results, thus, call for policies that protect abundant commons, which are often overlooked, and hence under-protected, by the regulator.

We consider a two period entry-deterrence game in which several incumbents independently select their initial appropriation of the resource in order to deter entry. Our model shows that an entry-deterring equilibrium can be sustained in which incumbents increase their first-period appropriation (relative to no entry threats) when the CPR is relatively abundant. Furthermore, a larger number of incumbents makes entry less attractive, shrinking the set of parameter values for which entry-deterring practices can be supported.

In order to evaluate the welfare consequences of the incumbents’ entry-deterring behavior, we first examine whether their increase in first-period appropriation yields an overexploitation of the resource relative to the social optimum. We show that the CPR is underexploited when the resource is relatively scarce and few incumbents operate. Firms’ appropriation levels are, nonetheless, higher than when entry threats are absent, thus entailing a welfare improvement.\(^4\) In contrast, our paper demonstrates that the commons is overexploited when the resource is abundant and several incumbents operate.\(^5\) Such overexploitation, however, is not necessarily welfare reducing. In particular,

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\(^1\)Similarly, more than 80 sawmill companies exploit forests mainly in central Sweden and some portions of the Norrland region, as reported by the Swedish Forest Industries Association (Skogsindustrierna).

\(^2\)See TNAU Agricultural University, http://agritech.tnau.ac.in/fishery/fish_fishing_methods.html.

\(^3\)Likewise, BP and ConocoPhillips exploit the Alaskan Prudhoe Bay oil field, with their subsidiaries BP Exploration Alaska Inc. and ConocoPhillips Alaska Inc. In addition, ExxonMobil and Chevron operate in the Alaskan North Pole oil field (with their subsidiaries Flint Hills Resources Alaska LLC and Petro Star Inc, respectively); as documented by the Resource Development Council for Alaska, Inc. (http://www.akrdc.org/issues/oilgas/overview.html.)

\(^4\)This underexploitation is observed in different fishing grounds. In particular, as reported by the United Nations Food and Agriculture Organization (2005), the incumbent fleet exploiting the silver hake in the North Atlantic (a relatively scarce resource) has consistently underexploited it below its annual sustainable catch since the late 1990s. This organization also found the underexploitation of other CPRs, such as the Argentine anchovy in the Southern Atlantic and the yellowfin sole in the Pacific Northwest.

\(^5\)Even in fishing grounds that are particularly abundant, overexploitation is commonly observed. For instance, the United Nations Food and Agriculture Organization (2005) identifies such overexploitation for the blue whiting in the Northeast Atlantic, the Chilean jack mackerel in the Southeast Pacific and the Peruvian anchovy in the Southeast
we identify a critical level of the stock below which overexploitation is relatively small and, hence, social welfare improves. If, in contrast, the stock is above such a critical level (i.e., very abundant), overexploitation is significant, thus becoming welfare reducing. Hence, relative to settings in which a single incumbent operates, the introduction of several firms that practice entry deterrence entails an unambiguous welfare loss. Our results are further emphasized when appropriation produces biodiversity losses, i.e., environmental damage, since in these settings overexploitation is more likely to become welfare reducing.

Different policy implications can be drawn from our findings, depending on the number of incumbents exploiting the commons and the abundance of the resource. Specifically, if few firms operate in a relatively scarce CPR, regulators should promote contexts in which incumbents face entry threats (e.g., by lowering entry costs) since firms’ entry-deterring practices can yield welfare gains. By contrast, when several firms exploit a very abundant commons, incumbents’ entry-deterring behavior entails a significant overexploitation, thus generating a welfare loss. In this setting, regulators should hinder entry threats by increasing fixed entry costs, e.g., more costly fishing permits, higher administrative costs, etc.

Summarizing, our equilibrium results suggest a policy implication that may go against common belief. In particular, policies that protect the CPR from further entry are unnecessary when the resource is scarce and few incumbents exploit the commons, since in this context their independent entry-deterring practices are welfare improving. In contrast, our results imply that the regulator should be especially protective with the resource when the CPR is abundant since, in this setting, the incumbents’ entry-deterring behavior yields a significant overexploitation of the commons, inducing welfare losses under large parameter conditions.

Related literature. Since the seminal work of Hardin (1968), several studies have analyzed the overexploitation of the commons in contexts in which a given set of firms are unthreatened by potential entry; for a detailed review of the literature, see Ostrom (1990), Ostrom et al. (1994) and Faysee (2005). Under the threat of entry, such overexploitation can be emphasized given the incumbent’s entry-deterring behavior, as shown by Mason and Polasky (1994) for the case of a single incumbent exploiting the CPR. Our paper contributes to this line of literature by examining equilibrium appropriation levels in commons where multiple incumbents operate. If few incumbents compete, we demonstrate that the overexploitation result identified by Mason and Polasky (1994) can be sustained if the resource is moderately abundant. In contrast, when several firms operate in the CPR, such overexploitation can only be supported if the resource becomes very abundant. In addition, our policy implications suggest that, while an abundant resource exploited by a single incumbent may not require protection from further entry, the presence of several firms threatened by entry recommends protection, since otherwise their aggregate appropriation would yield a significant overexploitation of the CPR.

Other studies, such as Cornes et al. (1986), Mason et al. (1988) and Mason and Polasky (1997), analyze overexploitation, and examine the optimal number of firms under different settings. Our model, in contrast, allows incumbents to strategically increase appropriation in order to deter entry.
Our model also connects to entry-deterrence games with multiple incumbents, as in Gilbert and Vives (1986). Unlike this paper, however, we consider a CPR which partially regenerates over time and, hence, second-period costs are negatively affected by first-period output decisions.\(^7\) Finally, a recent line of studies investigates entry-deterring practices under incomplete information settings (Polasky and Bin, 2001; Espinola-Arredondo and Munoz-Garcia, 2011), whereby potential entrants are uninformed about the level of the stock while the single incumbent is informed.\(^8\) The presence of several firms helps nonetheless to disseminate information about the stock and, hence, CPRs with multiple incumbents can be more appropriately analyzed under contexts of complete information, as in our paper.

The following section describes the model. Sections three and four examine equilibrium appropriation in the second and first-period, respectively. Section five analyzes under which conditions the commons are overexploited, while section six identifies when such an overexploitation entails welfare gains. Finally, section seven concludes.

### 2 Model

Consider a common pool resource (CPR) initially exploited by a set of \(N\) incumbents. A potential entrant decides whether or not to exploit the CPR given an entry cost, \(F > 0\), which represents the investment required to start exploiting the commons. The initial stock of the CPR is represented by \(\theta \in [0, 1]\), which is perfectly observed by all players in the game. In particular, we study a two-stage complete-information game with the following time structure:

1. In the first stage, every incumbent independently chooses its appropriation level \(x_i > 0\).

2. Given the first-period aggregate appropriation \(X \equiv \sum_{i=1}^{N} x_i\), the potential entrant chooses whether or not to join the CPR.

   (a) If entry does not occur, the \(N\) incumbents independently and simultaneously choose their second-period appropriation levels \(q_1, \ldots, q_N\), while the entrant’s profits from staying out are normalized to zero.

   (b) If entry ensues, agents compete for the CPR, selecting their appropriations \(q_1, \ldots, q_N, q_{N+1}\), for the \(N\) incumbents and the entrant.

\(^7\) Importantly, Gilbert and Vives (1986) consider that incumbent firms commit to a given production level during the first period of the game, which is sold in the second period once the potential entrant has decided whether or not to enter. The potential entrant hence might be deterred if the incumbents’ aggregate production is relatively large, leaving an insufficient market share to the entrant. Unlike incumbent firms in our paper who produce in both time periods at different costs, Gilbert and Vives (1986) assume that incumbents make a single production decision during the first period.

\(^8\) Other authors have theoretically and experimentally analyzed uncertainty regarding the profitability of the CPR; see Suleiman and Rapoport (1988), Suleiman et al. (1996) and Apesteguia (2006). Unlike Polasky and Bin (2001) and Espinola-Arredondo and Munoz-Garcia (2011), this literature does not allow for informational asymmetries among players.
In the first stage of the game, incumbent \( i \in N \) appropriates \( x_i \), with an associated total cost of \( c(x_i, \theta) = (1 - \theta)x_i \), which is increasing in first-period appropriation, \( x_i \), and decreasing in the available stock, \( \theta \). Intuitively, a more abundant stock facilitates the exploitation of the resource. The inverse demand curve is \( p(X) = 1 - X \), where \( X \) satisfies \( 0 < X \leq \theta \leq 1 \).

In the second-period, appropriation costs are affected by first-period appropriation as well as the regeneration rate of the CPR. In particular, firm \( i \)’s second-period costs are given by \( c(q_i, X, \theta, \beta) = [1 - (\theta - (1 - \beta)X)]q_i \), which increase in firm \( i \)’s second-period appropriation level, \( q_i \), and in aggregate first-period appropriation, \( X \). In contrast, costs decrease in the initial stock, \( \theta \), and in the regeneration rate of the CPR, \( \beta \in (0, 1) \). Specifically, a larger appropriation during previous periods, \( X \), depletes the resource, thus increasing second-period costs. Furthermore, such an increase becomes more severe when the resource does not regenerate across periods, i.e., \( \beta \rightarrow 0 \). However, if the CPR completely regenerates \( \beta \rightarrow 1 \) (i.e., biological regeneration offsets first-period appropriation), then \( X \) does not affect second-period costs and they converge to \( (1 - \theta)q_i \). Finally, consider that, similar to the first-period game, firms face a linear inverse demand curve. We next solve the game operating by backward induction.

3 Second period

No entry. During the second period, if entry does not occur, the same set of \( N \) incumbents exploit the resource, each of them appropriating \( q_i > 0 \). Therefore, each firm solves

\[
\max_{q_i > 0} \pi_i(q_i, Q_{-i}) \equiv (1 - q_i - Q_{-i})q_i - [1 - (\theta - (1 - \beta)X)]q_i, \tag{1}
\]

where \( Q_{-i} = \sum_{j \neq i} q_j \), by selecting best response function \( q_i(Q_{-i}) = \frac{\theta - (1 - \beta)X}{2} - \frac{Q_{-i}}{2} \), entailing an individual equilibrium appropriation of \( q_i^{NE}(X) = \frac{\theta - (1 - \beta)X}{N+1} \), where \( NE \) denotes no entry. The aggregate equilibrium appropriation is thus \( Q^{NE}(X) = Nq^{NE}_i(X) \), which is increasing in the level of the stock, \( \theta \), and the regeneration rate, \( \beta \), but decreasing in first-period appropriation, \( X \). Let \( \pi_i^{NE}(X) = \left(\frac{\theta - (1 - \beta)X}{N+1}\right)^2 \) denote incumbent \( i \)’s second-period equilibrium profits.

Entry. If entry occurs in the second period, \( N + 1 \) firms (\( N \) incumbents and the entrant) compete for the common resource. Every individual firm selects the level of \( q_i \) that solves (1) by choosing \( q_i^E(X) = \frac{\theta - (1 - \beta)X}{N+2} \), implying an aggregate appropriation of \( Q^E(X) = (N + 1)q_i^E(X) \), where superscript \( E \) denotes entry.\(^9\) Equilibrium profits are therefore \( \pi_i^E(X) = \left(\frac{\theta - (1 - \beta)X}{N+2}\right)^2 \), which lie below those when entry does not occur, \( \pi_i^{NE}(X) \), for all \( X \). Note that profits \( \pi_i^E(X) \) are decreasing in first-period appropriation, \( X \), thus making entry unprofitable if \( X \) is sufficiently high; as figure 1 depicts. Intuitively, the resource becomes so depleted that the potential entrant’s profits from exploiting the CPR do not offset its entry costs, \( F \). Specifically, the entrant’s second-period profits satisfy \( \pi_{ent}^E(X) \leq F \) for all \( X \geq X_{ED} \), where \( X_{ED} = \frac{\theta - \sqrt{F(N+2)}}{1 - \beta} \). For compactness, we refer

\(^9\)Note that the incumbent considers that \( Q_{-i} \) captures the second-period appropriation from the other \( N - 1 \) incumbents and the entrant.
to $X_{ED}$ as the “entry-deterring appropriation level.”

**Fig 1.** Entry-deterring appropriation, $X_{ED}$.

### 4 First period

In the first stage of the game, given an aggregate appropriation from all firms $j \neq i$, $X_{-i}$, firm $i$ must select its individual appropriation $x_i$ taking into account that its choice might deter entry. The following lemma describes incumbents’ behavior as a function of $X_{-i}$.

**Lemma 1.** When the commons are initially exploited by $N$ incumbents, every firm $i \in N$ behaves as follows:

1. allows entry when $X_{-i} \leq \overline{X}_{-i}(X_{ED})$;
2. deters entry when $\overline{X}_{-i}(X_{ED}) < X_{-i} \leq \overline{X}_{-i}(X_{ED})$; and
3. blockades entry when $\overline{X}_{-i}(X_{ED}) < X_{-i}$.

Figure 2 summarizes our results about firm $i$’s strategic appropriation decision, by comparing its profits from deterring entry, $\hat{\pi}^E_i(X_{-i})$, with those from allowing entry, $\hat{\pi}^{AE}_i(X_{-i})$.\(^\text{10}\) In particular, the figure identifies three regions for firm $i$’s production: Region I, where other firms’ appropriation, $X_{-i}$, is so low that firm $i$ would need to significantly increase its first-period exploitation in order to deter entry. Such increase in production, however, is too costly for firm $i$ since $\hat{\pi}^E_i(X_{-i}) < \hat{\pi}^{AE}_i(X_{-i})$, leading this firm to allow entry. In Region II, $X_{-i}$ is relatively higher, inducing firm $i$ to deter entry by reaching $X_{ED}$, i.e., appropriating $x_i = X_{ED} - X_{-i}$, since profits satisfy $\hat{\pi}^E_i(X_{-i}) \geq \hat{\pi}^{AE}_i(X_{-i})$ for all $\overline{X}_{-i}(X_{ED}) > X_{-i} \geq \overline{X}_{-i}(X_{ED})$. Finally, in Region III, all

\(^{10}\)For more details about these profits, see the proof of Lemma 1.
other firms’ appropriation satisfies $X_{-i} > X_{E_D}$, allowing firm $i$ to behave as if entry threats were absent, and entry is blockaded.

![Graph showing profits vs. appropriation](image)

Fig 2. Incumbent $i$'s first-period appropriation decision.

Given lemma 1, the following proposition describes the subgame perfect equilibrium (SPE) of the entry game. (For more details about the equilibrium appropriation levels, see the proof of Proposition 1).

**Proposition 1.** In the entry game where $N$ incumbents exploit the commons, there exists a unique SPE, depending on the precise value of the entry-deterring output, $X_{E_D}$:

1. If $X_{E_D} \leq X_{NE}$, every incumbent produces as if entry threats were absent, and their aggregate appropriation, $X_{NE}$, is sufficient to blockade entry;

2. If $X_{NE} < X_{E_D} \leq X_{ED}$, a continuum of entry-deterring equilibria can be sustained, in which incumbents appropriate more than when entry threats are absent, and aggregate first-period appropriation reaches the entry-deterring level $X_{E_D}$;

3. If $X_{ED} > X_{ED}$, each incumbent’s appropriation anticipates that entry will occur in the subsequent period, the entry-deterring output is hence not reached, and entry is ultimately allowed;

where $X_{ED}$ solves $X_{-i}(X_{E_D}) = \frac{(N-1)X_{E_D}}{N}$, i.e., $\overline{X}_{E_D}$ denotes the largest value of $X_{E_D}$ for which deterring entry is an equilibrium.

The above proposition, hence, identifies equilibrium behavior in three different regions, depending on the value of the appropriation level that all $N$ incumbents must reach in order to prevent entry, $X_{E_D}$. First, when $X_{E_D}$ is relatively low, i.e., $X_{ED} \leq X_{NE}$, firms can blockade entry by
just selecting their output levels as if entry threats were absent. Second, if \( X_{ED} \) lies between \( X^{NE} \) and \( \overline{X}_{ED} \), then entry is successfully deterred. Specifically, in this entry-deterring equilibrium every individual incumbent does not produce more than what is strictly necessary to reach the entry-deterring output \( X_{ED} \).\(^{11}\) Finally, when \( X_{ED} \) is sufficiently large, practicing entry deterrence becomes too costly, and incumbents allow entry. The following subsection provides numerical examples of the regions identified in proposition 1, and analyzes how they are affected by changes in the number of incumbents, \( N \), the regeneration rate, \( \beta \), and entry costs, \( F \).

4.1 Comparative statics

**Number of incumbents.** Let us first examine how entry is affected by the introduction of more incumbents exploiting the CPR. The next corollary shows that the equilibrium where entry is allowed shrinks as the number of incumbents increases.

**Corollary 1.** The region of parameter values sustaining blockaded entry (allowed entry) expands (shrinks, respectively) as the number of incumbents increases.

For comparison purposes, the next figure illustrates the case in which a single incumbent exploits the commons; as in Mason and Polasky (1994).\(^{12}\) When the level of the stock, \( \theta \), is relatively scarce, i.e., \( \theta \leq \theta_1 \) in figure 3, \( X_{ED} \) satisfies \( X_{ED} \leq X^{NE} \) and entry is blockaded. However, when the stock becomes more abundant, i.e., \( \theta_1 < \theta \leq \theta_3 \), the unique incumbent increases its exploitation level to reach \( X_{ED} \), and successfully deters entry. Finally, when the stock is relatively high, i.e., \( \theta > \theta_3 \), \( X_{ED} \) lies above \( \overline{X}_{ED} \). In this setting, the incumbent would need to significantly raise its first-period appropriation level in order to prevent entry, which is not profitable, and thus acquiesces entry.

\(^{11}\) For instance, in a symmetric entry-deterring equilibrium all \( N \) incumbents select the same first-period appropriation \( x_i = \frac{X_{ED}}{N} \), while in asymmetric equilibria appropriation levels might differ among incumbents, still reaching the entry-deterring level \( X_{ED} \). The specific distribution of first-period appropriations, however, is inconsequential for our subsequent investigation of whether aggregate exploitation exceeds the social optimum, and thus overexploitation arises, and about its associated welfare consequences.

\(^{12}\) To facilitate the comparisons and graphical representation of our results, we consider \( \beta = 0.8 \) and no discounting in future payoffs. In addition, this combination of parameters implies that the entrant’s profits become \( \pi_{ent}^E(X) = \frac{\theta - 0.25X}{49} \), thus entailing that \( \pi_{ent}^E(X) \) originates at \( \pi_{ent}^E(0) = \frac{\theta}{49} \) and monotonically decreases in \( X \) (see figure 1). Hence, entry deterrence is only possible if entry costs satisfy \( F < \pi_{ent}^E(0) \), e.g., \( F = 0.0204 \). Different parameter values yield similar results, and can be provided by the authors upon request.
When we allow for $N = 2$ incumbents (figure 4a), the region where entry is blockaded expands. As a consequence, for entry deterrence to be profitable the stock must be more abundant when $N = 2$ than when $N = 1$. In addition, the set of parameter values under which entry is allowed shrinks. Intuitively, when the stock is relatively scarce and two firms exploit the resource, entry becomes unprofitable and incumbents do not need to overexploit the commons in order to deter entry (blockaded entry expands in figure 4a relative to figure 3). This tendency is further emphasized when more incumbents exploit the commons. As figure 4b depicts, when four firms exploit the CPR, $X_{ED}$ lies below $X^{NE}$ (and entry is thus blockaded) for all levels of the stock. Hence, there exists a minimal number of firms $N_{ED}$ for which the entry-deterring equilibrium cannot be supported, and hence blockaded entry becomes the unique equilibrium prediction for all $N > N_{ED}$, i.e., $N_{ED}$ solves $\theta_1 = 1$. 

Fig 3. Equilibrium with $N = 1$ incumbent.

Fig 4a. $N = 2$ incumbents.

Fig 4b. $N = 4$ incumbents.
Our results hence provide a direct comparison with those in Mason and Polasky (1994). In particular, the introduction of additional incumbents implies that the overexploitation of the resource needed to deter entry can only be sustained for more stringent parameter conditions. Furthermore, when the number of incumbents is sufficiently large, \( N > N_{ED} \), the entry-deterring outcome reported in Mason and Polasky (1994) cannot be sustained under any parameter values. Hence, concerns about the strategic overexploitation that a single incumbent practices in order to deter entry can be readily ignored when several firms exploit the CPR, since entry becomes unprofitable.

**Regeneration rate.** When the stock fully regenerates across periods, \( \beta \to 1 \), the incumbents’ costs remain constant over time, and hence their production decisions resemble those of firms producing goods whose current costs are unaffected by their production history; similarly as in the entry deterrence game examined by Gilbert and Vives (1986). In contrast, if the regeneration rate decreases, the CPR becomes less attractive, and equilibria in which entry is blockaded or deterred emerge under larger parameter conditions. The next figure depicts the case in which \( \beta \) decreases from \( \beta = 0.8 \) (as in figure 4a) to \( \beta = 0.5 \), thus expanding the region where entry is blockaded.

\[\text{Fig 5. } N = 2 \text{ incumbents when } \beta \text{ decreases.}\]

Let us finally generalize our previous findings by analyzing under which conditions blockaded entry can be supported as the unique equilibrium of the game. In particular, define \( \theta_1 \) as the lower bound of the entry-deterring equilibrium, i.e., the level of the stock that solves \( X_{ED} = X^{NE} \).

**Corollary 2.** **Cutoff** \( \theta_1 \) **increases in the number of incumbents,** \( N \). **In addition, it experiences an upward (downward) shift in entry costs,** \( F \) **(in the regeneration rate,** \( \beta \)).

Therefore, since entry is blockaded for the range of \( \theta \) in which \( \theta \leq \theta_1 \), an increase in the number of incumbents enlarges such range; as the shaded area in figure 6a illustrates.\(^{13}\) In addition, recall

\(^{13}\)The proof of Corollary 2 shows that cutoff \( \theta_1 \) is increasing, but non-linear, in \( N \). However, for the parameter combinations considered throughout the paper, where \( N < 10 \), this cutoff becomes almost linear, as depicted in figures 6a and 6b. A similar argument applies to cutoff \( \theta_3 \).
that $N_{ED}$ solves $\theta_1 = 1$. Hence, for all $N \geq N_{ED}$ cutoff $\theta_1 \geq 1$, and the only equilibrium outcome that can be sustained is that where entry is blockaded. Moreover, this minimal number of firms $N_{ED}$ decreases in $F$, since entry becomes less attractive for potential entrants, thus expanding the shaded region where entry is blockaded in equilibrium (see figure 6b). In contrast, an increase in $\beta$ shifts cutoff $N_{ED}$ rightward, thus shrinking the shaded region of blockaded entry. Intuitively, as the resource is more replenished in the second-period game, entry becomes more attractive, and hence blockaded entry can only be sustained under more restrictive conditions.

![Fig 6a. Region of blockaded entry.](image1)

![Fig 6b. Comparative statics of $\theta_1$.](image2)

5 Overexploitation of the commons

In order to develop welfare comparisons, let us first examine appropriation levels with and without entry threats. Hence, in this section we confine our analysis to the entry-deterring equilibrium, supported in the region $\theta \in (\theta_1, \theta_3]$. The welfare effects of exploiting the CPR in this equilibrium are far from obvious, since society must weigh the increase in first-period appropriation that arises under entry deterrence against the reduction in second-period exploitation once entry is prevented.

Lemma 2. Aggregate first-period (second-period) appropriation is larger (smaller, respectively) when entry threats are present than when they are absent. In addition, aggregate first-period appropriation under entry threats exceeds the social optimum (i.e., overexploitation arises) if and only if $\theta > \tilde{\theta}$, where $\tilde{\theta} \equiv \frac{(2+N)\sqrt{F}(1-(\beta-1)^2\delta)}{\beta}$.

Figure 7 depicts first- and second-period aggregate appropriation. Firms produce $X_{UI}$ when they are unthreatened by entry, where subscript $UI$ denotes unthreatened incumbents ($X_{UI}$ thus coincides with their appropriation under blockaded entry), while firms increase aggregate first-period appropriation to $X_{ED}$ when they face entry threats. Such output, however, lies above the
socially optimal output, $X_{SO}$, for stock levels $\theta > \bar{\theta}$. In the second-period game, the resource is more depleted under entry threats and, therefore, aggregate appropriation lies below that when incumbents are unthreatened, i.e., $Q_{ED} < Q_{UI}$. Importantly, both $Q_{ED}$ and $Q_{UI}$ are lower than the socially optimal appropriation level, and hence, we hereafter focus on first-period exploitation.\footnote{For completeness, the proof of Lemma 2 identifies first- and second-period equilibrium appropriation levels, both with and without entry threats, as well as the socially optimal appropriation levels. In addition, note that a one unit increase in first-period appropriation, $X$, produces a less-than-proportional reduction in second-period appropriation. Specifically, as described in section 3, an increase in $X$ produces a reduction of $\frac{X}{N_{ED} + 1}(1 - \beta)$ in $Q^{NE}(X)$, which is smaller than one for all parameter values.\footnote{Note that $\bar{\theta}$ originates above $\theta_1$ for all parameter values, and it also originates above $\theta_3$ (as depicted in figure 8) when the regeneration rate is sufficiently strong. In particular, under no discounting, $\bar{\theta}$ starts above $\theta_3$ if and only if $\beta > 2/3$. Otherwise, $\bar{\theta}$ originates between $\theta_1$ and $\theta_3$, and the region where underexploitation is sustained for all $\theta$’s does not exist. Finally, cutoffs $\bar{\theta}$ and $\theta_3$ are both increasing in $N$ for the relevant range of $N$, i.e., $N \in [1, N_{ED}]$ for all admissible parameter values.}}

The following corollary examines how the region of overexploitation in the entry-deterring equilibrium, i.e., $\theta_3 \geq \theta > \bar{\theta}$, is affected by the number of incumbents. Figure 8 illustrates our findings by breaking the area of stock levels where the entry-deterring equilibrium is sustained, $\theta_1 < \theta \leq \theta_3$, into two subareas: one in which entry deterrence yields an overexploitation of the resource (the shaded region where $\theta > \bar{\theta}$, as described in Lemma 2), and another in which entry deterrence entails an underexploitation of the CPR, i.e., $\theta \leq \bar{\theta}$.

\textbf{Corollary 3.} Let $N^*$ be the number of firms that solves $\bar{\theta} = \theta_3$. Then, for all $N < N^*$, underexploitation is supported under all $\theta$. Similarly, let $N^{**}$ be the number of firms that solves $\bar{\theta} = \theta_1$. Then, for all $N > N^{**}$, overexploitation is sustained under all $\theta$.

As depicted in figure 8, corollary 3 divides the number of incumbents into three regions, where: (1) underexploitation holds for all parameter values,\footnote{Note that $\bar{\theta}$ originates above $\theta_1$ for all parameter values, and it also originates above $\theta_3$ (as depicted in figure 8) when the regeneration rate is sufficiently strong. In particular, under no discounting, $\bar{\theta}$ starts above $\theta_3$ if and only if $\beta > 2/3$. Otherwise, $\bar{\theta}$ originates between $\theta_1$ and $\theta_3$, and the region where underexploitation is sustained for all $\theta$’s does not exist. Finally, cutoffs $\bar{\theta}$ and $\theta_3$ are both increasing in $N$ for the relevant range of $N$, i.e., $N \in [1, N_{ED}]$ for all admissible parameter values.} $N < N^*$; (2) under- and overexploitation
can be sustained, depending on the stock’s abundance, $N^* \leq N \leq N^{**}$; and (3) overexploitation is supported for all parameter conditions, $N > N^{**}$.

![Diagram of Fig 8. Under- and overexploitation in the ED equilibrium](image)

**Example.** For parameter values $\beta = 0.8$, $F = 0.005$ and no discounting, these cutoffs become $N^* = 1$, $N^{**} = 6$, and $N_{ED} = 10$.\(^{16}\) That is, if a single incumbent operates in the commons, entry-deterring practices only lead to an underexploitation of the CPR for all $\theta$’s. Otherwise, from $N = 1$ to $N = 6$ we observe that entry deterrence can lead to overexploitation, but only if the stock is sufficiently abundant, i.e., $\theta > \bar{\theta}$, where $\bar{\theta} = \frac{6+3N}{25\sqrt{2}}$.\(^{17}\) Finally, if $6 < N \leq 10$, the commons is overexploited for all $\theta$’s.\(^{18}\)

### 6 Welfare comparisons

In this section we examine whether the increase in appropriation due to entry deterrence yields welfare gains or losses. In particular, let us define social welfare in a given period $t$ as the sum of consumer and producer surplus,\(^{19}\) and overall welfare as its discounted sum across both periods,

\(^{16}\)Note that, for each of these critical number of firms, $N^*$, $N^{**}$ and $N_{ED}$, we report the next largest integer by using the ceiling function $\lceil N \rceil$.

\(^{17}\)Intuitively, the resource becomes attractive for potential entrants and incumbents can only deter entry by substantially increasing their first-period appropriation, ultimately yielding an overexploitation of the CPR relative to the social optimum.

\(^{18}\)If the regeneration rate of the CPR, $\beta$, decreases, cutoffs $N^*$, $N^{**}$ and $N_{ED}$ experience a leftward shift, approaching them to the origin. For instance, in our previous parametric example, if $\beta$ decreases from 0.8 to 0.3, $N^{**}$ is reduced from 6 to 2, while $N_{ED}$ decreases from 10 to 5. (Regarding cutoff $N^*$, it decreases from 0.9 to 0.7, implying that the largest integer for which overexploitation can be sustained under all values of $\theta$ is still $N^* = 1$). Intuitively, the commons become less attractive for the potential entrant, implying that incumbents’ entry-deterring practices only arise in equilibrium when $N < 5$.

\(^{19}\)This social welfare function has been considered in similar studies analyzing exploitation of the commons, such as Mason and Polasky (1994), and hence allows for more direct comparisons. Nonetheless, section 6.2 below extends
i.e., \( SW = SW_1 + \delta SW_2 \).

**Proposition 2.** Social welfare under entry threats exceeds that under no threats when the stock level is sufficiently scarce, i.e., \( \theta < \bar{\theta} \), where

\[
\bar{\theta} = \frac{\sqrt{FT} \left[ V^5 - D^2 NV^2 (4 + N (3 + N)) \delta + 2D^4 N^2 T \delta^2 \right]}{V^4 (BT - 1) + D^2 NV^2 (T + \beta (NT - 4)) \delta}
\]

where \( T = (2 + N), D = (\beta - 1) \) and \( V = (1 + N) \).

Figure 9 superimposes the social welfare cutoff \( \bar{\theta} \) on the regions of over- and under-exploitation identified in figure 8. As described in lemma 2, when incumbents are unthreatened, aggregate appropriation lies below the social optimum. Under entry threats, entry-deterring practices still yield an underexploitation of the resource if \( \theta \leq \bar{\theta} \) (Region A in figure 9). However, since output approaches the social optimum, welfare unambiguously increases.²⁰

However, if entry deterrence generates overexploitation, \( \theta > \bar{\theta} \), such welfare result becomes dependent on the stock abundance: specifically, if \( \theta < \bar{\theta} \) overexploitation is small, and thus social welfare with entry threats is larger than without threats (region B), while if \( \theta \geq \bar{\theta} \) overexploitation becomes larger, and welfare is lower when incumbents are threatened by potential entrants than otherwise (region C).

²⁰Note that \( \bar{\theta} \) originates above \( \bar{\theta} \) (as depicted in figure 9) when the regeneration rate of the CPR is relatively strong. Specifically, under no discounting, \( \bar{\theta} \) originates above \( \bar{\theta} \) under relatively large parameter conditions, i.e., for all \( \beta \geq 0.2 \).
6.1 Discussion

Our results suggest that, when a relatively small number of incumbents exploit the CPR, their entry-deterring practices can entail welfare improvements, both when they under- and overexploit the commons relative to the social optimum. However, when the number of firms increases, the threat of entry yields an overexploitation of the resource that entails a welfare loss for stock levels above \( \overline{\theta} \). In addition, if the number of firms is further augmented, entry deterrence yields an unambiguous welfare loss under all parameter values.\(^{21}\)

Our equilibrium predictions, hence, lead to different policy implications depending on the number of incumbents exploiting the commons and the abundance of the resource. Specifically, if few incumbents operate in a relatively scarce CPR (regions \( A \) and \( B \)), regulators should promote entry threats (e.g., by lowering entry costs) since firms’ entry-deterring practices can yield welfare gains under relatively large parameter conditions. By contrast, when several firms exploit a very abundant commons (region \( C \)), incumbents’ entry-deterring behavior yields a significant overexploitation, thus generating a welfare loss. In this setting, regulators should hinder entry threats by increasing the fixed entry cost, e.g., more costly fishing permits, higher administrative costs, etc.

6.2 Introducing environmental damage

Let us now consider the case in which firms’ exploitation produces a loss in the biodiversity of the commons. Specifically, we include environmental damage \( EnvD \equiv d \times (X^2 + \delta Q^2) \) into the social welfare function, implying that welfare becomes \( SW(d) = SW_1 + \delta SW_2 - EnvD \), where \( d \in [0, 1] \). In this context, firms’ equilibrium behavior coincides with that in previous sections (and so do the bounds of the entry-deterring equilibrium, \( \theta_1 \) and \( \theta_3 \)), but such output entails a different equilibrium welfare level. The following proposition examines how our equilibrium results are affected by the introduction of environmental damages.

**Proposition 3.** Cutoffs \( \theta_1 \) and \( \theta_3 \) are constant in the loss of environmental quality, \( d \). However, cutoffs \( \overline{\theta} \) and \( \overline{\theta} \) are both decreasing in \( d \).

Since \( \overline{\theta} \) decreases in \( d \), the region of stock levels for which entry-deterring practices yield an overexploitation of the resource expands, i.e., the shaded area in figure 8 enlarges. Intuitively, society assigns a larger value to the biodiversity of the resource, and thus the incumbents’ appropriation exceeds the socially optimal level under larger parameter conditions than when environmental damage is absent. In addition, cutoff \( \overline{\theta} \) also decreases in \( d \), implying that the region of overexploitation that entails a welfare loss expands (region \( C \) in figure 9). Hence, incumbents’ entry-deterring appropriation becomes more harmful under larger parameter values.

\(^{21}\)For instance, for the parameter values considered in our previous example, \( \overline{\theta} \) crosses \( \theta_1 \) at \( N = 5.26 \), and hence welfare gains arise for all \( N \leq 6 \). Otherwise, overexploitation entails welfare losses under all stock levels, \( \theta \).
7 Conclusions

Our paper analyzes an entry-deterrence model with multiple incumbents who strategically increase appropriation in order to prevent entry. We find that, when the number of firms is relatively small, entry deterrence can be sustained if the resource is moderately abundant. In addition, when few firms operate in a CPR, we demonstrate that entry-deterring appropriation levels are higher than those in contexts where firms are unthreatened by entry (but still lie below the social optimum, i.e., underexploitation), thus yielding a welfare improvement. However, when several firms compete for the resource, entry deterrence entails an overexploitation of the commons, which generates welfare losses if, in addition, the CPR is abundant.

Our model can be extended in several directions. First, we introduce environmental damages by assuming that, for simplicity, appropriation imposes a flow externality —whose negative effects fully dissipate at the end of the first period— rather than a stock externality, whose impacts remain across time. An extension of the paper could consider that a proportion of the damage caused by first-period appropriation persists in future periods. In such a setting, firms’ entry-deterring practices would entail an overexploitation of the resource under larger parameter conditions, thus yielding a welfare reduction in more circumstances. Second, we consider that potential entrants are perfectly informed about the stock’s abundance. However, as pointed out by Polasky and Bin (2001), it might be especially difficult for potential entrants to accurately assess the profitability of a CPR that has been exploited by a small number of firms during long periods of time. Under such information structure, potential entrants would use incumbents’ aggregate appropriation levels (or market prices) as a signal to infer the stock level.
8 Appendix

8.1 Proof of Lemma 1

Depending on the precise level of all other firms' first-period appropriation, $X_{-i}$, firm $i$ chooses a different exploitation level. Let us separately analyze each case.

**Blockaded entry.** When other incumbents' appropriation satisfies $X_{-i} > X_{ED}$ entry is blockaded, and every firm $i$ selects $x_i$ in order to maximize its overall discounted profits

$$\max_{x_i > 0} \pi^N_i(x_i; X_{-i}) = (1 - x_i - X_{-i})x_i - (1 - \theta)x_i + \delta \pi^N_i(X_{-i}), \quad (A.1)$$

where $X \equiv x_i + X_{-i}$, $\pi^N_i(X)$ represents incumbent $i$'s second-period profits, and $\delta \in [0, 1]$ denotes its discount factor. The above maximization problem hence yields the best response function

$$x^*_i(X_{-i}) = \frac{[(1 + N)^2 - 2(1 - \beta)\delta] \theta}{2(1 + N)^2 - 2(\beta - 1)^2\delta} \cdot X_{-i} \quad (A.2)$$

entailing overall profits of $\pi^{BE}_i(X_{-i}) = \pi^N_i(x^*_i(X_{-i}); X_{-i})$, where $BE$ reflects blockaded entry. Similarly, when $X_{-i} \leq X_{ED}$ and firm $i$ produces according to $x_i^NE(X_{-i})$ such that $X_{-i} + x_i^NE(X_{-i}) \geq X_{ED}$, then entry is blockaded and firm $i$ produces ignoring the threat of entry. In particular, let $\overline{X}_{-i}(X_{ED})$ denote the appropriation level from all firms $j \neq i$, $X_{-i}$, that solves $X_{-i} + x_i^NE(X_{-i}) = X_{ED}$. Using expression (A.2) of the best response function $x_i^NE(X_{-i})$, we have

$$X_{-i} + \left[ \frac{[(1 + N)^2 - 2(1 - \beta)\delta] \theta}{2(1 + N)^2 - 2(\beta - 1)^2\delta} \cdot X_{-i} \right] = X_{ED}$$

and solving for $X_{-i}$ yields $\overline{X}_{-i}(X_{ED}) = \frac{2[(1 + N)^2 - (\beta - 1)^2\delta]}{(1 + N)^2} X_{ED} - \frac{\theta[(1 + N)^2 + 2(\beta - 1)^2\delta]}{(1 + N)^2}$ for a general value of $X_{ED}$. Plugging the specific value of $X_{ED}$, $X_{ED} \equiv \frac{\theta - \sqrt{F(N + 2)}}{1 - \beta}$, we obtain

$$\overline{X}_{-i}(X_{ED}) = \frac{2\sqrt{F}[(1 + N)^2 - (\beta - 1)^2\delta] - \theta(1 + \beta)(1 + N)^2}{(\beta - 1)(1 + N)^2}.$$

**Deterred entry.** When $X_{-i} \leq X_{ED}$ and firm $i$ deters entry by reaching the entry-detering appropriation level, i.e., firm $i$'s appropriation, $x_i$, satisfies $x_i = X_{ED} - X_{-i}$, implying that its profits from deterring entry

$$\pi^N_i(x_i; X_{-i}) = (1 - x_i - X_{-i})x_i - (1 - \theta)x_i + \delta \left( \frac{\theta - (1 - \beta)(x_i - X_{-i})}{N + 1} \right)^2$$

become

$$\tilde{\pi}^{ED}_i(X_{-i}) = (1 - X_{ED})(X_{ED} - X_{-i}) - (1 - \theta)(X_{ED} - X_{-i}) + \delta \left( \frac{\theta - (1 - \beta)X_{ED}}{N + 1} \right)^2.$$
**Allowed entry.** Finally, when other incumbents’ appropriation is still relatively low, \( X_{-i} \leq X_{ED} \), and firm \( i \)'s exploitation of the CPR allows entry, i.e., \( x_i < X_{ED} - X_{-i} \), its maximization problem becomes

\[
\max_{x_i > 0} \pi_i^E(x_i; X_{-i}) = (1 - x_i - X_{-i})x_i - (1 - \theta)x_i + \delta \pi_i^E(X), \quad \text{(A.3)}
\]

which yields the best response function

\[
x_i^E(X_{-i}) = \left[ \frac{(2 + N)^2 - 2(1 - \beta)\theta}{2(2 + N)^2 - 2(\beta - 1)^2\delta} - \frac{(2 + N)^2 - 2(\beta - 1)^2\delta}{2(2 + N)^2 - 2(\beta - 1)^2\delta} X_{-i} \right]
\]

\text{(A.4)}

It is straightforward to show that the slope of best response function \( x_i^E(X_{-i}) \) is larger in absolute value than that of \( x_i^{NE}(X_{-i}) \), indicating that a given increase in \( X_{-i} \) leads firm \( i \) to more significantly reduce its first-period appropriation when entry ensues than when it does not, implying that its profits become

\[
\tilde{\pi}_i^{AE}(X_{-i}) = (1 - x_i^E(X_{-i}) - X_{-i})x_i^E(X_{-i}) - (1 - \theta)x_i^E(X_{-i}) + \delta \left( \frac{\theta - (1 - \beta)}{N + 2} \right) X_{-i}^E(X_{-i})
\]

and using expression (A.4), the above profits from allowing entry can be more compactly expressed as

\[
\tilde{\pi}_i^{AE}(X_{-i}) = \left[ \frac{(2 + N)^2 + 4\beta\delta}{4(2 + N)^2 - 4(\beta - 1)^2\delta} \right] x_i^E(X_{-i}) - (1 - \theta)x_i^E(X_{-i}) + \delta \left( \frac{\theta - (1 - \beta)}{N + 2} \right) X_{-i}^E(X_{-i}) + \left( \frac{2 + N}{2 + 3N + N^2} \right) X_{-i}^2
\]

where \( AE \) denotes that entry is allowed. In addition, let \( X_{-i}(X_{ED}) \) solve \( \tilde{\pi}_i^{ED}(X_{-i}) = \tilde{\pi}_i^{AE}(X_{-i}) \), i.e., the level of \( X_{-i} \) that makes firm \( i \) indifferent between deterring and allowing entry. In particular,

\[
X_{-i}(X_{ED}) = \frac{2A(X_{ED} + \sqrt{B}((\beta - 1)X_{ED} + \theta))}{(2 + 3N + N^2)^2}
\]

where \( A \equiv 2(2 + 3N + N^2)^2 - (\beta - 1)^2(N - 1)^2\delta \) and \( B \equiv (1 + N)^2(3 + 2N)d[(2 + N)^2 - (\beta - 1)^2\delta] \).

\[ \blacksquare \]

### 8.2 Proof of Proposition 1

Let \( X_{ED} \) indicate the entry-deterring appropriation level that solves

\[
X_{-i}(X_{ED}) = (N - 1)x_i^E.
\]

(A.5)

That is, when all other \((N - 1)\) incumbents produce \( x_i^E \), firm \( i \) is indifferent between allowing and preventing entry when the entry-deterring appropriation level, \( X_{ED} \), that firm \( i \) must reach is exactly that solving A.5. In addition, let \( \overline{X}_{ED} \) denote the largest value of \( X_{ED} \) for which deterring
entry is an equilibrium. That is, $X_{ED}$ solves

$$X_{-i}(X_{ED}) = \frac{(N - 1)X_{ED}}{N},$$

where entry is deterred by having all $j \neq i$ incumbents equally share the burden of reaching the entry-deterring appropriation $X_{ED}$.

In addition, cutoffs $X_{ED}$ and $X_{ED}$ satisfy $X_{ED} > X_{ED}$ since function $X_{-i}(X_{ED})$ is linear and increasing in $X_{ED}$, and $\frac{(N-1)X_{ED}}{N} > (N - 1)x_i^E$, as figure A1 depicts. [In the special case of $N = 1$, both expressions A.5 and A.6 become $X_{-i}(X_{ED}) = 0$, implying that cutoffs $X_{ED}$ and $X_{ED}$ coincide.

Therefore, three equilibria can be identified depending on the value of $X_{ED}$ (for presentation purposes, figure A.2 provides a numerical example): (1) one where entry is blocked for all $X_{ED} \leq X^{NE}$ (where $X^{NE}$ denotes the aggregate first-period output when entry threats are absent, as found in lemma 1); (2) one where entry is prevented when $X_{ED}$ satisfies $X^{NE} < X_{ED} \leq X_{ED}$; and (3) one where entry is allowed for all $X_{ED} > X_{ED}$. As a consequence, in the region where $X_{ED} < X_{ED} \leq X_{ED}$ two types of equilibrium outcomes coexist (entry deterrence and allowing entry).
These three equilibria can be alternatively expressed in terms of $\theta$. First, note that $X_{ED}$ can be rewritten as a linear function of $\theta$ as follows

$$X_{ED} = \frac{-\sqrt{F(N+2)}}{1-\beta} + \frac{1}{1-\beta} \theta$$

and similarly $X^{NE}$ can be expressed as

$$X^{NE} = \frac{N \left[ (1+N)^2 + 2(\beta-1)\frac{\delta}{(1+N)(1+\beta N)} \right]}{(1+N)(1+\beta N)}$$

Let $\theta_1$ denote the value of $\theta$ at which $X_{ED}$ crosses $X^{NE}$. This crossing point is unique, since both $X_{ED}$ and $X^{NE}$ are linear in $\theta$, but $X_{ED}$ originates in the negative quadrant, $\frac{-\sqrt{F(N+2)}}{1-\beta}$, while $X^{NE}$ originates at zero (see figure A2). Hence, entry is blockaded for all $\theta \leq \theta_1$ where

$$\theta_1 = \frac{\sqrt{F(N+2)} \left[ (1+N)^2 - 2(\beta-1)\frac{\delta}{(1+N)(1+\beta N)} \right]}{(1+N)^2(1+\beta N)}$$

Similarly, let $\theta_2$ represent the value of $\theta$ at which $X_{ED}$ crosses $\bar{X}_{ED}$. Cutoff $\bar{X}_{ED}$ also originates at zero, and is linear in $\theta$. Hence, entry is deterred for all $\theta_1 < \theta \leq \theta_2$, where

$$\theta_2 = \frac{\sqrt{F(N+2)}}{(1-\beta) \left[ \frac{1}{1-\beta} - D + E \right]}$$
and 

\[ D \equiv \frac{(1 + N)(1 + \beta N)((2 + N)^2 - (\beta - 1)^2\delta)\sqrt{(3 + 2N)\delta(2 + N)^2 - (\beta - 1)^2\delta}}{((1 + N)(2 + N)^2 - (\beta - 1)^2N\delta)[((1 + N)^2 - (\beta - 1)^2\delta)((2 + N)^2 - (\beta - 1)^2\delta)]} \]

and \( E \equiv \frac{-N(1+N)^2(2+N)^2+\beta(1+N)(N-3(-5+\beta-(-3+\beta)N))\delta+2(\beta-1)^3N\delta^2}{(1+N)^3(2+N)^3-\beta(1+N)(4+3N(2+N))\delta+2(\beta-1)^4N\delta^2} \). Finally, let \( \theta_3 \) represent the value of \( \theta \) at which \( X_{ED} \) crosses \( \overline{X}_{ED} \). Cutoff \( X_{ED} \) also originates at zero, and is linear in \( \theta \). Hence, entry is allowed for all \( \theta > \theta_3 \), where

\[ \theta_3 \equiv \frac{\sqrt{F(N+2)}}{(1-\beta)\left[1 + \frac{G_1G_2G_3}{G_4}\right]} \]

where \( G_1 \equiv -2\sqrt{N^2(1+N)^2(3+2N)(1+\beta N)^2(2+N)^2 - (\beta - 1)^2\delta}, G_2 \equiv N(-(1+N)^3(2+N)^2 + 2(\beta - 1)(-1 + N(\beta + (1+N)(2+N))(5+2N))\delta, G_3 \equiv 4(\beta - 1)^3N\delta^2, \text{ and } G_4 \equiv (1+N)^4(2+N)^2 - 4(\beta - 1)^2N(1+N(2+N)(3+N))\delta + 4(\beta - 1)^4N^2\delta^2 \). In addition, \( \theta_1 > \theta_2 > \theta_3 \), since \( \overline{X}_{ED}, \underline{X}_{ED} \) and \( X^{NE} \) originate at zero, and \( \overline{X}_{ED} > \underline{X}_{ED} > X^{NE} \), while \( X_{ED} \) originates at \( \frac{-\sqrt{F(N+2)}}{1-\beta} < 0 \).

Finally, let us show that, within the coexistence region \( \theta \in [\theta_2, \theta_3] \), only the equilibrium in which entry is prevented is undominated in terms of firms’ profits.

**Lemma A.** In the coexistence region, \( \theta \in [\theta_2, \theta_3] \), every firm i’s profits in the equilibrium where entry is prevented are larger than in that where entry is allowed.

**Proof of Lemma A.** In the equilibrium where entry is allowed, all firms use the best response function \( x_i^E(X_{-i}) \), and by symmetry \( x_i^E = \left[\frac{(2+N)^2 -(\beta-1)^2\delta}{(1+N)(2+N)^2 - (\beta-1)^2N\delta}\right]^\theta \). Therefore, firm i’s equilibrium profits become

\[ \pi^{AE} \equiv \frac{T^4 + T^2(-1 + B(2 + (4 + \beta(-2 + N))N))\delta - 4D^2\beta N\delta^2\theta^2}{[VT^2 - 2D^2N\delta]N} \]

where \( T = (2 + N), D = (\beta - 1) \text{ and } V = (1 + N) \). A continuum of equilibria exist in which entry is deterred, whereby firms’ aggregate appropriation is \( X_{ED} \) and \( x_i \) satisfies \( X_{ED} - X_{-i}(X_{ED}) \geq x_i \geq x_i^{NE}(X_{-i}) \). For simplicity, we focus on the symmetric entry-deterring equilibrium where every firm selects \( x_i = \frac{X_{ED}}{N} \). In this setting, firm i’s equilibrium profits from deterring entry become

\[ \pi^{DE} \equiv \frac{FT^2(D^2N\delta - V^2) + V^2\theta \left[1 + \beta\right]\sqrt{FT - \beta\theta}}{D^2NV^2} \]

In addition, \( \pi^{AE} < \pi^{DE} \) for all \( \theta > \theta_2 \), where

\[ \tilde{\theta}_2 \equiv \frac{L_1 + \sqrt{D^2FV^2(VT^2 - 2D^2N\delta)^2(T^4(N^2 - 1)^2 - 4NT^4[\beta^2(1 + N(-3 - 4N + N^2)) - L_2] \delta + 4D^2N^2L_3\delta^2)}}{2V^2(T^3(\beta + N)(1 + \beta N) + D^2NT(-1 + \beta(-2 + \beta(-3 + 2 + N))N)\delta)} \]
and \( L_1 \equiv (1 + \beta) \sqrt{F((2 + 3N + N^2)^2 - 2D^2NV\delta)^2}, \) \( L_2 \equiv N(3 + 2N) + \beta[4 + NT(1 + N^2)] \) and \( L_3 \equiv -3 - 2N + \beta[1 + NT(N^2 - 3)]. \)

Furthermore, cutoff \( \hat{\theta}_2 \) satisfies \( \hat{\theta}_2 < \theta_2 \). Hence, for all \( \theta \) in the coexistence region \( \theta \in [\theta_2, \theta_3] \), profits satisfy \( \pi^{AE} < \pi^{DE} \). We can therefore identify a unique equilibrium prediction for each value of \( X_{ED} \): blockaded entry when \( X_{ED} \leq X^{NE} \) (i.e., \( \theta \leq \theta_1 \)), entry deterrence when \( X^{NE} < X_{ED} \leq X_{ED} \) (i.e., \( \theta_1 < \theta \leq \theta_3 \)), and allowing entry when \( X_{ED} > X_{ED} \) (i.e., \( \theta > \theta_3 \)).

### 8.3 Proof of Corollary 1

As shown in the proof of Proposition 1, cutoffs \( X_{ED}, X_{ED} \) and \( X^{NE} \) originate at zero, and \( X_{ED} > X_{ED} > X^{NE} \), while \( X_{ED} \) originates at \( \frac{-\sqrt{F(N+2)}}{1-\beta} < 0 \). Hence, \( \theta_1 > \theta_2 > \theta_3 \). In addition, since \( \theta_1 \) increases in \( N \), the region of parameter values supporting the equilibrium where entry is allowed (blockaded) shrinks (expands, respectively).

### 8.4 Proof of Corollary 2

From the proof of Proposition 1, cutoff \( \theta_1(N) \) is \( \theta_1(N) = \frac{\sqrt{F(T(V^2 - 2D^2N\delta)}}{V^2(1+\beta N)} \), where \( T = (2 + N), \) \( D = (\beta - 1) \) and \( V = (1 + N) \). Cutoff \( \theta_1 \) increases in \( N \) since

\[
\frac{\partial \theta_1}{\partial N} = \frac{\sqrt{F} \left[ V^3(3 + 2N + \beta(N^2 - 2)) + 2D^2(\beta N^2(3 + N) - 2)\delta \right]}{V^3(1 + \beta N)^2}
\]

which is strictly positive for all admissible values of \( \beta, \delta \in (0, 1) \) and \( N \geq 1 \). In addition, \( \theta_1 \) increases in \( F \) given that

\[
\frac{\partial \theta_1}{\partial F} = \frac{T \left[ V^3 - 2D^2N\delta \right]}{2\sqrt{FV^2(1 + \beta N)}}
\]

which is strictly positive for all admissible values of \( \beta, \delta \in (0, 1) \) and \( N \geq 1 \). Finally, \( \theta_1 \) decreases in \( \beta \) since

\[
\frac{\partial \theta_1}{\partial \beta} = -\frac{\sqrt{FNT} \left[ V^3 + 2D(2 + N + \beta N)\delta \right]}{V^2(1 + \beta N)^2}
\]

which is strictly negative for all admissible parameter values.

### 8.5 Proof of Lemma 2

**First-period output.** From the proof of Proposition 1, we know that aggregate first-period appropriation \( X_{ED} \) (in the entry-deterring equilibrium when entry threats are present) is strictly larger than \( X^{NE} \) (i.e., the aggregate first-period appropriation when entry threats are absent) for all \( \theta > \theta_1 \). Since the entry-deterring equilibrium can only be supported if \( \theta \) satisfies \( \theta_3 \geq \theta > \theta_1 \), we can then conclude that \( X_{ED} > X^{NE} \) holds.

**Second-period output.** In the entry-deterring equilibrium, entry does not ensue, and we can, hence, evaluate the second-period appropriation \( Q^{NE}(X) \) at the entry-deterring output \( X_{ED} \), yielding an equilibrium appropriation level of \( Q^{NE}(X_{ED}) = \frac{\sqrt{F(N+2)}}{1+N} \) when entry threats are present.
Similarly, when entry threats are absent, we evaluate \( Q^{NE}(X) \) at \( X^{NE} \) to obtain an equilibrium appropriation level of \( Q^{NE}(X^{NE}) = \frac{N(1+N)(1+\beta N)\theta}{(1+N)^2 - 2(1-\beta)^2 N\delta} \). Comparing \( Q^{NE}(X_{ED}) \) and \( Q^{NE}(X^{NE}) \) yields that \( Q^{NE}(X_{ED}) < Q^{NE}(X^{NE}) \) for all \( \theta > \theta_1 \). Since \( \theta_3 \geq \theta > \theta_1 \) holds in the entry-deterrent equilibrium, \( Q^{NE}(X_{ED}) < Q^{NE}(X^{NE}) \) must be satisfied.

**Socially-optimal output.** In order to compare first-period appropriation in the entry-deterrent equilibrium, \( X_{ED} \), with the socially optimal output, \( X_{SO} \), let us first find the value of \( X_{SO} \). Starting in the second period, it is straightforward to show that, for a given first-period appropriation, \( X \), \( Q_{SO}(X) = \theta + (\beta - 1)X \) maximizes second-period welfare

\[
\frac{Q^2}{2} + (1 - Q)Q - [1 - (\theta - (1 - \beta) X)]Q,
\]

entailing a second-period welfare of \( SW_2(X) = \frac{[\theta + (\beta - 1)X]^2}{2} \). Given this second-period welfare, the social planner selects the first-period appropriation level, \( X \), that maximizes overall social welfare across both periods, by solving

\[
\max_{X \geq 0} \frac{X^2}{2} + (1 - X)X - (1 - \theta) X + \delta \frac{[\theta + (\beta - 1)X]^2}{2},
\]

which is maximized at \( X_{SO} = \frac{(1-\delta+\delta\theta)\theta}{1-(\beta-1)^2\delta} \). Comparing \( X_{ED} = \frac{\theta - \sqrt{F(N+2)}}{1-\beta} \) and \( X_{SO} \), and solving for \( \theta \), we obtain that \( X_{ED} > X_{SO} \) for all \( \theta > \tilde{\theta} \), where \( \tilde{\theta} = \frac{2 + 2\sqrt{F}[1 - (\beta - 1)^2\delta]}{\beta} \). Let us finally compare the second-period appropriation under no entry threats, \( Q_{UI} \), with the social optimum. The latter can be obtained from evaluating the social optimum second-period appropriation \( Q_{SO}(X) \) at \( X_{SO} \), i.e., \( Q_{SO}(X_{SO}) = \frac{\theta\beta}{1-(\beta-1)^2\delta} \). In particular, \( Q_{UI} < Q_{SO}(X_{SO}) \) for all parameter values. Furthermore, since \( Q_{ED}(X_{ED}) < Q_{UI} = Q^{NE}(X^{NE}) \) for all stock levels in the entry-deterring equilibrium \( \theta_3 \geq \theta > \theta_1 \), then \( Q_{ED}(X_{ED}) < Q_{UI} < Q_{SO}(X_{SO}) \). ■

**8.6 Proof of Proposition 2**

In order to analyze the welfare effect of entry threats, we need to compare social welfare in the entry-deterrent equilibrium (arising when \( \theta \) satisfies \( \theta_1 < \theta \leq \theta_3 \)) with that under no entry threats using the same range of \( \theta \). Under entry deterrence, equilibrium social welfare is

\[
SW^{ED} = \frac{FT^2 (NT (D^2\delta - 1) - 1) + V^2 \theta \left(2\beta\sqrt{F}T + (1 - 2\beta) \theta\right)}{2D^2V^2}
\]

whereas under no entry threats, social welfare coincides with that emerging under blockaded entry, entailing

\[
SW^{BE} = \frac{N \left[V^4T + V^2 (2 - 3N + \beta (4 + N (2 (6 + N) + \beta (-4 + NT)))) \delta - 4D^4 (2\beta - 1) N\delta^2\right] \theta^2}{2(V^3 - 2D^2N\delta)^2}
\]
Hence, the difference $SW^{ED} - SW^{BE}$ for $\theta \in (\theta_1, \theta_3]$ is positive for all $\theta \leq \bar{\theta}$, where

$$
\bar{\theta} = \frac{\sqrt{FT} [V^5 - D^2 NV^2 (4 + N (3 + N)) \delta + 2 D^4 N^2 T \delta^2]}{V^4 (BT - 1) + D^2 NV^2 (T + \beta (NT - 4)) \delta}
$$

and $T = (2 + N)$, $D = (\beta - 1)$ and $V = (1 + N)$. ■

### 8.7 Proof of Proposition 3

**Output comparisons.** In order to compare first-period appropriation in the entry-deterring equilibrium, $X_{ED}$, with the socially optimal output considering environmental damage, $X_{SO}(d)$, let us first find the value of $X_{SO}(d)$. Starting in the second period, it is straightforward to show that, for a given first-period appropriation, $X$, $Q_{SO}(X) = \frac{\theta + (\beta - 1)X}{1 + 2d}$ maximizes second-period welfare

$$
\frac{Q^2}{2} + (1 - Q)Q - [1 - (\theta - (1 - \beta)X)]Q - dQ^2,
$$

entailing a second-period welfare of $SW_2(X) = \frac{[\theta + (\beta - 1)X]^2}{2 + 4d}$. Given this second-period welfare, the social planner selects the first-period appropriation level that maximizes overall social welfare across both periods

$$
\max_{X \geq 0} \frac{X^2}{2} + (1 - X)X - (1 - \theta)X - dX^2 + \delta \frac{[\theta + (\beta - 1)X]^2}{2 + 4d},
$$

which is maximized at $X_{SO}(d) = \frac{(1 + 2d + (\beta - 1)\delta)\theta}{(1 + 2d)^2 - (\beta - 1)^2 \delta}$. Hence, evaluating the second-period appropriation at $X_{SO}(d)$, we find that the socially optimal second-period appropriation is $Q_{SO}(X_{SO}) = \frac{\theta (\beta + 2d)}{1 + 4d(1 + d) - (\beta - 1)^2 \delta}$. Comparing $X_{ED} = \frac{\theta - \sqrt{F(N + 2)}}{1 - \beta}$ and $X_{SO}(d)$, and solving for $\theta$, we obtain that $X_{ED} > X_{SO}$ for all $\theta > \bar{\theta}(d)$, where $\bar{\theta}(d) = \frac{(2 + N)\sqrt{F(1 + 2d)^2 - (\beta - 1)^2 \delta}}{(1 + 2d)(\beta + 2d)}$. Finally, note that $\frac{\partial \bar{\theta}(d)}{\partial d} < 0$ for all admissible parameter values.

**Welfare comparisons.** Comparing social welfare in the entry-deterring equilibrium, $SW^{ED}(d)$, with that under no entry threats (which coincides with that under blockaded entry), $SW^{BE}(d)$, we obtain that $SW^{ED}(d) > SW^{BE}(d)$ for all $\theta < \bar{\theta}(d)$, where

$$
\bar{\theta}(d) = \frac{\sqrt{FT} (2D^2 N \delta - V^3) ((1 + 2d) V^2 + D^2 N(-T + 2dN)\delta)}{V^2(V(1 - 2d(1 + 2N) + \beta(-T + 2dN)) + D^2 N(-T + 2d(4 + N) + \beta(4 + N(-T + 2dN)))\delta)}
$$

where $T = (2 + N)$, $D = (\beta - 1)$ and $V = (1 + N)$. Finally, note that $\frac{\partial \bar{\theta}(d)}{\partial d} < 0$ for all admissible parameter values. ■

### References


